Modulated Coded Zero-Forcing Decision Feedback Equalizer

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Abstract

Modulated codes (MC) are error correction codes (ECC) over the real/complex field, which are used for mitigating the intersymbol interference (ISI). The arithmetic operations of the MC encoder are the same as the ones of an ISI channel, therefore, an ISI channel and an MC can be naturally combined together. With the combination, the MC can be optimally designed for the mitigation of the ISI. In this paper, we propose a new coded zero-forcing decision feedback equalizer (ZF-DFE) using MC. For the MC coded ZF-DFE we present the performance analysis and the optimal MC design for a given ISI channel, where the coding gain based on the joint ZF-DFE is formulated comparing to the uncoded AWGN channel. Our numerical results show that the MC coded ZF-DFE outperforms the conventional uncoded/coded ZF-DFE and the uncoded/coded Tomlinson-Harashima (TH) precoding at, at least, low channel SNR, where the BPSK is used.

1 Introduction

Decision feedback equalizers (DFE) have been used in various communication systems including modem design, see for example [1, 2, 3, 7]. The DFE, however, has the error propagation problem, in particular for spectral null channels. Recently, error correction codes (ECC) have been combined with the DFE, see for example [8] using high rate block codes. As pointed out in [8] the error propagation of the DFE makes it hard to be combined with an ECC. Trellis coded modulations (TCM) have also been studied for the intersymbol interference (ISI) channels, see for example [3, 4]. For a given ISI channel, it is usually not an easy task to design a good TCM for mitigating the ISI. The main reason behind the difficulties for the current ECC or TCM assisted ISI mitigation methods is that the arithmetic operations of the ECC encoding and the ISI channels are over different fields with one over finite fields and the other over the real/complex field. Based on this observation, ECC over the real/complex field, called *modulated codes* (MC), have been recently studied in [9, 10, 11] for mitigating the ISI. Using MC, a spectral null channel can be converted into an invertible multi-input and multi-output (MIMO) system [9]. It has been proved in [10] that an MC does not have any coding gain over the AWGN channel and it may, however, have coding gain over the ISI channel. It has been proved in [9] that for any ISI channel with two or more taps, there always exist MC such that a coding gain can be achieved. Furthermore, the joint maximum likelihood sequence equalizers (MLSE) have been studied in [11] with a superior performance over the conventional coded modulation schemes for spectral null channels.

In this paper, MC coded zero-forcing decision feedback equalizer (ZF-DFE) is proposed. We present the performance analysis for the MC coded ZF-DFE. Based the performance analysis, we present the optimal MC design for the MC coded ZF-DFE given an ISI channel or its statistics. The coding gain is also formulated over the uncoded system in the AWGN channel. Simulation results indicate that the MC coded ZF-DFE outperforms significantly over the conventional uncoded/coded ZF-DFE and the uncoded/coded TH precoding for both spectral null and none spectral null channels. The computational complexity of the MC coded ZF-DFE is similar to the one of the uncoded ZF-DFE. MMSE based optimal design of the joint transmitter and receiver using nonmaximally decimated multirate filterbanks as precoders, which are basically the MC, has been recently studied, see for example [12]. Some related works can be found in [13].

This paper is organized as follows. In Section 2, we describe MC and their combinations with ISI channels. In Section 3, we describe the MC coded ZF-DFE, present the performance analysis and the optimal MC design for the MC coded ZF-DFE. In Section 4, we present simulation and theoretical results.

2 Modulated Code Description

For the completeness, in this section we describe MC and some of their properties obtained in the previous papers [9, 10, 11].

In what follows, we use boldfaced capital English letters, $\mathbf{X}(z)$, $\mathbf{Y}(z)$, ..., to denote polynomial matrices/vectors or z transforms, captial English letters, $X(n)$, $Y(n)$,..., to denote constant matrices and vectors that are formed from the scalar sequences $x(n)$,y(n),..., after the serial to parallel conversion unless specified otherwise, and small case English letters to denote scalar values, $x(n)$, $y(n)$, ... Since we deal with error correction codes defined over the complex field, instead of using D we use z^{-1} as

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the delay variable. The D transforms become the z transforms.

A rate K/N *modulated code* (MC) is an N by K polynomial matrix

$$\mathbf{G}(z) = \begin{bmatrix} g_{11}(z) & \cdots & g_{1K}(z) \\ \vdots & \vdots & \vdots \\ g_{N1}(z) & \cdots & g_{NK}(z) \end{bmatrix} = \sum_{l=0}^{Q_G} G(l)z^{-l}, \quad (2.1)$$

where $g_{nk}(z)$ is a polynomial of z^{-1} with complex-valued coefficients, $G(l)$ is an $N \times K$ constant matrix with complex entries, and Q_G is a nonnegative integer. The constraint length ν of a K/N MC is defined the same as the conventional convolutional codes. Let $s(n)$ be a binary information sequence and $x(n)$ be the complex symbol sequence after the binary-to-complex symbol mapping of $s(n)$. Let $X(n)$ be the K by 1 vector sequence of $x(n)$ after the serial to parallel conversion. Their z transforms are $\mathbf{x}(z)$ and $\mathbf{X}(z)$, respectively. Then the encoding of an MC is

$$\mathbf{Y}(z) = \mathbf{G}(z)\mathbf{X}(z), \quad (2.2)$$

where $\mathbf{Y}(z)$ is the z transform of the encoded N by 1 vector sequence.

Since, in the encoding of an MC, the coded signal mean power may be different from the information signal mean power. For convenience, an MC is normalized such that the mean power of the encoded signal $y(n)$ is the same as the one of the information sequence $x(n)$. This can be achieved by normalizing the magnitude squared sum of all the coefficients of all the polynomials $g_{n1}(z)$, $g_{n2}(z)$,..., $g_{nK}(z)$ in $\mathbf{G}(z)$ as follows. Let

$$g_{nk}(z) = \sum_{l} g_{nk}(l)z^{-l}, \quad 1 \le n \le N, 1 \le k \le K. \quad (2.3)$$

Then,

$$\sum_{n=1}^{N}\sum_{k=1}^{K}\sum_{l} |g_{nk}(l)|^2 = N. \quad (2.4)$$

If an MC $\mathbf{G}(z)$ satisfies (2.4), it is called a *normalized* MC. For an MC $\mathbf{G}(z)$, its *free distance* can be defined as the minimum Euclidean distance between all two different encoded sequences $y_1(n)$ and $y_2(n)$ in (2.2). Comparing to an uncoded system in AWGN channel, the *coding gain* of a normalized rate K/N MC with its free distance d_{free}^2 in AWGN channel is

$$\gamma = \frac{d_{free}^2 K}{N d_{min}^2}, \quad (2.5)$$

where d_{min}^2 is the minimum distance between the complex symbols of the information sequence $x(n)$. Notice that the above coding gain is defined based on the maximum-likelihood decoding, i.e., the Viterbi decoding. Later, we will have another coding gain definition based on the ZF-DFE decoding. The following result was obtained in [11].

Theorem 1 *A modulated code does not have any coding gain in an AWGN channel, i.e.,* $\gamma \le 1$ *in (2.5).*

In the following, we consider an ISI channel. Without loss of generality, in what follows we only consider normalized MC. Let $H(z)$ be the z transform of an ISI channel with finite taps $h(n)$, $0 \le n \le \Gamma - 1$, for $\Gamma > 1$ and $h(0) \ne 0$ and $h(\Gamma - 1) \ne 0$. For convenience, the ISI channel $h(n)$ is normalized to have a unit energy:

$$\sum_{n=0}^{\Gamma-1} |h(n)|^2 = 1. \quad (2.6)$$

Let $\mathbf{G}(z)$ be a normalized rate K/N MC. Then, the combination of the MC $\mathbf{G}(z)$ and the ISI channel $H(z)$ becomes another MC $\mathbf{C}(z)$:

$$\mathbf{C}(z) = \mathbf{H}(z)\mathbf{G}(z), \quad (2.7)$$

where $\mathbf{H}(z)$ is the following N by N pseudo-circulant polynomial matrix (see [14]):

$$\mathbf{H}(z) = \begin{bmatrix} h_0(z) & z^{-1}h_{N-1}(z) & \cdots & z^{-1}h_1(z) \\ h_1(z) & h_0(z) & \cdots & z^{-1}h_2(z) \\ \vdots & \vdots & \vdots & \vdots \\ h_{N-2}(z) & h_{N-3}(z) & \cdots & z^{-1}h_{N-1}(z) \\ h_{N-1}(z) & h_{N-2}(z) & \cdots & h_0(z) \end{bmatrix}, \quad (2.8)$$

where $h_n(z)$ is the nth polyphase component of $H(z)$

$$h_n(z) = \sum_{l} h(Nl+n)z^{-l}, \quad 0 \le n \le N-1.$$

Let $Q(n)$ be the received N by 1 vector sequence with its z transform $\mathbf{Q}(z)$. Then,

$$\mathbf{Q}(z) = \mathbf{C}(z)\mathbf{X}(z) = \mathbf{H}(z)\mathbf{G}(z)\mathbf{X}(z). \quad (2.9)$$

An important consequence of the above combined MC is its invertibility even when the ISI channel $H(z)$ is spectral null, see [9].

Although the encoding at the transmitter is based on the normalized MC $\mathbf{G}(z)$, the decoding is based on the combined MC $\mathbf{C}(z)$, where $\mathbf{G}(z)$ is over the ISI channel while $\mathbf{C}(z)$ is over the AWGN channel. *The coding gain* γ_{ISI} *of the MC* $\mathbf{G}(z)$ *at the transmitter over the ISI channel is defined as the coding gain of the combined MC* $\mathbf{C}(z)$ *over the AWGN channel*, i.e.,

$$\gamma_{ISI} = \frac{d_{free,\mathbf{C}}^2}{d_{min}^2}\frac{K}{N}, \quad (2.10)$$

where $d_{free,\mathbf{C}}$ is the free distance of the combined MC $\mathbf{C}(z)$. When the ISI channel is fixed, the MLSE performance of the uncoded ISI channel is always not as good as the one of the uncoded AWGN channel. This implies that the above coding gain γ_{ISI} is a portion of the real coding gain compared to the uncoded ISI channel, i.e., the real coding gain is the sum of γ_{ISI} (dB) and the difference (dB) between the uncoded ISI and AWGN channels.

By the normalization, the mean power of the transmitted signal $y(n)$ is the same as the one of the information signal $x(n)$. A very important observation is that the mean

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power of the received signal $q(n)$ may be different. It turns out that, by properly choosing an MC $\mathbf{G}(z)$, the mean power of the received signal $q(n)$ after the ISI channel may be greater than the information signal mean power. This is the key for the existence of a coding gain for an MC in an ISI channel, which is stated as follows.

Theorem 2 *For any ISI channel $h(n)$, $0 \le n \le \Gamma - 1$, with $\Gamma > 1$ and $h(0) \neq 0$, $h(\Gamma - 1) \neq 0$, there exists a normalized modulated code $\mathbf{G}(z)$ such that the combined modulated code $\mathbf{C}(z)$ in (2.7) has a coding gain, i.e., $\gamma > 1$ in (2.5), over the uncoded system in the AWGN channel.*

This result was obtained in [10].

3 MC Coded Zero-Forcing Decision Feedback Equalizer

In this section, we propose the MC coded ZF-DFE and the performance analysis. We then present the optimal MC design especially for the MC coded ZF-DFE.

3.1 MC Coded ZF-DFE and Performance Analysis

The block diagram for the MC coded ZF-DFE is shown in Fig. 1, where I_K is the K by K identity matrix, and the K by 1 vector decision takes the best K by 1 vector of all the possible K by 1 information symbol vectors, and η is the channel additive white Gaussian noise with zero mean and the variance $\sigma_\eta^2 = N_0/2$.

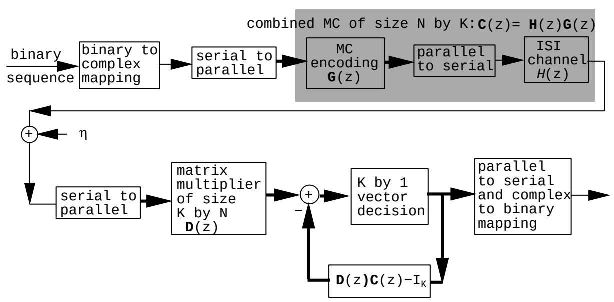

Figure 1: Modulated-coded zero-forcing decision feedback equalizer.

The role of the matrix multiplier $\mathbf{D}(z)$ at the receiver in Fig. 1 for the MC coded ZF-DFE is to convert the nonsquare matrix polynomial $\mathbf{C}(z)$ of the combined MC into a square matrix polynomial so that the DFE can be implemented as shown in Fig. 1. It is usually the case that the higher of the order of the ISI channel to equalize is, the worse of the DFE performance is. To make the order of the overall ISI system $\mathbf{F}(z)$ after the matrix multiplier as low as possible, where

$$\mathbf{F}(z) \triangleq \mathbf{D}(z)\mathbf{C}(z) = \mathbf{D}(z)\mathbf{H}(z)\mathbf{G}(z), \qquad (3.1)$$

and $\mathbf{H}(z)$ is from (2.8), the matrix multiplier $\mathbf{D}(z)$ simply takes a K by N constant matrix. It also suggests that the MC $\mathbf{G}(z)$ takes a block code, i.e., $\mathbf{G}(z)$ is an N by K constant matrix. We next want to study the MC design rule for the ZF-DFE. Consider an N by K block MC $\mathbf{G}(z) = G$ and a constant K by N matrix multiplier $\mathbf{D}(z) = E$. The combined MC becomes

$$\mathbf{C}(z) = H(0)G + H(1)Gz^{-1} + \cdots + H(P)Gz^{-P}, \quad (3.2)$$

where $\mathbf{H}(z) = \sum_{p=0}^{P} H(p)z^{-p}$ for some $P \ge 0$ and

$$H(0) = \begin{bmatrix} h(0) & 0 & \cdots & 0 \\ h(1) & h(0) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ h(N-1) & h(N-2) & \cdots & h(0) \end{bmatrix}, \quad (3.3)$$

which is nonsingular when $h(0) \neq 0$. From the feedback loop in the ZF-DFE in Fig. 1, we want to have $EH(0)G = I_K$, i.e., the feedback does not depend on the current vector. Therefore, the matrix multiplier $\mathbf{D}(z) = E$ is the right inverse (pseudo inverse), $(H(0)G)^+$, of the N by K constant matrix $H(0)G$.

Since the matrix multiplier $\mathbf{D}(z) = E$ is implemented at the receiver, the channel additive noise η is also multiplied by the matrix E. Let $E = (e_{ij})_{K \times N}$. Then the mean power of the multiplied noise $\tilde{\eta}$ of η is

$$\sigma_{\tilde{\eta}}^2 = \frac{\sum_{i=1}^{K}\sum_{j=1}^{N}|e_{ij}|^2}{K}\sigma_\eta^2 = \frac{\sum_{i=1}^{K}\sum_{j=1}^{N}|e_{ij}|^2}{2K}N_0. \quad (3.4)$$

By the normalization condition of the MC G, the mean transmitted signal power is still σ_x^2. Similar to the conventional ZF-DFE for invertible ISI channel, see, for example, [7], the signal-to-noise ratio (SNR) after the MC coded ZF-DFE for the invertible $\mathbf{C}(z)$ is

$$\text{SNR} = \frac{\sigma_x^2}{\sigma_{\tilde{\eta}}^2} = \frac{2K\sigma_x^2}{\sum_{i=1}^{K}\sum_{j=1}^{N}|e_{ij}|^2 N_0}. \quad (3.5)$$

Based on this SNR analysis at the receiver, to maximize the SNR we have the following **optimal MC design rule**:

$$\min_G \sum_{i=1}^{K}\sum_{j=1}^{N}|e_{ij}|^2 \ \text{ under the condition } \ EH(0)G = I_K, \quad (3.6)$$

where the MC G satisfies the normalization condition

$$\sum_{i=1}^{N}\sum_{j=1}^{K}|g_{ij}|^2 = N. \quad (3.7)$$

Let the singular value decomposition of the matrix $H(0)G$ be

$$U_l V U_r = H(0)G, \quad (3.8)$$

where U_l and U_r are $N \times N$ and $K \times K$ unitary matrices, respectively, and

$$V = \begin{pmatrix} \mathrm{diag}(\lambda_1, \cdots, \lambda_K) \\ 0_{(N-K)\times K} \end{pmatrix}, \quad (3.9)$$

and λ_i for $i = 1, 2, ..., K$ are the singular values of the matrix $H(0)G$. Then the matrix multiplier E should be

$$\mathbf{D}(z) = E = U_r^\dagger V^{-1} U_l^\dagger, \tag{3.10}$$

where † denotes the conjugate transpose and

$$V^{-1} = (\mathrm{diag}(1/\lambda_1, \cdots, 1/\lambda_K), 0_{K\times(N-K)}). \tag{3.11}$$

Thus, the total energy of the matrix E is

$$\sum_{i=1}^{K}\sum_{j=1}^{N} |e_{ij}|^2 = \sum_{i=1}^{K} \frac{1}{\lambda_i^2}. \tag{3.12}$$

Therefore, using the elementary inequality on the right hand side of (3.12) we have

$$\sum_{i=1}^{K}\sum_{j=1}^{N} |e_{ij}|^2 \geq K \left(\prod_{i=1}^{K} \frac{1}{\lambda_i^2} \right)^{1/K}, \tag{3.13}$$

where the equality (the minimum) is reached if and only if

$$\lambda_1 = \lambda_2 = \cdots = \lambda_K = \lambda. \tag{3.14}$$

The optimality condition (3.14) is the one to design the MC G that whitens the matrix $H(0)$ generated from the ISI channel. In the next subsection, we propose a method to design such MC G given an $H(0)$.

We now study the error probability for the MC coded ZF-DFE in Fig. 1. Let us consider the vector decision block in Fig. 1. For a general MC G at the transmitter and the matrix multiplier E with the form in (3.10), each $K \times 1$ multiplied noise vector $\tilde{\eta}$ for a fixed time may be colored when $K > 1$. In this case, the vector decision is necessary for the optimal detection. If the MC G whitens $H(0)$, i.e., the condition (3.14) holds, then it is not hard to see that each $K \times 1$ multiplied noise vector $\tilde{\eta}$ for a fixed time is white too. Thus, the vector decision in Fig. 1 can be reduced to the symbol-by-symbol detection as shown in Fig. 2.

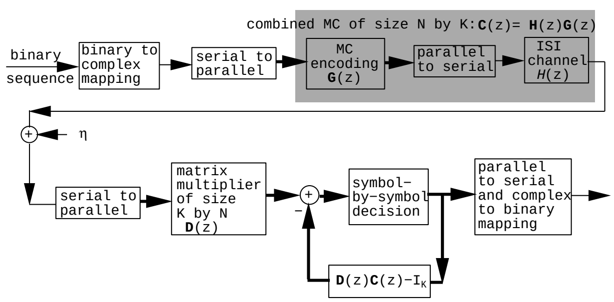

Figure 2: Modulated-coded zero-forcing decision feedback equalizer with optimal MC.

Assume that the condition (3.14) for the MC encoding holds, which is always possible to design as we shall see later. In this case,

$$\sum_{i=1}^{N}\sum_{j=1}^{K} |e_{ij}|^2 = \frac{K}{\lambda^2}.$$

Let $P_s(\gamma_s)$ denote the symbol error probability at the symbol SNR γ_s for the binary-to-complex symbol mapping used at the transmitter in Fig. 1. For convenience, in what follows we only consider the BPSK binary-to-complex symbol mapping. In this case, the symbol error probability is $P_s(\gamma_s) = Q(\sqrt{2\gamma_s})$, where γ_s is the SNR before the decision block in Fig. 1. Using the SNR (3.5), the corresponding γ_s is

$$\gamma_s = \frac{\sigma_x^2}{2\sigma_{\tilde{\eta}}^2} = \frac{K\sigma_x^2}{\sum_{i=1}^{K}\sum_{j=1}^{N}|e_{ij}|^2 N_0} = \frac{\lambda^2\sigma_x^2}{N_0} = \frac{\lambda^2 K}{N}\frac{E_b}{N_0}. \tag{3.15}$$

Then, the bit error rate (BER) for the MC coded ZF-DFE at the E_b/N_0 is

$$\mathrm{BER} = P_s(\gamma_s) = Q(\sqrt{2\frac{E_b}{N_0}\gamma}), \tag{3.16}$$

where γ is the **coding gain** over the uncoded BPSK in AWGN channel as follows:

$$\gamma = \frac{\lambda^2 K}{N}, \tag{3.17}$$

where λ is defined in (3.14).

3.2 The Optimal MC Design

In this subsection, we present the optimal MC design such that the optimality condition (3.14) is satisfied.

Let the singular value decomposition of the $N \times N$ matrix $H(0)$ defined in (3.3) as

$$H(0) = W_l \Lambda W_r, \tag{3.18}$$

where W_l and W_r are two $N \times N$ unitary matrices and

$$\Lambda = \mathrm{diag}(\xi_1, \cdots, \xi_N), \tag{3.19}$$

where $\xi_1 \geq \cdots \geq \xi_N > 0$ are the N singular values of $H(0)$. Then, the optimal normalized MC G is

$$G_{opt} = W_r^\dagger \bar{G} U_r = W_r^\dagger \Lambda^{-1} U \begin{pmatrix} \lambda I_K \\ 0_{(N-K)\times K} \end{pmatrix} U_r, \tag{3.20}$$

where U_r is an arbitrary $K \times K$ unitary matrix, U =diag(U_1, U_2), and U_1 and U_2 are two arbitrary $K \times K$ and $(N-K)\times(N-K)$ unitary matrices, respectively, W_r is the $N \times N$ unitary matrix defined in (3.18), Λ is the diagonal matrix defined in (3.19), and $\lambda = N/(\sum_{i=1}^{K}\xi_i^{-2})$. The optimal coding gain formula in (3.17) for the BPSK signaling, we have the following **optimal coding gain** using the optimal rate K/N MC G_{opt} in (3.20) for a given channel:

$$\gamma_{opt} = \frac{K}{\sum_{i=1}^{K}\xi_i^{-2}}, \tag{3.21}$$

where ξ_i, $i = 1, 2, ..., K$, are the first K largest singular values of $H(0)$.

4 Simulation Results

In this section, we want to present some simulation results to illustrate the theory for the optimal MC design for the MC coded ZF-DFE developed in the previous sections.

Consider a simple channel Channel A: $[1/\sqrt{2}, 1/\sqrt{2}]$. We compare four equalization techniques, namely (i) conventional convolutionally coded and uncoded ZF-DFE; (ii) conventional convolutionally coded and uncoded TH precoding; (iii) MC coded ZF-DFE; (iv) MC coded joint MLSE. Theoretical BER vs. E_b/N_0 curves for BPSK in AWGN channel and the MC coded ZF-DFE with BPSK signaling are also compared with the simulation results. In all the following optimal normalized MC G_{opt} in (3.20), the unitary matrices U and U_r are set to the identity matrices. In the following conventional convolutionally coded ZF-DFE and the TH precoding methods, the rate 1/2 and constraint length 2 with the optimal $d_{free} = 5$ convolutional code $(5, 7)$ is used. The optimal MC in (3.20) is

$$G_{opt} = \begin{pmatrix} 1.2030 \\ 0.7435 \end{pmatrix}. \qquad (4.1)$$

The largest singular value of $H(0)$ is $\xi_1 = 1.1441$ and the optimal coding gain in (3.21) for the MC coded ZF-DFE is $\gamma_{opt} = 1.17$dB. For the MC in (4.1), the squared free Euclidean distance of the combined MC with the ISI Channel A is $d_{free}^2 = 11.58$. Thus, the coding gain of the joint MLSE method is $\gamma_{MLSE} = 1.6$dB.

In Fig. 3, the BERs vs. E_b/N_0 for the conventional uncoded ZF-DFE are plotted with the solid line marked by o; the BERs vs. E_b/N_0 for the convolutionally coded ZF-DFE are plotted with the solid line marked by □; the BERs vs. E_b/N_0 for the uncoded TH precoding are plotted with the solid line marked by Δ; the BERs vs. E_b/N_0 for the convolutionally coded TH precoding are plotted with the solid line marked by $*$; the BERs vs. E_b/N_0 for the MC coded ZF-DFE with the above optimal MC code in (4.1) are plotted by the solid line. The theoretical BERs vs. E_b/N_0 for uncoded BPSK in the AWGN channel are plotted with the dashed line. The BERs vs. E_b/N_0 of the joint MLSE for the MC in (4.1) and Channel A are plotted with the solid line marked by $+$.

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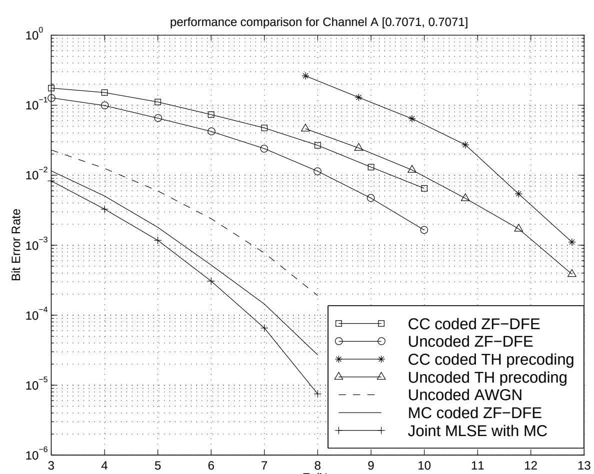

Figure 3: Performance comparison for different equalization methods: Channel A and MC code rate 1/2.