Performance analysis of integrated space-time receiver with combined full-and reduced-complexity for DS/CDMA systems

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Abstract - An integrated space-time receiver is proposed to suppress co-channel interference and increase the capacity for DS/CDMA systems. The receiver employs an array of antennas, which allows it to exploit the spatial domain information in the received signals, and at the same time, each antenna incorporates an adaptive filter with varying complexity and operated in a manner similar to an adaptive equaliser, to further suppress interference in the temporal domain. Numerical results are presented using MMSE criterion, showing that the receiver offers significant performance improvement relative to corresponding time-domain-only counterpart in terms of near-far resistance and capacity in multipath channels. A comparison in MSE between simulation results using the RLS algorithm and analytical results using optimum Wiener solution is also presented.

I. INTRODUCTION

CDMA has received considerable interest in recent years. Much of this work has addressed the near-far problem in which strong interfering signals can overwhelm a weak desired signal in the detection process. The conventional matched-filter receiver is susceptible to the near-far effect, and its output corresponding to the desired user has strong interference terms. This is mainly because of nonzero cross-correlation between the set of signature waveforms. The minimum mean-squared error (MMSE) receivers (1),(2) are robust to the near-far problem and only require code timing on the one desired signal. In general, the path signals arrive at the receiver not only with different delays, but also from different directions of arrival in space. The spatial characteristics of the multipath channel can not be exploited with single-antenna MMSE receivers.

The use of antenna array has been shown to increase system capacity (3)-(7), and to improve the performance and capability of the MMSE receivers, two adaptive cascaded space-time MMSE receivers are presented in (8) by combining an adaptive antenna array with the existing time-domain MMSE receivers. The adaptive antenna array adaptively updates its weights by using the desired spreading code as a reference for beamforming to suppress interfering signals with DOA’s different from that of a desired signal, and the adaptive filter in the time domain eliminates any residual interference and high-level interference having the same DOA as that of the desired signal, which the adaptive antenna array cannot suppress. These cascaded space-time equalisers with feedback in DS/CDMA have been shown to significantly improve the performance over their time-domain-only counterparts. Intuitively, however, one would expect that these cascaded architectures are not optimal, because the spatial and temporal adaptive equalisations are performed sequentially instead of concurrently. As spatial equalisation by itself is not effective against delay-spread problems, the adaptive spatial equalisation stage may seriously affect the desired signal in an attempt to minimise unwanted delayed interference signals in or near the direction of the desired signal. The end effect is that, even though these interference signals may be finally eliminated by the time-domain equalisation stage, the output signal-to-noise ratio of the cascaded space-time equaliser is far from being optimal.

In this paper, an integrated hybrid space-time receiver is proposed (9) to improve the performance as well as capacity of the time-domain-only MMSE receiver. The proposed receiver is near-far resistant in the sense that the performance without power control is almost identical to the performance with perfect power control. The receiver can be implemented when interference parameters as well as the direction of arrivals from desired user and interference are unknown. With integrated space-time architecture, where the adaptation is carried out concurrently in both the spatial and temporal domains, these delayed interference signals will be suppressed by the temporal equalisation without affecting the desired signal, and at the same time, interference signals coming from other directions will be eliminated or minimised by the spatial equalisation. As a result, the output signal-to-noise ratio of the integrated space-time equalisers will be much stronger. The proposed receiver is shown to outperform the corresponding time-domain-only counterpart, in terms of reduced mean-squared error (MSE), near-far resistance and capacity.

The remainder of the paper is organised as follows. Section II gives a description of the proposed adaptive integrated space-time receiver. Simulation and analytical results are described in Section III with a comparison between the existing single-antenna MMSE receiver and the proposed receiver. The conclusion is given in Section IV.

II. SYSTEM DESCRIPTION

A block diagram of the integrated hybrid space-time receiver in DS/CDMA is shown in Fig. 1. The receiver consists of an array of M antenna elements, and after down-conversion the signal at each antenna element is chip-matched and sampled at the chip rate. The first element is then
followed by an $N$-tap delay line (TDL), with tap delay equal to the chip period and $N$ being the period of the spreading sequence. Each of the remaining elements is followed by a bank of $V$ filters with $V < N$, which are cyclic shifts of the desired spreading sequence. Thus, it is seen that the complexity of the adaptive filtering is proportional to $M \times N$, and the complexity in each element of the array can be reduced by pre-multiplying the $N$-dimensional received vector with an $N \times V$ matrix. The length of the resulting vector is thereby reduced from $N$ to $V$. The outputs of TDL and these filters are then summed and sampled at the bit rate. During training, the error signal is formed as the difference between the soft decision and the desired user's data bit. Once the mean-squared error (MSE) is at an acceptable level, training is terminated and data transmission begins. At this time, the error signal is formed as the difference between the soft decision and the desired user’s data bit. The weights of the TDL and these filters are adapted to minimize the MSE according to an adaptive algorithm.

The received signal due to the $k$th user at the $m$th element of the array is expressed as

$$ r_{m,k}(t) = \sqrt{2L} \sum_{i=1}^{\infty} \sqrt{P_{k,i}} \sum_{j=-\infty}^{\infty} b_{i}(i) s_{k}(t - iT - \tau_{k,i}) \times \cos(\omega_{m}t + \theta_{k,i}) \exp(j(m-1) \frac{2\pi d \sin \phi_{k,i}}{\lambda}), \quad k=1,2,...,K $$

(1)

where $L$ is the number of paths per user, and $b_{i}(i)$ is the $i$th bit (±1) of the $k$th user; $\theta_{k,i}$ and $P_{k,i}$ are the carrier phase and received power of the $i$th path of the $k$th user, respectively; $T$ is the bit period, and $\omega_{m}$ is the carrier frequency. The delay of the $i$th path of the $k$th user, defined as $\tau_{i,k} = p_{k,i} T_{c} + \delta_{i,k}$, where $p_{k,i}$ is an integer, $T_{c}$ is the chip period and $0 \leq \delta_{i,k} \leq T_{c}$; $d$, $\lambda$, and $\phi_{k,i}$ are the element spacing, free-space wavelength, and direction of arrival (DOA) of the $i$th path of $k$th user, respectively. The spreading waveform $s_{k}(t)$ of the $k$th user, assumed to be periodic with period $T = N T_{c}$, is given by

$$ s_{k}(t) = \sum_{j=0}^{N-1} a_{k}(j) \Psi(t - jT_{c}). $$

(2)

The chip waveform $\Psi(t)$ is assumed to be a rectangular pulse with unit height and duration $T_{c}$. The spreading code vector $a_{k}$ of the $k$th user is defined as

$$ a_{k} = [a_{k}(0), a_{k}(1),...,a_{k}(N-1)]^{T}. $$

(3)

The assumption that the length of the spreading waveforms is equal to the length of data bit is not necessary for a DS/CDMA system, but it is used to simplify the analysis. The received signal at the $m$th element is the sum of $K$ simultaneous CDMA transmissions plus the additive white Gaussian noise (AWGN)

$$ r_{m}(t) = \sum_{k=1}^{K} r_{m,k}(t) + n_{m}(t), \quad m = 1,2,...,M. $$

(4)

It is also assumed that the receiver is synchronised to the desired signal ($k = 1$) with strongest path ($l = 1$), and the carrier phase $\theta_{1,1}$ of the desired signal is perfectly tracked. The $m$th sample taken at the chip rate at the output of the chip matched filter at the $m$th antenna element of the array is given by

$$ r_{m}(n) = \sqrt{2} \int_{nt_{c} + \tau_{1,1}}^{(n+1)t_{c} + \tau_{1,1}} r_{m}(t) \Psi(t) \cos(\omega_{m}t + \theta_{1,1}) dt. $$

(5)

The total received sample signal vector of the $i$th bit interval at the $m$th antenna element with $m = 1,2,...,M$ is represented by

$$ r^{(i)}_{m} = [r_{m}(iN), r_{m}(iN+1),...,r_{m}(iN+N-1)]^{T}. $$

(6)

Having obtained a sampled data model, the demodulation of the $i$th bit $b_{i}(i)$ of the 1st user is now considered, in which the total received signal vector $r^{(i)}_{m}$ at the $m$th antenna element can be expressed in vector form as

$$ r^{(i)}_{m} = \sum_{k=1}^{K} \sum_{i=1}^{L} \sqrt{P_{k,i}} \cos(\theta_{k,i}) [b_{k}(0)a_{k}^{(i)} + b_{k}(-1)a_{k}^{(i-1)}] \times \exp(j(m-1) \frac{2\pi d \sin \phi_{k,i}}{\lambda}) + n^{(i)}_{m}, $$

(7)

where

$$ a_{k}^{(i)} = \frac{\delta_{k,l}}{T_{c}} f_{k}(N - p_{k,l} - 1) + (1 - \frac{\delta_{k,l}}{T_{c}}) f_{k}(N - p_{k,l}), $$

(8a)

$$ a_{k}^{(i-1)} = \frac{\delta_{k,l}}{T_{c}} g_{k}(N - p_{k,l} - 1) + (1 - \frac{\delta_{k,l}}{T_{c}}) g_{k}(N - p_{k,l}), $$

(8b)

$$ f_{k}(e) = (0,0,...,a_{k}(0),...,a_{k}(e-1))^{T}, $$

(8c)

$$ g_{k}(e) = (a_{k}(e),a_{k}(e+1),...,a_{k}(N-1),0,0,...,0)^{T}. $$

(8d)

where $a_{k}^{(i)}$, $a_{k}^{(i-1)}$, $f_{k}(e)$, $g_{k}(e)$ are $N$-dimensional vectors, $b_{k}(i)$ is the current data bit, and $b_{k}(i-1)$ is the previous data bit. The vector $n^{(i)}_{m}$ is a zero-mean Gaussian random vector uncorrelated in both time and space with variance $\sigma^{2}$, where $\sigma^{2} = (2E_{b}/N_{0}N)^{-1}$ with $E_{b}/N_{0}$ representing the data-bit energy to one-sided noise power spectral density. It is assumed that $\theta_{1,1} = 0$ and $\tau_{1,1} = 0$, i.e., $p_{k,l} = 0$, $\delta_{1,1} = 0$, and therefore $a_{k}^{(i)}$ is a zero vector. For asynchronous system, each $r^{(i)}_{m}$ will contain part of the current bit and part of the previous bit of each path for every user.
where $r(t)$ is sampled at the bit rate, and $\alpha$ is the forgetting factor, and $\alpha = \beta \leq 1$.

Successive shifts are spaced by $\Delta = N/V$. The filter outputs are sampled at the bit rate, and combined by means of $V$ taps. Defining

$$h_l(n) = a_l [(n + l\Delta) \mod N], \quad 0 \leq n \leq N - 1,$$

the input and weight vectors are expressed as

$$\mathbf{z}(i) = \{z_1(i), z_2(i), \ldots, z_{N-1}(i), z_{N}(i)\},$$

$$\mathbf{w}(i) = \{w_1(i), w_2(i), \ldots, w_{N-1}(i), w_{N}(i)\}.$$  

The optimum weight vector $\mathbf{w}_{\text{opt}}$ from the Wiener-Hopf solution is given by

$$\mathbf{w}_{\text{opt}} = \mathbf{R}^{-1} \mathbf{q},$$

where $\mathbf{R}$ is the auto-correlation matrix of the input vector,

$$\mathbf{R} = \mathbf{E}[\mathbf{z}(i)\mathbf{z}(i)^H].$$

and $q$ is the correlation between the desired response and input vector,

$$q = \mathbf{E}[b_1(i)z(i)].$$

The MMSE $J_{\text{min}}$ produced by optimum Wiener filter associated with $\mathbf{w}_{\text{opt}}$ is defined [10] as

$$J_{\text{min}} = 1 - q^H \mathbf{w}_{\text{opt}} = 1 - q^H \mathbf{R}^{-1} \mathbf{q}.$$  

In practice, since the DOA's of the desired signal, interference, and the signal power are unknown, the computation of the optimum weight vector $\mathbf{w}_{\text{opt}}$ is not possible, and an adaptive algorithm is used to approximate this solution. Using the Recursive Least-Squares (RLS) algorithm to find the updated solution for the weight vector $\mathbf{w}(i)$, the iterative relations [10] are

$$u(i) = \frac{\alpha^{-1}P(i-1)z(i)}{1 + \alpha^{-1}z^H(i)P(i-1)z(i)},$$

$$\mathbf{w}(i) = \mathbf{w}(i-1) + u(i)\tilde{\xi}^H(i),$$

$$P(i) = \alpha^{-1}P(i-1) - \alpha^{-1}u(i)z^H(i)P(i-1),$$

where $\alpha$ is the forgetting factor, and $P(0) = \mathbf{I}$, the identity matrix. The weight vector $\mathbf{w}(i)$ is updated at the bit rate, and is chosen to minimise the mean-squared error $\text{MSE} = \mathbf{E}[\tilde{\xi}(i)^2]$.

The outputs of all equalisers are then summed and hard-limited to form an estimate of the transmitted data bits.

### III. Numerical Results

The code sequences are chosen arbitrarily from a Gold set of length $N = 31$. The element spacing is one-half wavelength, and the desired signal is received at an $E_b/N_0$ of 20dB. The number of filters $V$ in each branch with $m \times 1$ of the array is set to 1, 3, and 4. The number of antenna elements is 3, and the multipath channel has 3 paths for each user. Path delays are uniformly distributed within 8 chips, and the results are averaged over 100 runs. There are one desired and four interference signals, each with strongest path’s DOA of $35^\circ$, $45^\circ$, $10^\circ$, $0^\circ$, and $60^\circ$ with respect to broadside, respectively. The remaining two paths for each user have DOA’s uniformly distributed in $[-90^\circ, 90^\circ]$. The interference-to-desired signal power ratio is set to 10dB for strongest paths, and is uniformly distributed for the remaining paths.

The analytical results correspond to the situation where the receiver has perfect knowledge of the auto-correlation matrix of the tap-input vector and the cross-correlation between the

Fig. 1. Integrated hybrid space-time receiver.
desired signal and tap-input vector, and hence the optimal solution to the Wiener-Hopf equations for the tap-weight vector. Fig. 2 illustrates that, compared with the full-length, time-domain-only counterpart \((M=1)\), better MSE convergence is obtained for the proposed integrated hybrid architecture. In addition, increasing the number of filters \(V\) in each branch with \(m=1\) will further reduce the MSE, hence provides higher system capacity. The results also show that the MSE produced by the RLS algorithm converges to the MMSE \(J_{\min}\) attained by the Wiener solution. In Fig. 3, the MSE from the RLS algorithm is attained in about 100 iterations, and both simulation and analytical results show that the proposed architecture is near-far resistant with no degradation in MSE if power control is neglected. A comparison between the analytical results using (16) and simulation results using RLS shows a good agreement. The analytical results in Figs. 4 and 5 indicate that MSE vs \(E_b/N_o\) and number of users can be further reduced by increasing the processing gain \(N\), and the number of elements \(M\) in the array, respectively.

This structure takes advantage of integrated space-time processing and combined full- and reduced-length adaptive filters, and has the flexibility of increasing the number of filters in the array elements to further reduce the MSE and to increase the system capacity. The results also illustrate that not only has the MSE been significantly reduced, but, more efficiently, the improvement in near-far resistance can also be achieved, thus further increasing the system performance and capacity.

![Fig. 2. MSE convergence and MMSE \(J_{\min}\).]

**Time-domain:** (i) learning curve, (ii) MMSE \(J_{\min}\)

**Integrated space-time** \(M=3\):

- Learning curves (iii) \(V=1\), (iv) \(V=3\), (v) \(V=4\);
- MMSE \(J_{\min}\): (vi) \(V=1\), (vii) \(V=3\), (viii) \(V=4\).

![Fig. 3. Near-far effect.]

**Time-domain:** (i) MSE from RLS (ii) MMSE \(J_{\min}\)

**Proposed integrated hybrid space-time** \(M=3\):

- MSE from RLS (iii) \(V=1\), (iv) \(V=3\), (v) \(V=4\);
- MMSE \(J_{\min}\): (vi) \(V=1\), (vii) \(V=3\), (viii) \(V=4\).

![Fig. 4. MMSE \(J_{\min}\) vs. signal-to-noise ratio.]

**Time-domain:** (i) \(N=31\), (ii) \(N=63\)

**Integrated space-time** \(M=3\):

- \(N=31\): (iii) \(V=1\), (v) \(V=3\);
- \(N=63\): (iv) \(V=1\), (vi) \(V=3\).
Although a fully integrated space-time equaliser will have $M \times N$ adaptive weights, as each of the $M - 1$ elements with $m \neq 1$ in the array will require $N$ matched filters in the corresponding time-sector, the results show that not all $N$ matched filters need be used in each of these elements. The receivers use the discrimination afforded by the antenna array to improve the performance of the interference suppression, and effectively compensate for non-zero cross-correlations between the users’ spreading waveforms thereby avoiding the reduction in performance which occurs in the single antenna. Degrees of freedom provided by space-time processing and by varying complexity in each element of the array are exploited to combat the near-far effect.

**IV. CONCLUSIONS**

Our simulation and analytical results clearly demonstrate that, in comparison with the time-domain structure, the proposed integrated space-time architecture provides significantly better performance due to the adaptation carried out concurrently in both the spatial and temporal domains. It is found to have the near-far insensitivity, MSE reduction, and offers the flexibility of increasing the number of filters in all elements of the array to further reduce the MSE and to increase the system capacity.

**REFERENCES**