# MMSE Based Fully Distributed Power Control Algorithm

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*Abstract*— **In this paper, a fully distributed power control algorithm (PCA) based on the minimum mean-squared error (MMSE) receiver is introduced. We study the performance of a synchronous MMSE based DS-CDMA system in which the proposed PCA is implemented. The receiver filter coefficients are obtained using the Wiener solution or an adaptive algorithm. We also study the convergence of the SINR and the total transmitted power and we compare, in terms of the capacity, the performance of a system in which the proposed power control has been implemented with a system without power control.**

## I. INTRODUCTION

The main reason for using power control in a conventional receiver based DS-CDMA system is to combat the near-far problem which occurs when an undesired user's signal overpowers the desired user's signal. The Minimum Mean Square Error (MMSE) receiver is shown in Figure 1. This receiver, which has been presented in [1], [2], [3], and others, is known to be near-far resistant but power control can still be used to reduce multiuser interference, increase the system capacity, compensate for channel loss, as well as to minimize the transmitted power and hence prolong the battery life.

As shown in [1], the MMSE receiver can achieve many of the performance levels of other multi-user receivers without the need for side information like user sequences, clock offsets, and the received powers of all the interfering signals. This receiver offers a strong potential for capacity improvement over a conventional receiver-based CDMA system.

To understand the advantages of the MMSE receiver, we need to describe briefly how it works. The received signal which consists of the desired user's signal, MAI, and Gaussian noise is fed at the chip rate into the equalizer until the N-tap delay line becomes full. After one symbol time, the equalizer contents are correlated with the tap weights, $\mathbf{a}$, and the result of this correlation is used to make a decision about which symbol was sent. These tap weights are updated every symbol interval to minimize the mean square error between the output of the filter and the desired output. In practice, the filter is trained for a reasonable period of time by a known training sequence to reach a tap weight vector that is close to the optimum weights. After the training period, the receiver switches to decision feedback mode.

In an MMSE receiver based CDMA system, it is natural to develop a power control algorithm based on the mean square error (MSE) at the output of the filter. In fact, since the MMSE receiver is near-far resistant, one can increase the transmitted power of a user experiencing a low SINR without having a major effect on other CDMA users. Likewise, one can decrease the transmitted power of a user enjoying a high SINR to conserve battery life and to decrease adjacent cell interference for other cells. The filter coefficients of the MMSE receiver can be obtained analytically by the Weiner solution and practically by adaptive algorithms like the Least Mean Square (LMS) and the Recursive Least Square (RLS). The major problem with many of the power control techniques presented in the literature is the need, with varying degree, for side information such as channel gains, spread sequences, bit error rate, received powers and the SINRs of all users.

In this paper, a fully distributed PCA is presented that is based on the MMSE. We study the capacity improvements that can be gained by an MMSE receiver-based CDMA system implementing this power control algorithm. We investigate the performance of this power control algorithm when the MMSE receiver filter coefficients are obtained through the Weiner solution or adaptive algorithms like the LMS and the RLS. We also look at the convergence of the SINR and the total transmitted power. In addition we study the practical implementation of the proposed power control algorithm. In [4], a power control algorithm for an MMSE multiuser receiver was presented where the measurements of the MSE of a given user are communicated to the corresponding transmitter which updates the power accordingly. A power control algorithm was proposed in [5] for a multiuser detector in which the performance of conventional and MMSE receivers are compared when the proposed power control algorithm is used. The PCA presented in this paper and the ones presented in [4] and [5] converge to the same transmitted power solution.

## II. SYSTEM MODEL

The system studied in this paper consists of a single cell of K users in an asynchronous complex baseband DS-CDMA system. All transmissions use BPSK modulation and the processing gain (N) is 31. The mobiles and the base station are assumed to be using the MMSE receiver and the same notation

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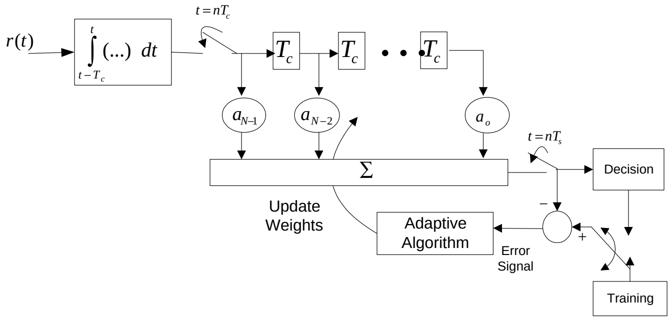

Fig. 1. The MMSE receiver structure

presented in [1] is used. The $ith$ user's complex baseband signal is given by

$$s_j(t) = \sqrt{p_j}\, d_j(t) c_j(t) \tag{1}$$

where $p_i$ is the transmitted power, $d_i(t)$ is a polar baseband data format, $c_i(t)$ is the spreading waveform of user $i$ and is assumed to be in a polar form with period N and chip interval $T_c$. The received signal consists of the transmitted signals of all users delayed by an amount $\tau_i$ which is uniformly distributed over a bit interval, $T_b$, plus a complex Gaussian noise with zero mean and variance of $\sigma^2$. The received signal at the $ith$ receiver is given as

$$r_i(t) = \sum_{j=1}^{K} \sqrt{h_{ij}}\, s_j(t-\tau_j) + n(t) \tag{2}$$

where $h_{ij}$ is the channel gain of user $j$ to the assigned base station of user $i$. The contents of the N-tap delay line (TDL) $\mathbf{r}_i(m)$ is given by

$$\mathbf{r}_i(m) = \sqrt{p_i(m)}\sqrt{h_{ii}}\, d_i(m)\mathbf{c}_i + \text{MAI} + \mathbf{n}(m) \tag{3}$$

where

$$\begin{aligned} \text{MAI} = \sum_{j \neq i}^{K} \Big[ & \sqrt{p_j(m)}\sqrt{h_{ij}}\, d_j(m)\widetilde{\mathbf{f}}_j(l,\delta) \\ & + \sqrt{p_j(m-1)}\sqrt{h_{ij}}\, d_j(m-1)\widetilde{\mathbf{g}}_j(l,\delta)\Big] \end{aligned} \tag{4}$$

In the above equation, $\tau_j = l_j T_c + \delta_j$ where $l_j$ is an integer and $0 \leq \delta_j < T_c$. The vectors $\widetilde{\mathbf{f}}_j$ and $\widetilde{\mathbf{g}}_j$ are defined as follows

$$\widetilde{\mathbf{f}}_j(l,\delta) = \frac{\delta}{T_c}\mathbf{f}_j(N-l-1) + \left(1 - \frac{\delta}{T_c}\right)\mathbf{f}_j(N-l)$$

$$\widetilde{\mathbf{g}}_j(l,\delta) = \frac{\delta}{T_c}\mathbf{g}_j(N-l-1) + \left(1 - \frac{\delta}{T_c}\right)\mathbf{g}_j(N-l)$$

where

$$\mathbf{f}_j(l) = (\mathbf{c}_j^{(l)} + \widehat{\mathbf{c}}_j^{(l)})/2$$

$$\mathbf{g}_j(l) = (\mathbf{c}_j^{(l)} - \widehat{\mathbf{c}}_j^{(l)})/2$$

$$\mathbf{c}_j^{(l)} = (c_{j,N-l}, c_{j,N-l+1}, \ldots, c_{j,N-1}, c_{j,0}, c_{j,1}, \ldots, c_{j,N-l-1})^T$$

$$\begin{aligned} \widehat{\mathbf{c}}_j^{(l)} = (& -c_{j,N-l}, -c_{j,N-l+1}, \\ & \ldots, -c_{j,N-1}, -c_{j,0}, c_{j,1}, \ldots, c_{j,N-l-1})^T \end{aligned}$$

The output of the MMSE receiver filter corresponding to the $jth$ user is

$$z_j(m) = \mathbf{a}_j^H \mathbf{r}_j(m) \tag{5}$$

where $\mathbf{a}_i$ is the filter coefficient vector that correspond to the $ith$ user's received signal. The optimum tap weights, $\mathbf{a}_i$, can be obtained as

$$\mathbf{a}_j = \mathbf{R}^{-1}\mathbf{P}_j \tag{6}$$

The autocorrelation matrix, $\mathbf{R}$, of the equalizer contents is defined as $\mathbf{R} = E\left[\mathbf{r}(m)\mathbf{r}^H(m)\right]$ and the correlation between the desired user response and the received signal is given by $\mathbf{P}_j = E\left[d_j(m)\mathbf{r}(m)\right]$.

## III. FULLY DISTRIBUTED POWER CONTROL ALGORITHM

Power control algorithms are based on the fact that the SINR at the receiver is directly proportional to the desired user's transmitted power and inversely proportional to the sum of the interfering signals' transmitted powers. The goal of power control algorithms is to equalize the SINR to minimize the total transmitted power in the system. This reduces the interference level in the CDMA system and hence increases the capacity. In general, power control algorithms are classified as centralized or distributed power control algorithms. In a centralized algorithm, there is a controller that has complete knowledge of all active radio links and their terminal powers [6] and is responsible for adjusting the transmitted powers at the transmitting terminals. On the other hand, in a distributed power control algorithm, each radio link adjusts its own transmitted power based on its own measurements [7].

In this section, we present a fully distributed power control algorithm. Define the desired MMSE ( $\overline{\text{MMSE}}$ ) as the value of the MMSE which corresponds to the maximum desired SINR. This relation is given [3] as

$$\text{MSINR}_i = \frac{1}{\text{MMSE}_i} - 1 \tag{7}$$

The SINR for the $ith$ user is given by

$$\text{SINR}_i = \frac{p_i h_{ii}\left|\mathbf{a}_i^H \mathbf{c}_i\right|^2}{\left|(\mathbf{a}_i^H \text{MAI})\right|^2 + 2\sigma^2(\mathbf{a}_i^H \mathbf{a}_i)} \tag{8}$$

Let $\overline{\text{SINR}}$ be the target SINR for a required performance. We define the capacity of the system as the number of users the system can support for a given $\overline{\text{SINR}}$. The MMSE is obtained by the Wiener solution for the tap weights as described in [8], [1], [2], [3]. The MMSE is given by

$$\text{MMSE}_i = 1 - \sqrt{p_i}\sqrt{h_{ii}}\mathbf{a}_i^H\mathbf{c}_i \tag{9}$$

From Eqn. 9, we can write the transmitted power in terms of $\overline{\text{MMSE}}$, the tap wieghts, and the spreading sequence as follows

$$p_i = \frac{\left(1 - \overline{\text{MMSE}}\right)^2}{h_{ii}\left|\left(\mathbf{a}_i^H\mathbf{c}_i\right)\right|^2} \tag{10}$$

Then, we can update the transmitted power according to the following

$$p_i(n+1) = \frac{\left(1 - \overline{\text{MMSE}}\right)^2}{h_{ii}\left|\left(\mathbf{a}_i^H(n)\mathbf{c}_i\right)\right|^2} \tag{11}$$

It is clear that the transmitter needs to know $\left(\mathbf{a}_i^H\mathbf{c}_i\right)$ and $h_{ii}$ to update its power. The value of these terms can be calculated by the receiver and then sent to the transmitter. The channel gain estimation can be approached as follows. The transmitter sends a pilot symbol at the beginning of each transmission period with a known power. The receiver uses the output of the MMSE receiver that corresponds to these pilot symbols to get a noisy estimate of the channel gain as follows

$$z_i = d_i\sqrt{p_i}\sqrt{h_{ii}}\mathbf{a}_i^H\mathbf{c}_i + \tilde{\mathbf{n}} \tag{12}$$

where $\tilde{\mathbf{n}}$ consists of the output of the filter due to the noise and the multiuser interference. Let the denominator of eqn. (11) be $\eta$. Assuming the pilot symbol transmitted power is 1 watt, a noisy estimate of $\eta$ can be given by

$$\eta = h_{ii}\left|\left(\mathbf{a}_i^H(n)\mathbf{c}_i\right)\right|^2 \approx \left|(d_i z_i)\right|^2 \tag{13}$$

The value of $\eta$ is then sent to the transmitter. The transmitter uses this value to update its transmitted power according to eqn. (11).

## IV. NUMERICAL RESULTS

In this section, we present some simulation results for a single-cell synchronous MMSE receiver-based DS-CDMA system using the MMSE-based PCA presented here. The system model used in this simulation has a channel gain modeled as a lognormal distribution with 8 dB standard deviation, a processing gain of 31, and a noise variance of 1. The transmitted powers of all users are initialized to 1. Figure 2 shows the convergence of the SINR and the total transmitted power for the system when the PCA presented in the previous section is implemented with the tap weights of the MMSE receiver obtained using the Wiener solution. There are 10 users in the system all with a target SINR of 10 dB. All users' SINRs converge to the $\overline{\text{SINR}}$ in a few iterations.

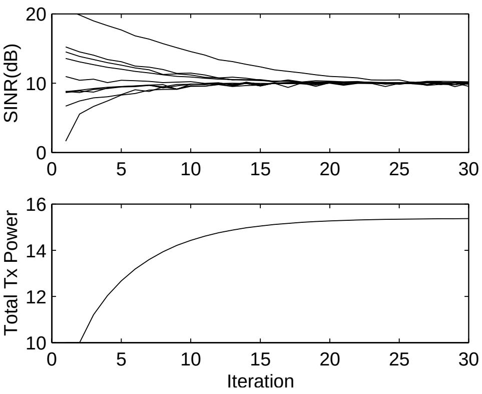

Fig. 2. A typical SINR and total transmitted power for MMSE receiver based CDMA system with PCA for 10 users and $\overline{\text{SINR}}$ = 10 dB.

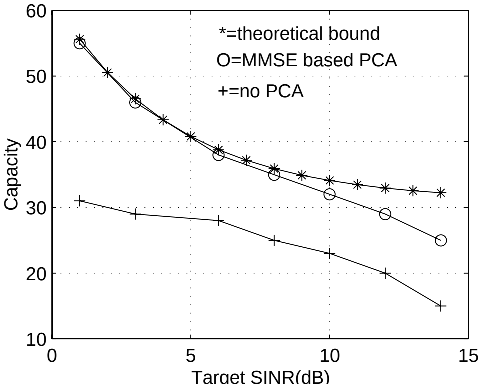

Fig. 3. The capacity improvement due to the use of the proposed power control algorithm.

To evaluate the advantage of implementing the proposed PCA, a comparison is made between two MMSE receiver-based CDMA systems in terms of their capacities for different values of $\overline{\text{SINR}}$. One system uses the PCA presented here while the other transmits with a constant power. For the system with a constant total transmitted power (no power control is used), the total transmitted power is taken to be the mean of the total transmitted power for an MMSE-based power control system.To illustrate the improvement offered by using power control, the results of this comparison are shown in Figure 3. As shown in Figure 3, on average about 52% improvement in the capacity can be gained by using the power algorithm presented in this paper. To obtain these results the weights of the MMSE receiver were found using the Wiener solution as given in Eqn. (6). and the blocking probability (the probability that SINR is less than the target SINR) was 2%. These simulation results shown in Figure 3 are found to be in agreement with the theoretical upper bound given in [9] despite the fact that, for our results, random rather than optimal sequences have been used but at the expense of increasing the power.

The power control algorithm performance in adaptive envi-

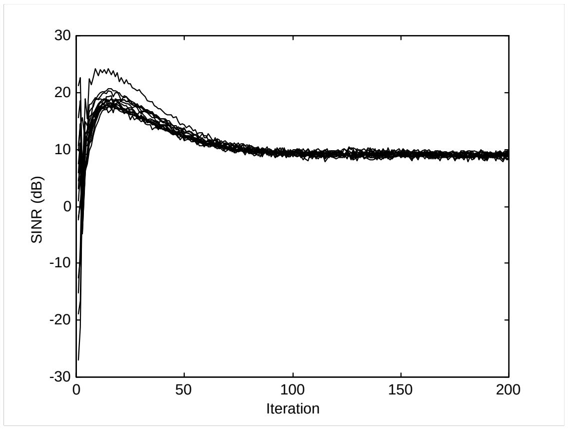

Fig. 4. A typical SINR convergence $\overline{\text{SINR}}$ = 9.5 dB for 15 users using LMS algorithm

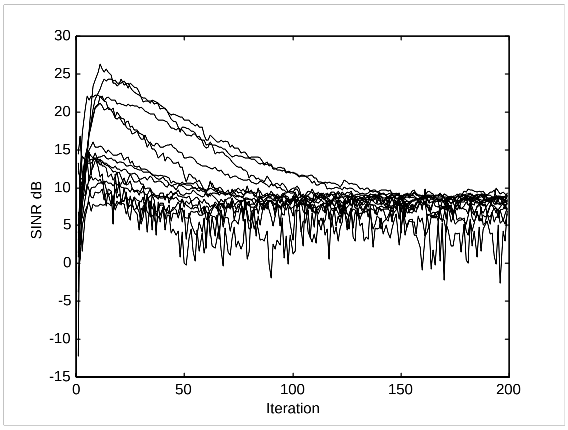

Fig. 5. A typical SINR convergence $\overline{\text{SINR}}$ = 9.5 dB for 15 users using RLS algorithm

ronments in which the LMS and RLS algorithm are used to update the filter weights was also studied and the results are shown in Figures 4, and 5. As expected, the convergence of the SINR in the adaptive cases is slower than when the receiver filter tap weights are obtained by the Wiener solution. The SINR converges to a value close to, but not exactly equal to, the target SINR due to the fact that the proposed power control algorithm has been developed assuming the tap weights of the filter were obtained by the Weiner solution. Simulations show that the LMS algorithm has a better tracking capability than that of the RLS algorithm for such nonstationary environments where the signal power is changing. This tracking superiority of the LMS may be attributed to the fact there is an inherent dependence of the step size of the LMS algorithm on the total power of the input to the adaptive filter. An adaptive step size based on the total power of the tap weights has been used to obtain Figure 4. The forgetting factor for the RLS algorithm used in Figure 5 is 0.99.

In the previous figures, the channel gain and the parameter $\left(\mathbf{a}_j^H \mathbf{c}_i\right)$ are assumed to be known exactly by the transmitter. Figure 6 shows the convergence of the SINR and the total transmitted power when $\eta$ is estimated using eqn. (13) at the receiver and the value of the estimates are then sent to the transmitter. In this case the estimates of $\eta$ are updated every $10th$ symbol. The results here show that the PCA can be implemented practically and only an estimate of the denominator of eqn. (11) needs to be sent to the transmitter. Practically, the estimates of $\eta$ need to be quantized and then sent to the transmitter. The accuracy of these estimates depends on the overhead that can be tolerated by the system.

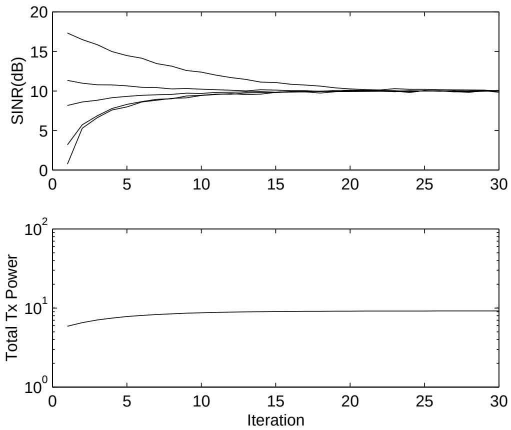

Fig. 6. A typical SINR and total TX power convergence for a practical implementation of the PCA for 5 users

## V. CONCLUSION

In this paper, we have shown that despite the fact that the MMSE receiver is near-far resistant, its performance can be improved by using power control. We introduced a fully-distributed power control algorithm based on a desired MSE value for an MMSE receiver based-CDMA system. By using the power control presented in this paper, the capacity of the system was increased and the transmitted power was minimized and hence the interference caused by the other users was reduced. On average, a capacity improvement of more than 50% can be gained.

The convergence speed of the power algorithm varies depending on the way the tap weights are updated. The convergence and tracking performance of the LMS algorithm are superior to those of the RLS algorithm. This may be due to the fact that the step size of the LMS algorithm is updated for each power update while the RLS parameter is kept constant. An adaptive step size for the LMS is essential to improve the tracking capability of these adaptive algorithms. The tracking of the RLS is very sensitive to the frequency of updating the power. Compared to the LMS, the RLS can not keep up with very frequent updates of the power. One may resort to an adaptive memory RLS or Kalman filtering theory to improve the performance of the RLS algorithm. Haykin in [8] presents a detailed study of the tracking performance of these algorithms.

A practical implementation of the power control algorithm has been suggested in this paper. This implementation is based on estimating the channel gain by the receiver. The convergence of the SINR for the practical implementation is shown to be comparable to that of the ideal PCA where all the parameters needed to update the power are available at the transmitter.

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