An Analysis on the Effect of the Number of Phases in Multipath Fading DS-CDMA Signature Sequences

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Abstract— Direct sequence code division multiple access (DS-CDMA) communication systems using random polyphase sequences with coherent reception in multipath fading channels are considered. By examining the asymptotic characteristics of the multiple access interference and multipath interference, we show that the DS-CDMA system performance with random m_p-phase sequences of $m_p \geq 3$ is better than that with random binary sequences in multipath fading environment.

I. Introduction

It has been shown that polyphase sequences have good correlation properties [1]-[3], allowing useful applications in direct sequence code division multiple access (DS-CDMA) systems. Hence, there has been much interest in using polyphase sequences in DS-CDMA communication systems [4],[5]. In most cases of the investigations, the performance of a system is evaluated for specific signature sequences: the results are therefore applicable only to systems using the particular sequences considered, but cannot be directly generalized for systems using other sequences. In order to obtain more widely applicable results, the random sequence model has been adopted in [6]: in [6], however, only even phase sequences have been considered and no consideration has been given to fading environment.

In this paper, DS-CDMA communication systems using *random polyphase* sequences with coherent reception in asynchronous *multipath fading* channels are considered. The DS-CDMA system model is presented in Section II, performance analysis is given in Section III, and simulation results are shown in Section IV. Some concluding remark is included in Section V.

II. System Model

A. Transmitter

The system model to be considered in this paper consists of K+1 simultaneous users (the zeroth user being the reference user whose performance is to be evaluated in this paper), and each user is assigned a unique CDMA signature sequence. The signature sequence given to each user is a random m_p-phase sequence, where $m_p \in \{2, 3, 4, \cdots\}$.

The signature waveform of the kth user can be written as

$$a_k(t) = \sum_{i=-\infty}^{\infty} a_{R(i,N)}^{(k)} p_{T_c}(t - iT_c), \tag{1}$$

where $a_i^{(k)} = e^{j\phi_i^{(k)}}$, $\{\phi_i^{(k)}\}_{i=0}^{N-1}$ are independent and identically distributed (iid) random variables uniformly distributed over the set $\{0, 2\pi/m_p, \cdots, 2(m_p-1)\pi/m_p\}$, $R(i,N)$ is the remainder of i when divided by the length N of the sequence, T_c is the chip duration, and $p_A(t)$ is the unit rectangular pulse defined by $p_A(t) = 1$ for $0 \leq t < A$ and 0 elsewhere.

We denote the data signal of the kth user by

$$b_k(t) = \sum_{i=-\infty}^{\infty} b_i^{(k)} p_T(t - iT), \tag{2}$$

where $T = NT_c$ is the symbol duration and the data symbols $\{b_i^{(k)}\}_{i=-\infty}^{\infty}$ are assumed to be iid random variables with $Pr\{b_i^{(k)} = 1\} = Pr\{b_i^{(k)} = -1\} = 1/2$. The transmitted signal for the kth user is, therefore,

$$s_k(t) = \sqrt{2P} a_k(t) b_k(t) e^{j(\vartheta_k + \omega_c t)}, \tag{3}$$

where P is the average transmitted power common to all users, ϑ_k is the phase of the kth carrier, and ω_c is the common carrier frequency. Without loss of generality, it is assumed that $\vartheta_0 = 0$.

B. Channel

A commonly used model for a frequency-selective multipath channel is the finite-length tapped delay line model [7] shown in Figure 1 for the kth user. In Figure 1, $n_k(t)$ is the additive white Gaussian noise (AWGN) in the channel of the kth user, and the lth tap weight $w_l = \alpha_l^{(k)} e^{j\psi_l^{(k)}}$ is the same as the fading term of the kth user in the lth path,

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$l = 0, 1, \cdots, L_p^{(k)} - 1$, where $L_p^{(k)}$ is the number of paths for the kth user. The number $L_p^{(k)}$ of paths is related to the maximum delay spread of the channel and is assumed to be much less than the number N of chips per information bit. The fading amplitudes $\{\alpha_l^{(k)}\}_{l=0}^{L_p^{(k)}-1}$ are assumed to be independent random variables, and the fading phases $\{\psi_l^{(k)}\}_{l=0}^{L_p^{(k)}-1}$ are iid random variables uniformly distributed over $[0, 2\pi)$ and are assumed to be independent of $\{\alpha_l^{(k)}\}_{l=0}^{L_p^{(k)}-1}$.

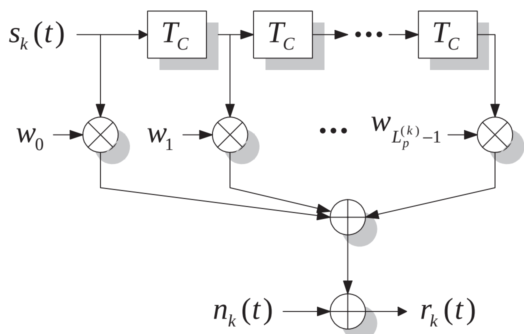

Fig. 1. A multipath channel model (tap weights: $w_l = \alpha_l^{(k)} e^{j\psi_l^{(k)}}$) for the kth user

C. Receiver

The receiver model considered in this paper is shown in Figure 2 for the reference user ($k = 0$). The receiver is essentially a coherent rake receiver, where the number L_r of branches is a variable parameter less than or equal to the number of paths.

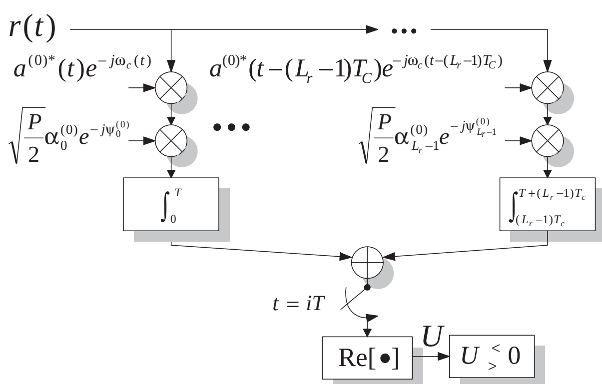

Fig. 2. The receiver model for the reference (zeroth) user.

The receiver is matched to the signature sequence of the reference user and is assumed to have achieved time synchronization with the initial path of the reference signal. The tap weights (both the amplitudes α_i and phases ψ_i) are assumed to be perfect estimates of the channel parameters: practically, these estimates can be obtained quite accurately from separate circuits, such as those given in [7], and then fed to the demodulator. The sampling times of the receiver are iT, $i = 1, 2, \cdots$.

The received signal in a single-cell system can be written as

$$r(t) = \sqrt{2P}\sum_{k=0}^{K}\sum_{l=0}^{L_p^{(k)}-1} a_k(t-\tau_l^{(k)}) b_k(t-\tau_l^{(k)}) \cdot \alpha_l^{(k)} e^{j(\omega_c t + \theta_l^{(k)})} + n(t). \quad (4)$$

In (4), the carrier phases $\{\theta_l^{(k)} = \vartheta^{(k)} + \psi_l^{(k)} - \omega_c \tau_l^{(k)}\}_{k=0}^{K}$ are iid random variables uniformly distributed over $[0, 2\pi)$, and the variance of $n(t)$ is assumed to be η_0. We will assume that $\tau_l^{(k)} = \tau_k + lT_c$, where $\tau_0 = 0$ and $\{\tau_k\}_{k=1}^{K}$ are iid random variables uniformly distributed over $[0, T)$.

Then, the real part U of the receiver output at each sampling time can be written as [8]

$$U = \sum_{n=0}^{L_r-1}\left\{U_{S,n} + U_{MAI,n} + U_{MP,n} + U_{GN,n}\right\}, \quad (5)$$

where

$$U_{S,n} = \{\alpha_n^{(0)}\}^2 b_0^{(0)} PT \quad (6)$$

is the desired signal component to be detected,

$$U_{MAI,n} = P\sum_{k=1}^{K}\sum_{l=0}^{L_p^{(k)}-1}\alpha_n^{(0)}\alpha_l^{(k)}\hat{B}_{ln}^{(k)} \quad (7)$$

is the noise component due to the multiple-access interference (MAI) from other users,

$$U_{MP,n} = P\sum_{l=0, l\neq n}^{L_p^{(0)}-1}\alpha_n^{(0)}\alpha_l^{(0)}\hat{B}_{ln}^{(0)} \quad (8)$$

is the noise component due to the multipath (MP) waveforms of the reference signal, and

$$U_{GN,n} = \int_{nT_c}^{T+nT_c}\sqrt{\frac{P}{2}} Re\left[n(t)a^{(0)*}(t-nT_c) \cdot \alpha_n^{(0)} e^{-j(\omega_c t + \theta_n^{(0)})}\right] dt \quad (9)$$

is the noise component due to the AWGN. In (7) and (8),

$$\hat{B}_{ln}^{(k)} \equiv Re\left[B_{ln}^{(k)} e^{j\theta_{ln}^{(k)}}\right], \quad (10)$$

where $\theta_{ln}^{(k)} = \theta_l^{(k)} - \theta_n^{(0)}$ and $B_{ln}^{(k)} = \int_0^T b^{(k)}(t-\tau_{ln}^{(k)}) a^{(k)}(t-\tau_{ln}^{(k)}) a^{(0)*}(t) dt$ with $\tau_{ln}^{(k)} = \tau_l^{(k)} - \tau_n^{(0)}$.

III. Performance Analysis

Let us first consider the normalized MP component

$$G_P = \frac{\hat{B}_{ln}^{(0)}}{\sqrt{N}T_c}. \quad (11)$$

In [9], it is shown that the asymptotic ($N \to \infty$) characteristic function (cf) of G_P is

$$\Phi_P^a(u) = \begin{cases} exp[-\frac{u^2}{4}]I_0(\frac{u^2}{4}) & \text{if } m_p = 2, \\ exp[-\frac{u^2}{4}] & \text{if } m_p \geq 3, \end{cases} \quad (12)$$

and the corresponding probability density function (pdf) is

$$f_P(x) = \begin{cases} f_{P2}(x) = \frac{1}{\sqrt{2\pi^3}} exp[-\frac{x^2}{4}] K_0(\frac{x^2}{4}) \\ \quad \text{if } m_p = 2, \\ f_{P3}(x) = \frac{1}{\sqrt{\pi}} exp[-x^2] \\ \quad \text{if } m_p \geq 3, \end{cases} \quad (13)$$

where

$$I_0(x) = \frac{1}{\pi}\int_0^{\pi} e^{x\cos\theta} d\theta \quad (14)$$

is the first kind modified Bessel function of order 0, and

$$K_0(x) = \int_0^{\infty} \frac{\cos(xt)}{\sqrt{t^2+1}} dt, \quad x > 0, \quad (15)$$

is the second kind modified Bessel function of order 0. In Figure 3, the graphs of f_{P2} and $f_{P3} = N(0, \frac{1}{2})$ are plotted for easy reference, where $N(m, \sigma^2)$ denotes Gaussian pdf with mean m and variance σ^2. It is clear from Figure 3 and Table I that the tail of $f_{P2}(x)$ is heavier than that of $f_{P3}(x)$ although the variances of $f_{P2}(x)$ and $f_{P3}(x)$ are both $\frac{1}{2}$.

Next, let us consider the normalized MAI component

$$G_I = \frac{\hat{B}_{ln}^{(k)}}{\sqrt{N}T_c}. \quad (16)$$

In [9], it is shown that the asymptotic cf of G_I is

$$\Phi_I^a(u) = \begin{cases} \int_0^1 exp\left[-\frac{u^2}{4}(1-2r+2r^2)\right] \\ \quad \cdot I_0\left(\frac{u^2}{4}(1-2r+2r^2)\right) dr & \text{if } m_p = 2, \\ \int_0^1 exp\left[-\frac{u^2}{4}(1-2r+2r^2)\right] dr & \text{if } m_p \geq 3, \end{cases} \quad (17)$$

and the corresponding pdf is

$$f_I(x) = \begin{cases} f_{I2}(x) = \frac{1}{\sqrt{2\pi^3}} \int_0^1 \frac{1}{\sqrt{1-2r+2r^2}} exp\left[-\frac{x^2}{4(1-2r+2r^2)}\right] \\ \quad \cdot K_0\left(\frac{x^2}{4(1-2r+2r^2)}\right) dr \\ \quad \text{if } m_p = 2, \\ f_{I3}(x) = \frac{1}{\sqrt{\pi}} \int_0^1 \frac{1}{\sqrt{1-2r+2r^2}} exp\left[-\frac{x^2}{1-2r+2r^2}\right] dr \\ \quad \text{if } m_p \geq 3. \end{cases} \quad (18)$$

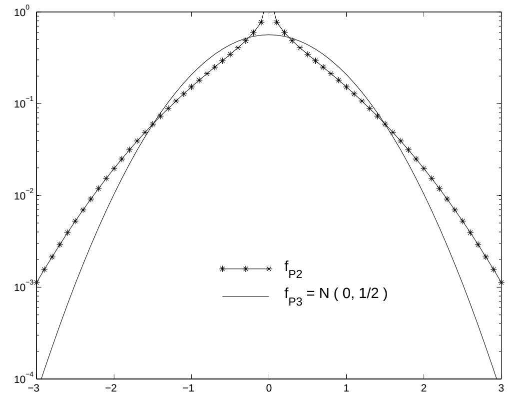

Fig. 3. A plot of f_{P2} and f_{P3}

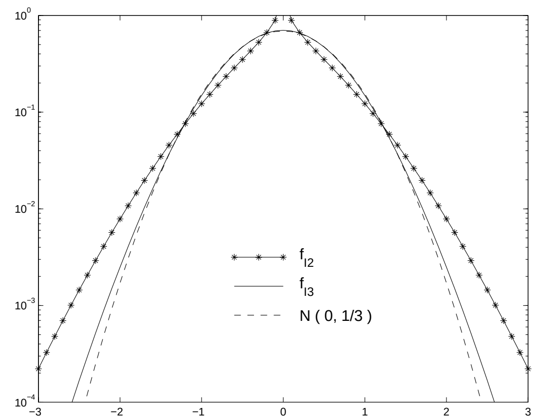

Fig. 4. A plot of f_{I2} and f_{I3}

In Figure 4, the graphs of f_{I2} and f_{I3} are plotted, together with the Gaussian pdf $N(0, \frac{1}{3})$: we can show [9] that the variances of f_{I2} and f_{I3} are both $\frac{1}{3}$ (more easily by using (17) than by using (18)). As shown in Table II and Figure 4 using the pdf (18), f_{I2} has a heavier tail than f_{I3}.

Let us now define $\beta_{ln}^{(k)} \equiv \alpha_l^{(k)}\alpha_n^{(0)}$ and $\mu \equiv \sqrt{N}T_c P$. Then, the total MP component $U_{MP} = \sum_{n=0}^{L_r-1} U_{MP,n}$ and the total MAI component $U_{MAI} = \sum_{n=0}^{L_r-1} U_{MAI,n}$ can be expressed as

$$U_{MP} = \mu \sum_{n=0}^{L_r-1} \sum_{l=0, l\neq n}^{L_p^{(0)}-1} \beta_{ln}^{(0)} G_P \quad (19)$$

and

$$U_{MAI} = \mu \sum_{k=1}^{K} \sum_{n=0}^{L_r-1} \sum_{l=0}^{L_p^{(k)}-1} \beta_{ln}^{(k)} G_I, \quad (20)$$

TABLE I

TAIL PROBABILITIES $Pr\{X > t\}$ OF f_{P2} AND f_{P3}

t	f_{P2}	$f_{P3} = N(0, \frac{1}{2})$
0.5	3.450×10^{-1}	4.394×10^{-1}
1.0	1.525×10^{-1}	2.076×10^{-1}
1.5	5.995×10^{-2}	5.947×10^{-2}
2.0	1.967×10^{-2}	1.033×10^{-2}
2.5	5.250×10^{-3}	1.089×10^{-3}
3.0	1.125×10^{-3}	6.963×10^{-5}
3.5	1.920×10^{-4}	2.700×10^{-6}
4.0	2.596×10^{-5}	6.349×10^{-8}

TABLE II

TAIL PROBABILITIES $Pr\{X > t\}$ OF f_{I2}, f_{I3}, AND A GAUSSIAN PDF

t	f_{I2}	f_{I3}	$N(0, \frac{1}{3})$
0.5	1.566×10^{-1}	1.901×10^{-1}	1.932×10^{-1}
1.0	4.693×10^{-2}	4.158×10^{-2}	4.163×10^{-2}
1.5	1.151×10^{-2}	5.467×10^{-3}	4.687×10^{-3}
2.0	2.309×10^{-3}	4.710×10^{-4}	2.660×10^{-4}
2.5	3.849×10^{-4}	2.761×10^{-5}	7.451×10^{-6}
3.0	5.388×10^{-5}	1.080×10^{-6}	1.017×10^{-7}
3.5	6.301×10^{-6}	2.743×10^{-8}	6.715×10^{-10}
4.0	6.083×10^{-7}	4.446×10^{-10}	2.131×10^{-12}

respectively. Using that $\Phi_Y(u) = \Phi_X(cu)$ when the two random variables X and Y are related as $Y = cX$ for a constant c, we obtain the asymptotic cf's of U_{MP} and U_{MAI} as

$$\Phi^a_{MP}(u : \underline{\beta}) = \prod_{n=0}^{L_r-1} \prod_{l=0, l\neq n}^{L_p^{(0)}-1} \Phi^a_P(\mu\beta^{(0)}_{ln} u) \quad (21)$$

and

$$\Phi^a_{MAI}(u : \underline{\beta}) = \prod_{k=1}^{K} \prod_{n=0}^{L_r-1} \prod_{l=0}^{L_p^{(k)}-1} \Phi^a_I(\mu\beta^{(k)}_{ln} u), \quad (22)$$

respectively, where $\underline{\beta} = \{\beta^{(0)}_{00}, \cdots, \beta^{(K)}_{L_p^{(K)}-1, L_r-1}\}$. Now, using (21) and (22), we can in principle obtain the asymptotic cf Φ^a_U of U from (5) as

$$\Phi^a_U(u : \underline{\beta}) = e^{juU_S} \cdot \Phi^a_{MP}(u : \underline{\beta}) \Phi^a_{MAI}(u : \underline{\beta}) \Phi_{GN}(u), \quad (23)$$

and the asymptotic pdf f_U of U as

$$f_U(t) = \frac{1}{2\pi} \int \int_{-\infty}^{\infty} \Phi^a_U(u : \underline{\beta}) e^{-jut} du f_{\underline{\beta}}(\underline{\beta}) d\underline{\beta}, \quad (24)$$

where

$$\Phi_{GN}(u) = exp(-\frac{u^2\eta_0}{2}) \quad (25)$$

is the cf of the total AWGN component $U_{GN} = \sum_{n=0}^{L_r-1} U_{GN,n}$, $U_S = \sum_{n=0}^{L_r-1} U_{S,n}$ is the total desired signal component, and $f_{\underline{\beta}}$ is the pdf of $\underline{\beta}$: although it seems not possible to obtain the explicit closed-form expression of the pdf f_U, it is anticipated from the above discussions on the statistical characteristics of the MAI and MP components that the tail of f_U would be heavier for $m_p = 2$ than for $m_p \geq 3$. Consequently, the system performance for $m_p \geq 3$ is expected to be different from (more specifically, better than) that for $m_p = 2$. This is a natural consequence considering the increased complexity for $m_p \geq 3$.

IV. SIMULATION RESULTS

We showed analytically that the system performance using m_p-phase, $m_p \geq 3$, sequences would be different from that using binary phase sequences by showing that the tail of f_U would be heavier for $m_p = 2$ than for $m_p \geq 3$ in Section 3: we used the asymptotic characteristic and probability density functions derived under the assumption that the number N of chips approached infinity. As the number of chips is finite in practical systems, we now show and discuss in this section some simulation results obtained for various system parameters to see the effect of the number m_p of phases on the system performance when the number of chips is finite. In these simulations, we assume that the fading amplitudes are Rayleigh with the exponential multipath intensity profile, $\Omega^{(k)}_l = \Omega^{(k)}_0 e^{-\delta l}$, $\delta \geq 0$, $l = 1, 2, \cdots, L_p^{(k)} - 1$, where $\Omega^{(k)}_l = E\{(\alpha^{(k)}_l)^2\}$ and δ is the rate of average power decay. For simplicity we also let $L_p^{(k)} = L_p$ for $k = 0, 1, \cdots, K$.

Figure 5 depicts the bit error probability $\bar{P}_b$ as a function of the average received SNR per bit $E\Omega^{(0)}_0/\eta_0$ to show the effect of the number m_p of phases on the system performance when $\delta = 0.2$, $K = 5$, and $L_p = L_r = 3$, for $N = 63$. Each point in the figures is obtained from a Monte Carlo run of 2 million iterations. We can clearly see that the BER for $m_p \geq 3$ differs from (is lower than) that for $m_p = 2$, which becomes clearer as the channel environment gets more favorable: as the SNR gets higher and the value of N gets larger, the outperformance of the system with $m_p \geq 3$ over the system with $m_p = 2$ becomes higher.

Figure 6 is to show the effect of the number m_p of phases on the average bit error probability when the number K of interfering users varies, with all the other parameters the same as those in Figure 5. In this figure, it is again clearly observed that the number m_p of phases has influence on the average bit error probability when the number of interfering users varies: the difference between the system performance for $m_p \geq 3$ and that for $m_p = 2$ gets larger as the number of interfering users decreases and the SNR gets higher. We would like to mention that the performance of the system with $m_p \geq 3$ is almost the same as that with $m_p = 2$ as K gets larger (relatively to the value of N), since in such a case the number of convolutions of the non-Gaussian pdf's increases and consequently the pdf f_U for $m_p \geq 3$ and $m_p = 2$ both approach Gaussian pdf's.

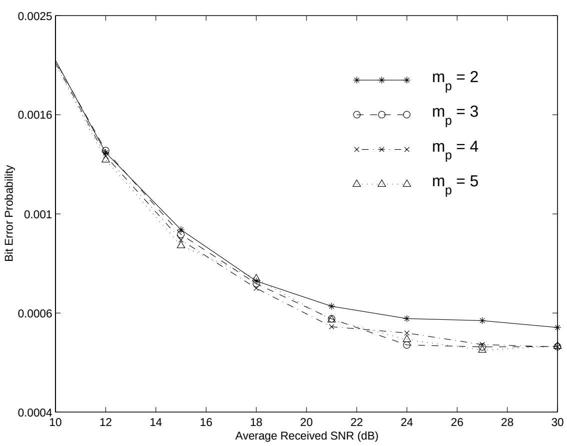

Fig. 5. Bit error probability when $\delta = 0.2$, $K = 5$, $L_p = L_r = 3$, and $N = 63$.

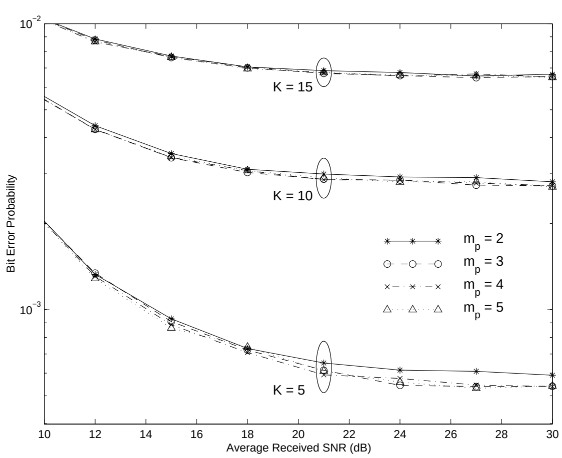

Fig. 6. Bit error probability when $\delta = 0.2$, $L_p = L_r = 3$, $K = 5, 10, 15$, and $N = 63$.

V. Concluding Remark

The performance of asynchronous DS-CDMA communication systems using random polyphase sequences in multipath fading channel was analyzed. In the analysis, a number of simultaneous BPSK transmitters and a coherent rake receiver were employed. The channel was assumed to be a frequency-selective multipath channel.

The asymptotic (when the number of chips approached infinity) characteristic and probability density functions of the MAI and MP components were derived for $m_p = 2$ and $m_p \geq 3$. Then, using the results, we reasoned that the pdfs of the MAI and MP components and consequently of the decision statistic for $m_p = 2$ had heavier tails than those for $m_p \geq 3$, resulting in possible better performance for $m_p \geq 3$.

The asymptotic characteristic functions derived in Section 3 were obtained under the assumption that the number of chips approached infinity. To see more realistic results on the system performance when the number of chips was finite, we obtained some simulation results under various channel circumstances in Section 4. It was shown that the performance of the system for $m_p \geq 3$ was better than that for $m_p = 2$.

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