A Noncoherent Coded Modulation for 16QAM*

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Abstract

In this paper, we present a noncoherent coded 16QAM (NC-16QAM) scheme by modifying the trellis-coded 16QAM (TC-16QAM) scheme. Our simulation results show that the performance of the NC-16QAM with noncoherent detection is close to the one of the original TC-16QAM with coherent detection. The NC-16QAM in this paper is an extension of the NC-8PSK recently obtained by Wei and Lin. A noncoherent initial phase estimation algorithm is also proposed.

1 Introduction

In the conventional trellis coded modulation (TCM) schemes, the carrier phase recovery is needed before the decoding, which is usually ensured by inserting pilot signals or pilot channels. In band-limited applications, pilot signals are usually not desired. Although the differential encoding and decoding is an alternative for avoiding the carrier phase recovery problem, its noncoherent detection performance is significantly worse than the coherent detection. In order to reduce the performance difference between the noncoherent and coherent detections, Raphaeli [1] proposed noncoherent coded modulation (NCM) schemes, with which the decoding can be implemented noncoherently without the differential encoding or carrier phase recovery. Wei and Lin [2] proposed a construction of NCM for 8PSK (NC-8PSK) by modifying the conventional TCM structure.

In this paper, we extend the Wei-Lin’s construction from NC-8PSK to NC-16QAM. The concept of differential-coherently equivalent sequences is introduced to assist the noncoherent initial phase estimation without any initial known symbols in the noncoherent decoding algorithm. Our simulation results show that the NC-16QAM with noncoherent detection is close to the TC-16QAM with coherent detection.

2 Construction of NC-16QAM

Before going to the construction, let us first see a concept. For two different signal sequences (such as two codeword sequences from a TCM), if one of them can be obtained by a constant phase shift of the other, then the two signal sequences are called noncoherently equivalent. Clearly, if two codewords are noncoherently equivalent, then a noncoherent detector can not distinguish one from the other, i.e., an detection error occurs. This tells us that a TCM scheme with noncoherent detection should avoid noncoherently equivalent codewords.

We next want to present our NC-16QAM construction. In the following, similar to [2] we consider the equivalent baseband signal model:

\[ r_k = s_k e^{j\theta} + n_k, \]  

(2.1)

where \( r_k \) is a received signal, \( s_k \) is a 16QAM modula-
tion symbol, $\theta$ is an arbitrary phase error, and $n_k$ is an additive white complex Gaussian noise. The 16QAM signal constellation is shown in Fig. 1.

In [2], Wei and Lin have pointed out that a TCM with parallel branches in its trellis diagram cannot be converted into an NCM. This suggests that a TCM without parallel branches should be selected as a modification candidate of an NCM. In [4], Ungerboeck presented the optimal 16-state TC-16QAM. However, there exist parallel branches in its trellis diagram, which cannot be used to form an NCM. Alternatively, we construct a new 16-state TC-16QAM without any parallel branches as shown in Fig. 2(a). The corresponding TC-16QAM symbol mapping is labeled in Fig. 2(b). From Fig. 2, one can see that the codeword sequences $\{0, 0, \cdots\}$, $\{10, 10, \cdots\}$, $\{11, 11, \cdots\}$, and $\{1, 1, \cdots\}$ are noncoherently equivalent, and the codeword sequences $\{12, 12, \cdots\}$, $\{6, 6, \cdots\}$, $\{7, 7, \cdots\}$, and $\{13, 13, \cdots\}$ are also noncoherently equivalent. Therefore, it is not an NCM by itself. To make it an NCM, we adopt a similar modification as in [2] by Wei and Lin. The TCM output symbol mapping for 16-state TC-16QAM is

$$s_k = \begin{cases} a_k, & k = 2l \\ (16 - a_k) \mod 16, & k = 2l + 1, \end{cases}$$

(2.2)

where $a_k$ is a symbol ranging from 0 to 15 at the $k$th time unit in the trellis diagram in Fig. 2. With this symbol mapping, there is no $(p, p, \cdots)$ loops except the all 0 loop in the modified trellis diagram. It is not hard (by computer search) to see that the modified TC-16QAM is an NCM, which is called NC-16QAM. The main difference with the NC-8PSK constructed in [2] by Wei and Lin is that the symmetric property of the original trellis diagram is not kept. This implies that the distance property might change. Fortunately, the two free distances of the TC-16QAM and the NC-16QAM in this case are the same, which is 12 and the same as the one for the optimal 16 states TC-16QAM in [4].

3 A Noncoherent Decoding Algorithm

We also adopt the idea of the adaptive decoding algorithm used in [2]. The difference with the approach in [2] is that the initial phase is noncoherently estimated based on the property of the NC-16QAM itself.

3.1 Noncoherent Initial Phase Estimation

Assume that the signal is synchronized and the phase shift is a constant or nearly a constant in an observation interval. The problem here is to estimate the phase shift $\theta$ from a short segment of $r_k$, $k = 0, 1, \cdots, K$ of the received signal without any pilot symbols. Before going to the solution, we need to introduce another concept.

Let $\bar{a} = \{a_0, a_1, a_2, a_3, a_4, \cdots\}$ and $\bar{b} = \{b_0, b_1, b_2, b_3, \cdots\}$ be two different signal sequences. If sequence $\{a_0a_1^*, a_1a_2^*, a_2a_3^*, \cdots\}$ $\equiv$ sequence $\{k_0b_1^*, b_1b_2^*, b_2b_3^*, \cdots\}$, where $*$ stands for the complex conjugate, then the two signal sequences $\bar{a}$ and $\bar{b}$ are called differential-coherently equivalent. Clearly, noncoherently equivalent signal sequences are differential-coherently equivalent. However, differential-coherently equivalent sequences may not be noncoherently equivalent. By using the computer search, it can be proved that the NC-16QAM constructed in Section 2 does not have any two differential-coherently equivalent sequences up to sequence length 10,000. This property is useful in the following noncoherent initial phase estimation.

We define the following metric to first optimally detect the first $K + 1$ transmitted symbols $s_k$, $0 \leq k \leq K$:

$$M(\bar{r}, \bar{s}) = \sum_{k=0}^{K-1} |r_k r_{k+1}^* - s_k s_{k+1}^*|^2,$$

(3.3)
\[ \tilde{r} = \{ r_0, r_1, r_2, \cdots, r_K \} \]

is the first \( K + 1 \) received signal values, \( \tilde{s} = \{ s_0, s_1, s_2, \cdots, s_K \} \) is a candidate of the transmitted signal sequence in an NC-16QAM codeword. The signal \( \tilde{s} \) is selected by minimizing

\[
M(\tilde{r}, \tilde{s}_0) = \min_{\text{all codewords } \tilde{s}} \{ M(\tilde{r}, \tilde{s}) \}. \tag{3.4}
\]

Since there does not exist any differential-coherently equivalent signal sequences in the NC-16QAM, the above solution is unique when the SNR is high.

The corresponding Viterbi Algorithm for the above minimization solution is as follows. Similar to the conventional Viterbi Algorithm, the metric in (3.3) can be decomposed into

\[
M_i(\tilde{r}, \tilde{s}) = \sum_{k=0}^{i-2} \left| r_k r^*_s k+1 - s_k s^*_k+1 \right|^2 + \left| r_{i-1} r^*_i - s_{i-1} s^*_i \right|^2, \tag{3.5}
\]

where any two consecutive terms in the decomposition are not independent as in the conventional Viterbi algorithm. Therefore, the Viterbi algorithm needs to be modified. To do so, we construct a super trellis that combines the two consecutive states into one super state, where two consecutive states are defined as two substates of a super state. In the super trellis, the number of super states is 256. Each state has 64 branches in and 64 branches out. An example of symbol error rate versus SNR of the above noncoherent initial phase estimation is shown in Fig. 3, where the sequence length \( K \) is chosen as \( K = 340 \) and the phase error is \( \exp(j5\pi/36) \).

### 3.2 Noncoherent Decoding Algorithm

In this subsection, we want to use the above noncoherent initial phase estimation algorithm into the adaptive noncoherent decoding algorithm for the NC-16QAM. One might ask why we do not directly use the metric defined in (3.3) as the noncoherent decoding algorithm for the whole received sequence rather than only for the initial part of the received signal.

The reason is that the distance criterion for designing the NC-16QAM is not based on the metric (3.3) of the differential forms of the symbols but the symbols themselves. This limits the optimal performance of the NC-16QAM code. The following adaptive noncoherent decoding algorithm is similar to the one proposed in [2] for their NC-8PSK code but with the noncoherent initial phase estimation method proposed in Section 3.1.

The decoding branch metric for the NC-16QAM code is

\[
M_i(r_i, s_i) = |r_i r^*_i - s_i|^2, \tag{3.6}
\]

where \( R_i \) is the following estimate of the phase at the time \( i \):

\[
\hat{R}_i = \omega R_{i-1} + (1 - \omega)T_i,
\]

\[
R_i = \frac{R_i}{|R_i|},
\]

\[
T_i = \frac{\sum_{p=M}^{p+M} r_{i-p} s_{i-p}}{\sum_{p=M}^{p+M} |s_{i-p}|^2}
\]

where \( \omega \) is a parameter similar to [2], \( T_i \) denotes the estimate of the phase by using the most recent \( P \) decoded symbols at the output of the Viterbi decoder, \( P \) is the number of past symbols used in the estimate of the update part of the phase. In our simulations, we choose \( P \geq 10 \). The integer \( M \) (\( M > 5 \) times of the memory length of the encoder) denotes the traceback length in the Viterbi algorithm.

Fig. 4 shows some performance comparison results of the NC-16QAM with the coherent and noncoherent decoding and the TC-16QAM with the coherent decoding. These simulation results indicate that the performance of the NC-16QAM by using our proposed noncoherent decoding algorithm is close to the one of the NC-16QAM by using coherent detection and also close to the one of the original TC-16QAM by using the coherent decoding.
4 Conclusions

In this paper, we have constructed an NCM for 16QAM and proposed a noncoherent detection algorithm, which is an extension of the NCM for 8PSK recently proposed by Wei and Lin in [2]. Both NC-16QAM and TC-16QAM have the same free distance. We have proposed the concept of differential-coherently equivalent sequences, which is used in the noncoherent initial phase estimation for the noncoherent detection algorithm. Our simulation results have shown that the performance of the NC-16QAM by using our proposed noncoherent decoding algorithm is close to the one of the NC-16QAM by using the coherent detection and also close to the one of the original TC-16QAM by using the coherent decoding.

References


Figure 1: The constellation of 16-QAM
Figure 2: (a): The Structure of NC-16QAM; (b): Trellis Structure of TC-16QAM

Figure 3: Symbol error rate of the noncoherent initial phase estimation

Figure 4: Performances comparison of TC-16QAM and NC-16QAM