Power-Controlled MC-CDMA in the Presence of Nonlinearities — Part II: Coded System Performance

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Abstract— **In this paper we analyze the effect of a memoryless nonlinear amplifier on the performance of convolutionally coded multicarrier code division multiple access (MC-CDMA) systems in the presence of Rayleigh multipath fading. Analytical results for the upper bounds on the bit error rate of the systems are obtained.**

I. Introduction

The need for low power and high speed transmission is well recognized in current and future mobile personal communications. A multicarrier modulation scheme is an effective technique for such high speed data applications, especially in an interference limited environment such as a multipath fading channel [1]. This technique is currently being used in digital audio broadcasting [2] and European digital terrestrial broadcasting [3]. However, because of its high peak-to-mean-envelope-power ratio (PMEPR) [4] when linear amplification is required, the transmitter amplifier's average output power must be backed off. This usually results in low dc to RF power conversion efficiency and thus nonlinear amplification is allowed to a certain extent to conserve power, even for nonconstant envelope signals.

In [5], the effect of the nonlinear amplifier on a single cell uncoded MC-CDMA system has been analyzed. Because of huge performance degradation (as seen in [5] even without nonlinear distortion) from hostile mobile environments such as fading and multi-user interference, channel coding is usually required to make wireless communication acceptable for various applications under transmitter power constraints. In this paper, we extend the analysis to the convolutionally coded systems. For the system model, we consider the reverse link (communication from the mobile to the base), and also assume perfect power control (that the average power received from each user at the base station is the same).

The paper is organized as follows. In Section II, the coded MC-CDMA system and the channel model considered are given. The considered channel model is the same model used in [5]. In Section III, the system performance is analyzed including the upper bound on the bit error rate (BER). In Section IV, numerical results are discussed. Finally, Section V concludes the paper.

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II. System and Channel Model

A. Modulation

The convolutionally encoded information sequence is interleaved and converted from a serial stream to M parallel streams.The binary code stream for the q-th carrier of a k-th user is denoted by $\{d_{k,q}^{(j)}\}$ where $d_{k,q}^{(j)} \in \{\pm 1\}$. The signal on the q-th carrier of the user k is then given by $d_{k,q}(t) = \sum_{j=-\infty}^{\infty} d_{k,q}^{(j)} p_{T_s}(t - jT_s)$ where $T_s = MT_bR_c$ is a symbol duration and $p_{T_s}(t)$ is a unit rectangular pulse in $t \in [0, T_s]$. The information bit duration before the parallel to serial process is T_b and the code rate is R_c. The signal on q-th carrier is then multiplied by an pseudo-random spreading code $a_{k,q}(t)$, where $a_{k,q}(t) = \sum_{j=-\infty}^{\infty} \sum_{i=0}^{N-1} a_{k,q}^{(i)} p_{T_c}(t - iT_c - jT_s)$ and the chip sequence $\{a_{k,q}^{(i)}\} \in \{\pm 1\}$ is a sequence of i.i.d. random variables with equal probabilities. Each spreading code has a chip duration of $T_c = T_s/N$ where N is the spreading gain of the system. Finally, the signal on each carrier is summed before amplification. The output signal of the modulator of the k-th user

$$x_k(t) = \sqrt{2P_k} \sum_{q=1}^{M} a_{k,q}(t) d_{k,q}(t) \cos\{(w_c + w_q)t + \theta_{k,q}\} \quad (1)$$

where P_k is the power per carrier of the k-th user, w_c is the carrier frequency and $\theta_{k,q}$ is an i.i.d. random variable uniformly distributed over $[0, 2\pi)$. The separation between q-th carrier frequency and center frequency w_c is denoted by $w_q = 2\pi q/T_c$.

B. Channel Model

The nonlinearly amplified signal is first distorted by fading and then corrupted by additive white Gaussian noise (AWGN) with two-sided spectral density $N_0/2$.

B.1 Amplifier Model

A memoryless bandpass nonlinear model is considered. In this model, the relationship between the input and output signal of the amplifier is described by the two memoryless functions, namely amplitude (AM/AM) and phase (AM/PM) nonlinearities [6]. If the modulated input signal to the amplifier has a form, $A(t)\cos\{w_ct + \phi(t)\}$, the output of the amplifier is expressed as $\mathcal{F}(A(t))\cos\{w_ct + \phi(t) + \Phi(A(t))\}$. The functions $\mathcal{F}(A(t))$ and $\Phi(A(t))$ denote AM/AM and AM/PM, respectively. In this paper, no AM/PM effect is assumed (i.e.$\Phi(A(t)) \approx 0$). The AM/AM can be represented by the odd order polynomial

$$\mathcal{F}(A(t)) = \alpha_1 A(t) + \alpha_3 A^3(t) + \alpha_5 A^5(t) + \ldots + \alpha_n A^n(t) \quad (2)$$

where n is odd and the α_i's are the coefficients for the polynomial, which determine the degree of nonlinearity. The amplifier input in (1) can be expressed in terms of its envelope $A(t)$ and phase $\phi(t)$. Then from (3) in [5], the amplifier output signal

$$\begin{aligned} y_k(t) &= \{\alpha_1 + \alpha_3 A_k^2(t) + \cdots + \alpha_n A_k^{n-1}(t)\}\, x_k(t) \\ &= x_k^{(1)}(t) + x_k^{(3)}(t) + \cdots + x_k^{(n)}(t) \end{aligned} \quad (3)$$

where $x_k^{(1)}(t) = \alpha_1 x_k(t)$ denotes the linearly amplified signal term, and $x_k^{(n)}(t)$ (for $n \geq 3$) denotes the amplified signal term from the n-th order nonlinearity, given by

$$\begin{aligned} x_k^{(n)}(t) &= \alpha_n(\sqrt{2P_k})^n \sum_{l_1=1}^{M} \sum_{l_2=1}^{M} \cdots \sum_{l_n=1}^{M} \widetilde{a}_{k,n}(t) \widetilde{d}_{k,n}(t) \\ &\quad \cos\{(w_c + w_{\jmath_n})t + \psi_{k,\jmath_n}\} \end{aligned} \quad (4)$$

where

$$\widetilde{a}_{k,n}(t) = a_{k,l_1}(t)\, a_{k,l_2}(t) \cdots a_{k,l_n}(t) \quad (5)$$

$$\widetilde{d}_{k,n}(t) = d_{k,l_1}(t)\, d_{k,l_2}(t) \cdots d_{k,l_n}(t) \quad (6)$$

and the index $\jmath_n = l_1 + l_2 - l_3 + \cdots + l_{n-1} - l_n$, i.e., $w_{\jmath_n} = 2\pi \jmath_n / T_c = w_{l_1} + w_{l_2} - w_{l_3} + \cdots + w_{l_{n-1}} - w_{l_n}$, and the phase $\psi_{k,\jmath_n} = \theta_{k,l_1} + \theta_{k,l_2} - \theta_{k,l_3} + \cdots + \theta_{k,l_{n-1}} - \theta_{k,l_n}$.

We note that the signal term generated from the 3rd, 5th, ..., n-th order nonlinearity can each be divided into two signal components: the in-phase and the interference signal. The in-phase signal refers to the output signal of the amplifier which turns out to be a scaled replica of the input signal ($x_k(t)$) to the amplifier, adding coherently to the linearly amplified signal $\alpha_1 x_k(t)$, whereas the interference signal refers to the output signal that can possibly interfere with $x_k^{(1)}(t)$. In summary, the nonlinearly amplified signal terms $x_k^{(n)}(t)$ can be rewritten as the sum of the attenuated version of the original signal and the interfering signal as

$$\begin{aligned} x_k^{(n)}(t) &= \alpha_n(\sqrt{2P_k})^{n-1}\Lambda_n(M) x_k(t) + \alpha_n(\sqrt{2P_k})^n \\ &\underbrace{\sum_{l_1=1}^{M} \sum_{l_2=1}^{M} \cdots \sum_{l_n=1}^{M}}_{\mathcal{A}_n^c} \widetilde{a}_{k,n}(t) \widetilde{d}_{k,n}(t) \cos\{(w_c + w_{\jmath_n})t + \psi_{k,\jmath_n}\} \end{aligned} \quad (7)$$

where $\mathcal{A}_n^c$ is the complement of set $\mathcal{A}_n$. Set $\mathcal{A}_n$ and number $\Lambda_n(M)$ are defined and listed in [5] along with detailed descriptions.

B.2 Multipath Fading Model with a Propagation Loss

For the channel model, slowly varying Rayleigh fading for each carrier is assumed so that the fade level is constant over a symbol duration. In addition, the bandwidth for each carrier is assumed to be smaller than the coherence bandwidth so that each carrier undergoes frequency nonselective fading. The complex low-pass equivalent channel response for the q-th carrier of the k-th user is given as $h_{k,q}(t) = \widehat{\beta}_{k,q} \exp(j\varphi_{k,q})\delta(t - \tau_k)$ where $\widehat{\beta}_{k,q} = \sqrt{g_k}\beta_{k,q}$. $\{\beta_{k,q}\}$ is a sequence of i.i.d. Rayleigh distributed random variables with $E[\beta_{k,q}^2] = 1$ where $E[X]$ denotes the expectation of X. The independence of $\{\beta_{k,q}\}$ on transmitted symbols of the same carrier can be justified by the use of sufficient interleaving. The channel propagation gain g_k of the k-th user is defined as $g_k = r_k^{-4}$, and r_k is the distance (normalized to square root value of the product of the height of the antenna of the base and mobile) between the user k and base station. The phase $\{\varphi_{k,q}\}$ and delay $\{\tau_k\}$ are a sequence of i.i.d. random variables uniformly distributed over $[0, 2\pi)$ and over $[0, T_s]$, respectively. $\delta(t)$ is the Dirac delta function. Note that in the AWGN channel $\{\beta_{k,q} = 1\}$ and $\{\varphi_{k,q} = 0\}$, for all k and q.

The received signal after the fading and AWGN channel

$$r(t) = n(t) + \sum_{k=1}^{K} S_k(t - \tau_k) + \sum_{k=1}^{K} I_k(t - \tau_k) \quad (8)$$

where a desired signal of the k-th user, $S_k(t)$, is

$$H_k^{(1)} \sqrt{2P_k} \sum_{q=1}^{M} \widehat{\beta}_{k,q} a_{k,q}(t) d_{k,q}(t) \cos\{(w_c + w_q)t + \widehat{\theta}_{k,q}\} \quad (9)$$

and the interference signal of the k-th user nonlinearities

$$I_k(t) = I_k^{(3)}(t) + I_k^{(5)}(t) + \cdots + I_k^{(n)}(t) \quad (10)$$

where the interference from the n-th order nonlinearities

$$\begin{aligned} I_k^{(n)}(t) &= \alpha_n(\sqrt{2P_k})^n \sum_{l_1=1}^{M} \sum_{l_2=1}^{M} \cdots \sum_{l_n=1}^{M} \mathrm{I}_{\{\mathcal{A}_n^c\}} \widehat{\beta}_{\jmath_n} \\ &\quad \widetilde{a}_{k,n}(t) \widetilde{d}_{k,n}(t) \cos\{(w_c + w_{\jmath_n})t + \widehat{\psi}_{k,\jmath_n}\} \end{aligned} \quad (11)$$

and $H_k^{(1)} = \{\alpha_1 + \alpha_3(\sqrt{2P_k})^2 \Lambda_3(M) + \cdots + \alpha_n(\sqrt{2P_k})^{n-1}\Lambda_n(M)\}$, $\widehat{\theta}_{k,q} = \theta_{k,q} + \varphi_{k,q}$, and $\widehat{\psi}_{k,\jmath_n} = \psi_{k,\jmath_n} + \varphi_{k,\jmath_n}$. The indicator function, $\mathrm{I}_{\{\zeta\}}$, equals one if the indices in the summation belong to set ζ and zero otherwise.

III. Performance Analysis

In this section, we first analyze the demodulator output statistics and then derive the upper bound on the average bit error rate.

A. Demodulator and Its Output Statistics

The coherently demodulated symbol, after the integrate-and-dump filter of the q-th subcarrier of the l-th user

$$Z_{l,q} = \int_{\tau_l}^{T_s+\tau_l} r(t)\, a_{l,q}(t-\tau_l)\cos\{(w_c+w_q)(t-\tau_l)+\widehat{\theta}_{l,q}\}dt$$
$$= \eta + D_{l,q} + S_{l,q} + \widehat{S}_{l,q} + J_{non,l,q} + I_{non,l,q} + \widehat{I}_{non,l,q} \quad (12)$$

where η is a normally distributed random variable with mean 0 and variance $N_0T_s/4$. $D_{l,q}$ is the desired term for the q-th channel of the l-th user, and $J_{non,l,q}$ is the nonlinear interference term to the q-th carrier from the l-th user itself. $S_{l,q}$ is the linear interference from the q-th branch of the other users and $\widehat{S}_{l,q}$ is the linear interference from the other (not q) subcarriers of the other users, $I_{non,l,q}$ is the nonlinear interference from the q-th branch of the other users, and $\widehat{I}_{non,l,q}$ is the nonlinear interference from the other (not q) branch of the other users. Note that the out-of-band carrier components generated from the nonlinearities are not included in $\widehat{I}_{non,l,q}$. The analytical expressions for the terms in (12) are in (12)-(20) of [5] and from (27)-(29) in [5], the conditional mean of $Z_{l,q}$, $E[Z_{l,q}|\beta_{l,q}] = \sqrt{2P_l}\widehat{\beta}_{l,q}d_{l,q}^{(0)}T_sH_l^{(1)}/2$ and the conditional variance, $\text{Var}[Z_{l,q}|\beta_{l,q}]$, is obtained as

$$\frac{N_0T_s}{4} + \frac{\widehat{\beta}_{l,q}^2 P_l T_s^2}{4N}(\widetilde{H}_{l,q}^{(n)})^2 + \sum_{k=1,k\neq l}^{K}\frac{g_kP_kT_s^2}{6N}\{(H_k^{(1)})^2+(\widetilde{H}_{k,q}^{(n)})^2\}$$
$$+\sum_{k=1,k\neq l}^{K}\frac{g_kP_kT_s^2}{4N\pi^2}\sum_{m=1,m\neq q}^{M}\frac{1}{(m-q)^2}\{(H_k^{(1)})^2+(\widetilde{H}_{k,m}^{(n)})^2\}\} \quad (13)$$

where $(H_k^{(1)})^2$ and $(\widetilde{H}_{k,q}^{(n)})^2$ are defined in (24) and (28) of [5].

B. Conditional SNR

If we approximate the multiple access interference and nonlinear interference by a Gaussian random variable, the conditional $\text{SNR}(\beta_{l,q})$ is given by

$$\frac{E^2[Z_{l,q}|\beta_{l,q}]}{\text{Var}[Z_{l,q}|\beta_{l,q}]} = \frac{2\beta_{l,q}^2\bar{\gamma}}{(1+\bar{c}_l)(1+v_{l,q}^K\bar{\gamma})+\beta_{l,q}^2\bar{\gamma}\frac{c_{l,q}}{N}} \quad (14)$$

where

- $\bar{\gamma}_{l,q} = [(H_l^{(1)})^2 + (\widetilde{H}_{l,q}^{(n)})^2]g_lP_lT_s/N_0$: average received symbol-energy-to-noise power density ratio (E_s/N_0) on the q-th carrier of the l-th user.
- $\bar{\gamma}_l = \frac{1}{M}\sum_{q=1}^{M}\bar{r}_{l,q}$: E_s/N_0 per carrier of the l-th user.
- $c_{l,q} = [(\widetilde{H}_{l,q}^{(n)})^2]/(H_l^{(1)})^2$: normalized nonlinear interference variance on a q-th carrier of the l-th user.
- $\bar{c}_l = \frac{1}{M}\sum_{q=1}^{M}c_{l,q}$: normalized nonlinear interference variance of the l-th user per carrier.
- $v_{l,q}^K = \frac{1}{N}\sum_{k=1,k\neq l}^{K}\{(\frac{1}{1+\bar{c}_k})(\frac{2}{3}(1+c_{k,q})+\frac{1}{\pi^2}\sum_{m=1,m\neq q}^{M}\frac{1+c_{k,m}}{(m-q)^2})\}$ and

In (14), the fact that $\bar{\gamma}_l = \bar{\gamma}_k = \bar{\gamma}$ for all k, from the perfect power control assumption, is used. The average received bit-energy-to-noise power density ratio (E_b/N_0) per carrier is $R_c^{-1}\bar{\gamma}$.

C. Upper Bound on Bit Error rate

For the coded system, we consider a convolutional code of rate $R_c = 1/2$ with constraint length 7 with maximum likelihood decoding. We assume that a perfect channel estimate is available so that we can weight the output of the integrate-and-dump filter of user l by the factor $(g_{l,i})$. We also assume that we have a sufficiently large size interleaver that code symbols in the same carrier are independent to each other after the deinterlever. Bounds on the bit error probability of user l for a code can be expressed as follows [7, page 327]

$$P_b \leq \sum_{d=d_{free}}^{\infty} w_d P_{l,2}(d) \quad (15)$$

where w_d is the total number of nonzero information bits on a path with hamming distance d, $P_{l,2}(d)$ is the error probability between two codewords which differ in d symbols, and d_{free} is the free distance of the code. We use truncated w_d up to approximately $d = 30$ which are tabulated in [8]. The pairwise error probability $P_{l,2}(d)$ of user l is just the error probability of a repetition code of length d. Assuming we send the all-zero message, $P_{l,2}(d)$ is given as follows,

$$P_{l,2}(d) = P\{\sum_{i=1}^{d} g_iZ_{l,i} \leq 0\} = P\{Z_l(d) \leq 0\} \quad (16)$$

where $Z_l(d) = \sum_{i=1}^{d} g_{l,i}Z_{l,i}$ and $g_{l,i}$ is a weighting factor defined as

$$g_{l,i} = E[Z_{l,i}|\beta_{l,i}]/\text{Var}[Z_{l,i}|\beta_{l,i}]. \quad (17)$$

Since given $\beta_{l,i}$, the demodulator output values are Gaussian random variables, the statistics of $Z_l(d)$ given $\beta_{l,i}$'s is also Gaussian with the conditional mean and variance given by $E[Z_l(d)|\beta_{l,1},\beta_{l,2},\ldots,\beta_{l,d}] = \sum_{i=1}^{d} E^2[Z_{l,i}|\beta_{l,i}]/\text{Var}[Z_{l,i}|\beta_{l,i}]$ and $\text{Var}[Z_l(d)|\beta_{l,1},\beta_{l,2},\ldots,\beta_{l,d}] = \sum_{i=1}^{d} E^2[Z_{l,i}|\beta_{l,i}]/\text{Var}[Z_{l,i}|\beta_{l,i}]$, respectively. The conditional pairwise error probability is then

$$P\{Z_l(d) \leq 0|\beta_{l,1},\beta_{l,2},\ldots,\beta_{l,d}\} = Q\left(\sqrt{\sum_{i=1}^{d}\text{SNR}(\beta_{l,i})}\right) \quad (18)$$

where

$$\text{SNR}(\beta_{l,i}) = \frac{E^2[Z_{l,i}|\beta_{l,i}]}{\text{Var}[Z_{l,i}|\beta_{l,i}]} \geq \text{SNR}_{min}(\beta_{l,i}) \quad (19)$$

$$\mathrm{SNR}_{min}(\beta_{l,i}) = \frac{2\beta_{l,q}^2\bar{\gamma}}{(1+\bar{c}_l)(1+v_l^K\bar{\gamma})+\beta_{l,q}^2\bar{\gamma}\frac{c_l}{N}} \quad (20)$$

The c_l and v_l^K in the above equation is taken from the maximum value of $c_{l,q}$ and $v_{l,q}^K$ among the M carriers, respectively, i.e., from the center carrier ($c_l = c_{\hat{q}}$, and $v_l^K = v_{l,\hat{q}}^K$, for $\hat{q} = \lfloor (M+1)/2 \rfloor$). Since the Q function is monotonically decreasing, we obtain the following inequality

$$Q\left(\sqrt{\sum_{i=1}^{d}\mathrm{SNR}(\beta_{l,i})}\right) \leq Q\left(\sqrt{\sum_{i=1}^{d}\mathrm{SNR}_{min}(\beta_{l,i})}\right). \quad (21)$$

Now, we need to average the above equation over the random parameters, $\{\beta_{l,i}\}$'s. For the numerical convenience, we use the alternative representation for $Q(x)$ [9], [10], which is given by

$$Q(x) = \frac{1}{\pi}\int_0^{\pi/2}\exp\left(-\frac{x^2}{2\sin^2\theta}\right)d\theta. \quad (22)$$

With this representation, (16) is upper bounded by

$$P_{l,2}(d) \leq \frac{1}{\pi}\int_0^{\pi/2} I_{in}^d(\theta)\, d\theta \quad (23)$$

where

$$I_{in}(\theta) = \int_0^{\infty}\exp\{-\frac{1}{2\sin^2\theta}\mathrm{SNR}_{min}(\beta_{l,i})\}f_{\beta_{l,i}}(\beta_{l,i})d\beta_i \quad (24)$$

and $f_{\beta_{l,i}}(\beta_{l,i})$ is a pdf of the Rayleigh random variable. By a change of variable (letting $\mathrm{SNR}_{min}(\beta_{l,i}) = y$), (24) becomes

$$I_{in}(\theta) = \int_0^{\frac{1}{\nu}}\exp\{-\frac{y}{2\sin^2\theta}\}\left(\frac{u}{(1-\nu y)^2}\exp\{-\frac{uy}{1-\nu y}\}\right)dy \quad (25)$$

where $u = (1+\bar{c}_l)(1+v_l^K\bar{\gamma})/(2\bar{\gamma})$ and $\nu = c_l/(2N)$. Note that we have defined $N = MR_cT_b/T_c$. If we let the spreading gain of the uncoded system $N_u = MT_b/T_c$, then $N = R_cN_u$. Thus, the effective spreading gain N of the coded system is smaller than that of the uncoded system by a factor of R_c^{-1} for a given bandwidth.

D. Approximation of $P_{l,2}(d)$

In the multiple access spread spectrum systems, the spreading gain N is usually much larger than the number of users. In this case, (20) can be approximated by

$$\mathrm{SNR}_{min}(\beta_{l,i}) \approx \frac{2\beta_{l,q}^2\bar{\gamma}}{(1+\bar{c}_l)(1+v_l^K\bar{\gamma})} \quad (26)$$

and it can be shown that

$$P_{l,2}(d) \leq \frac{1}{2} - \frac{1}{2}\sum_{i=0}^{d-1}\binom{2i}{i}\frac{\lambda(1-\lambda^2)^i}{2^{2i}} \quad (27)$$

where

$$\lambda = \sqrt{\frac{\bar{\gamma}}{(1+\bar{c}_l)(1+v_l^K\bar{\gamma})+\bar{\gamma}}} \quad (28)$$

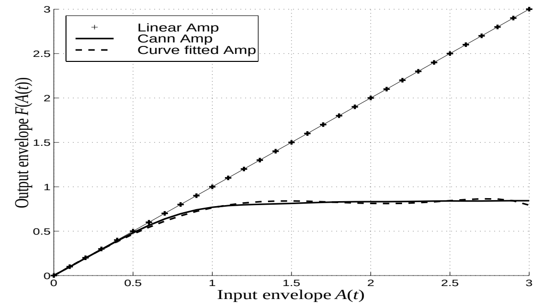

Fig. 1. AM/AM of the amplifier model

IV. Results

First, we define the output backoff (OBO) of user k as the ratio of maximum possible amplifier power to the average in-band output power of the amplifier,

$$\begin{aligned}\mathrm{OBO}_k &= \frac{1/2\max\{\mathcal{F}^2(A_k(t))\}}{\sum_{q=1}^{M}\{(H_k^{(1)})^2P_k + (\widetilde{H}_{k,q}^{(n)})^2P_k\}} \\ &= \frac{\max\{\mathcal{F}^2(A_k(t))\}}{2MP_k(H_k^{(1)})^2(1+\bar{c}_k)}. \end{aligned} \quad (29)$$

For a given saturation power, smaller OBO gives larger average output power but more signal distortion. Note that from the perfect power control assumption,

$$\mathrm{OBO}_l/\mathrm{OBO}_k = r_k^4/r_l^4. \quad (30)$$

The considered amplifier AM/AM is shown in Fig. 1 with α_1=1.000, $\alpha_3 = -0.274$, $\alpha_5 = 0.0394$, $\alpha_7 = -0.002$, and $\alpha_k = 0$ for $k \geq 7$. These coefficients are obtained by curve fitting (least square) Cann's bandpass solid state amplifier model used in [11]. Figs. 2–4 are obtained with $K = 30$ users, $M = 10$ carriers, $N_u = 160$ spreading gain, and convolutional code of rate $R_c = 1/2$ with constraint length 7. Two deterministic locations (r_1 and r_2 from the base station with $r_2/r_1 = 20$) are considered as in [5] with three possible configurations : (1) user A is at r_1 and 29 other users are at r_2, (2) user A is at r_2 and 29 other users are at r_1, and (3) all 30 users are at r_2. Note that users at r_1 should operate in more linear region than the users at r_2 from the power control requirement (30).

Figs. 2 and 3 show the upper bound on the BER of each user for different OBOs, when all users are at r_2 in the AWGN and fading channel, respectively. As expected, the BER increases as the OBO decreases because of more signal distortion. In the fading channel, the BER decay rate is not as steep as in the AWGN channel, resulting in a higher BER than for the AWGN channel. Also note that a BER floor occurs for large E_b/N_0 because of the multi-user interference.

The vertical axis of Fig. 4 is the increase in required E_b/N_0 of user A needed to meet a target BER of 10^{-4} in AWGN and fading, compared to the required E_b/N_0 with a perfectly linear amplifier. This is denoted as E_b/N_0 degradation. This E_b/N_0 degradation is plotted over the

OBO of the users at r_2. The performance in AWGN degrades only when user A has low OBOs. The additional degradation from the other users' low OBOs is minimal. However, note that the other users' low OBOs will degrade their own BER performance. This suggests that the main source of the performance degradation of user A is the self nonlinear interference of user A (low OBO of user A) and not the nonlinear interference of other users. This is also found to be the case for uncoded systems in [5]. Similar results are obtained for the fading channel except that the performance degradation at low OBOs is a little more sensitive to other users' low OBOs than in AWGN channel. This is because our considered BER (10^{-4}) is near the BER floor region so that a slight increase in BER from additional nonlinear effects will cause relatively large E_b/N_0 degradation, unlike in the AWGN case.

In summary, the additional performance degradation of the considered systems due to the nonlinearities, in terms of increase in E_b/N_0 required for the target BER of 10^{-4} is as large as 2.2 dB and 6.2 dB in the AWGN and fading channel, respectively when the amplifier OBO is close to 1 dB.

V. Conclusion

The bit error rate performance of single cell convolutionally coded MC-CDMA systems has been obtained analytically in the presence of multipath fading and nonlinearities.

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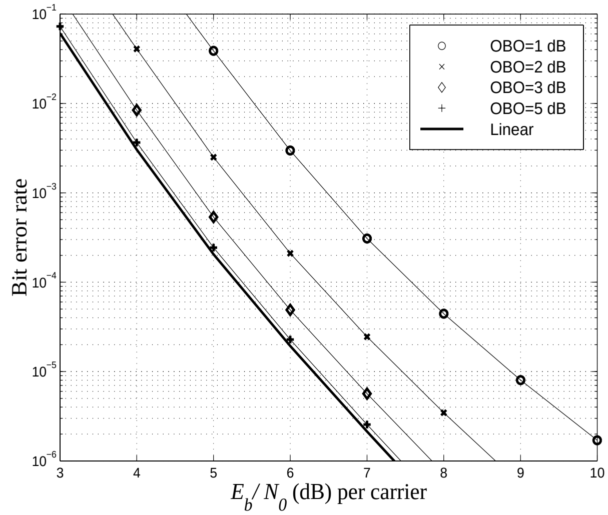

Fig. 2. BER when all users are at r_2: AWGN

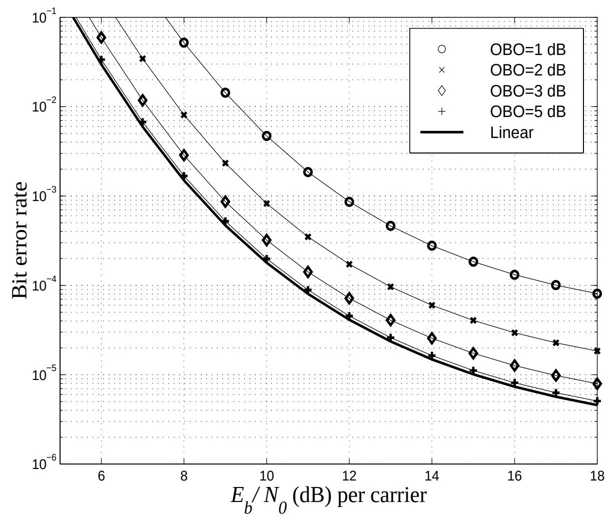

Fig. 3. BER when all users are at r_2: fading

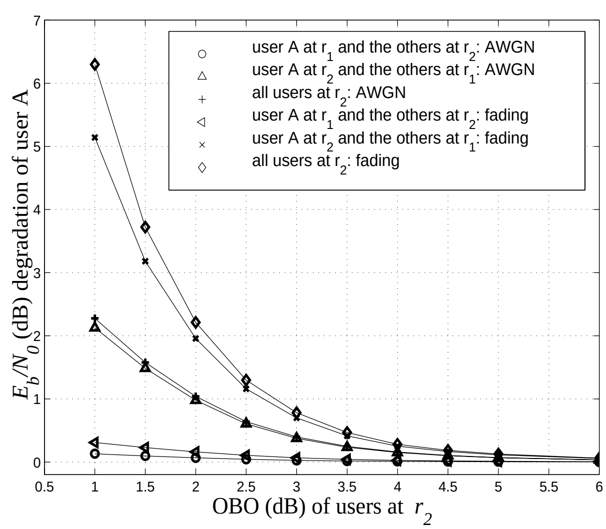

Fig. 4. E_b/N_0 (dB) degradation of user A