Performance Evaluation of the Forward Link of a CDMA Cellular System

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***Abstract* – In this paper, we present the results of evaluating the performance of the forward link of a CDMA cellular radio system. We use a semi-analytical approach consisting of the waveform level simulation of the analog channel followed by the hidden Markov modeling of the discrete channel. The performance parameters of the discrete channel were evaluated analytically using the hidden Markov model and compared with the results of simulation.**

I. INTRODUCTION

The purpose of this study was not to evaluate the performance of a specific system but rather to illuminate the use of novel simulation techniques that improve the accuracy of the simulation and/or make it faster. In this study we simulated the forward (base to terminal) link of a generic CDMA system which is closely related to IS-95 [1]. The reverse link system simulation was described in our earlier paper [2].

Two distinct simulation techniques were used to evaluate the performance of the system: waveform level simulation and discrete level simulation. The waveform level simulation is used to evaluate the performance of a system composed of modulators, channels, filters etc. It is used to study the effect of system parameters on overall performance. In this study, the simulation was used to evaluate the uncoded performance of the system as a function of time-varying fading channels, co-channel interference and noise. The system is assumed to be synchronized (i.e. acquisition and tracking were not simulated).

However, the waveform level simulation is computationally very expensive for evaluating the performance of the system digital signal processing components such as error correction coders, interleavers and voice coders. To simulate efficiently such systems, we need to use the discrete system simulation. We use Hidden Markov Models (HMMs) for the discrete level simulation in which the channel is represented by a probabilistic function of a Markov chain states. The waveform level simulation is used to characterize the discrete channel HMM. The output of the waveform level simulation is a time series of bit-errors that is long enough to represent accurately the statistical properties of the discrete channel. This time series is used to estimate parameters of the HMM.

II. FORWARD RADIO LINK SIMULATION

The forward radio link is the communication channel from the base station to each individual mobile user in a cell. A CDMA system consists of a voice encoder, channel encoder, interleaver, the radio link, deinterleaver, channel decoder and voice decoder [1]. The input signal is the output of the voice coder with data rate of 9.6 kb/s. This data is then encoded by a rate-1/2 convolutional code, resulting a coded data rate of 19.2 kb/s. The interleaver frame is 20 ms long. The data in each link to an individual user in a cell is spread by a specific Walsh function of order of 64 to ensure the selectivity of this user. Since every user in the cell is synchronized, the orthogonality of Walsh functions helps to mitigate both in-cell and out-of-cell interference. Therefore, the only interference detected are the residuals of the multipath fading and the out-of-cell signals. The modulated signal is then spread by a short PN sequence at 1.2288 Mega-chips per second (Mc/s) with a period of $2^{15}-1$ both in-phase and quadrature. The system is based on the IS-95, but the simulation methodology can be applied to modeling other communication systems.

The forward link is modeled as a multiplexed QPSK signal with coherent detection. The mobile channel is modeled by three multipath rays, each with an independent Rayleigh fading. The motion of the vehicle is modeled by a Doppler spread of 200 *Hz* which corresponds to a mobile traveling at 67 mph. The received signal is filtered and coherently detected by a Rake detector. Acquisition and tracking are not simulated, thus synchronization and phase rotation of the channel are derived from the channel. In this simulation we employ two distinct error estimation techniques where only the signal is simulated, and the effect of noise and interference to the system is analyzed during post-processing. These techniques will speed-up simulation for evaluating frame error rate under various $E_b/(N_o+I)$ for the described system. The discrete level modeling was then used to characterize the waveform level simulation channel. The performance effects of the

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discrete components such as coders, interleavers etc. were not evaluated. If a very large interferer from another cell is present, its effect on the system must be modeled and simulated together with the entire system.

The transmitter input is a coded voice signal, $S_I(t)$, with a bit rate of 19.2 kb/s, represented by random binary data. $S_I(t)$ is sampled at 8 samples/bit, resulting in a sampling rate $f_{s,i} = 153.6$ samples/s. This signal is then up-sampled and interpolated by a factor of 64 , and then spread by a complex pseudo-random (PN) sequence at 1.2288 *Mc/s*. The PN sequence is composed of two uncorrelated PN sequences, PN_1 and PN_2, each with maximal length sequence with period of $2^{15} - 1$, thus generating the two rails of the QPSK complex signal. Each chip in the PN sequences is then sampled at a rate of 8 samples/chip, resulting in a system signal sampling rate $f_{s,sys} = 9.8304$ Msamples/s. This signal is filtered by a square-root raised-cosine filter, $h_T(t)$, with .35 roll-off. The transmitted signal is

$$S_T(t) = S_I(t)(PN_1(t) + jPN_2(t)) * h_T(t)$$

The wireless propagation channel model selected for this simulation is one of the channels proposed by Telecommunication Industrial Association (TIA) Subcommittee TR45.4 for Personnel Communications Systems (PCS). The channel is the vehicular outdoor channel model with 3 Rayleigh paths, with 0, 1.5 and 14.0 μs delay respectively. The Doppler frequency is 200 *Hz*. The complex lowpass equivalent channel impulse response is,

$$c(\tau,t) = \sum_{i=1}^{3} \alpha_i(t) e^{j\Phi_i(t)} \delta(\tau - \tau_i) = \sum_{i=1}^{3} c_i(t)$$

where $c_i(t)$, τ_i , α_i and Φ_i are the i-th ray's signal, delay, amplitude and phase respectively. The relative strength of the rays is 0 dB, -3 dB and -6 dB. A complex Gaussian signal with a flat spectrum $P_f(f)$ is generated by the signal generator. This signal is shaped by a filter with a frequency response $H_i(f)$ to have Jakes normalized power spectrum [16] $P_j(f)$.

$$P_j(f) = 1 / [\pi f_d \sqrt{1 - (f/f_d)^2}\,]$$

and

$$H_i(f) \;=\; \sqrt{P_j(f)}$$

The modeling of $H_i(f)$ is described in [2].

The received signal for the i-th ray , $S_R(t)$, is filtered with a mirrored square-root raised-cosine filter of the transmitter and coherently detected.

$$S_{R_i}(t) = [S_T(t) * c_i(t)] * h_R(t)$$

where $h_R(t)$ is the transfer function of the receive filter. In reality, synchronization and carrier phase for the coherent receiver are estimated from a transmitted pilot signal common to all users. In this simulated system the carrier phase and the signal delays for each Rake finger are estimated from the simulated channel. In the phase and delay estimator, each ray's phase rotation is estimated and the phase of received signal is rotated by $\Phi_i(t)$ degrees to compensate for the channel effect on the i-th multipath ray. In the coherent Rake detector, $S_R(t)$ is despread with the appropriate delayed PN sequences for the in-phase and quadrature.

$$S_{D_i}(t) = S_{R_i}(t) e^{-j\Phi_i(t)} (PN_1(t - \tau_i) - jPN_2(t - \tau_i))$$

where *i* is the index of the appropriate Rake finger. The Rake detector delays each i-th ray by $\tau_{\max} - \tau_i$ so as to align all three multipath components, where τ_i is the delay for the i-th ray in the channel and $\tau_{\max} = \max(\tau_1, \tau_2, \tau_3)$. The signals are then added and down-sampled by a factor of 64. where *i* is the index of the appropriate Rake finger. The Rake detector delays each i-th ray by $\tau_{\max} - \tau_i$ so as to align all

$$S_D = \sum_{i=1}^{3} S_{D_i}(t - (\tau_{\max} - \tau_i))$$

S_D is then detected by an integrate-and-dump detector. The detected signal is stored in a file for post-processing.

III. Performance Evaluation

The error rate performance of the forward link is affected by interference and noise. Since the receiver is linear, the noise at the decision point can be regarded as additive white and Gaussian (AWGN). The interference from the residual multipath rays of the in-cell signals, and out-of-cell other base station signals can be regarded as a sum of many small interferers. Therefore, the interference can also be approximated by an AWGN at the decision point. Thus, we can simulate the system without injecting noise and interference at the receiver, while the effect of noise and interference is evaluated during post-processing.

There are two methods that this can be accomplished: the *direct method* and the *quasi-analytic method*. In the direct method, the received analog signal at the decision point V_R is added to the noise and interference sample $V_{n,i}$ generated by the Gaussian number generator with mean $\mu = 0$ and standard deviation σ.

$$\tilde{V}_R(k,t) = V_R(k,t) + V_{n,i}(k)$$

where $\tilde{V}_R(k,t)$ is the received k-th symbol sampled at relative time *t*, and $V_{n,i}$ is noise and interference sample for the k-th symbol.

The resultant signal $\tilde{V}_R$ is compared with the transmitted signal V_T. An error $E_k = 1$ is generated when these two signals are of opposite polarity, otherwise

$E_k = 0$. E_k is the error indicator for the k-th symbol.

In the quasi-analytic *(QA)* method, symbol error calculation [8] is a hybrid in that it combines both simulation and analysis. The analog output of the simulated system is stored in a file. The simulation is used to generate a noiseless waveform and assuming that the noise is AWGN, one can calculate the probability of error with the following formula for BPSK signal at the decision point.

$$P(k) = 0.5 erfc(|V_R(k,t)| / \sqrt{2}\ \sigma)$$

for $V_R(k,t) V_T(k,t) \geq 0$ and

$$P(k) = 1 - 0.5 erfc(|V_R(k,t)| / \sqrt{2}\ \sigma)$$

for $V_R(k,t) V_T(k,t) < 0$. It is common to express the argument of the error complementary function as E_b/N_o, where $E_b/N_o = (P_{av} T_b B_n)/\sigma^2$, E_b is the energy per bit, N_o is the noise power spectral density, and P_{av}, T_b, B_n are the average signal power, bit duration and noise bandwidth respectively.

To provide the temporal distribution of errors, we divide the received signal into segments of duration of T_s small enough so that they are correlated. The coherence time T_c (the channel is assumed to be locally time invariant) is computed from the space-time correlation function $R(\tau)$. In our simulation, we use Jakes channel model where $R(\tau)$ is the Fourier Transform of $P(f)$.

$$R(\tau) = F(P(f)) = J_0(2\pi f_d \tau)$$

where $J_o(\cdot)$ is the zero-th order Bessel function. A popular rule of thumb of the coherence time is [8] $T_c = 0.5/f_d$. The number of symbols within the coherence time is $N_s = T_c R_s$, where R_s is the symbol rate.

The performance measure of the system is the probability of frame error rate (FER). The frame size is equal to the interleaver depth. We assume that $j-1$ errors per frame can be tolerated by the system, because of interleaving and forward error correction. Thus, we are interested in estimating p_j, the probability of occurrence of j or more errors per frame. A good rule of thumb for estimating p_j in a simulation is $\tilde{p}_j = n_j/N$, where n_j is the number of frames with j or more errors, and N is the total number of frames.

The confidence limits of p_j as a function of N are described in [8, p 500]. p_j has a binomial distribution $B(N, n_j, \tilde{p})$. For n_j large enough this can be approximated with a Gaussian distribution. The independence of the errored frames is assumed if the time between error events is large enough so that the correlation between errored frames is small. This time duration, T_{uc}, can be estimated from the space-time correlation function $R(\tau)$ as described in the previous section. A good rule of thumb is $T_{uc} = 10/\pi f_d$ (in our simulation, $T_{uc} = 306 bits$). If $T_{uc} > T_f$, as is often the case, we segment the error sequence into frames by dropping out $T_{uc} - T_f$ number of bits to form every two consecutive uncorrelated frames.

IV. SIMULATION RESULTS

For each bit-error rate P_k obtained by QA at k-th symbol decision point, a sample is produced by the uniform random number generator $U(0,1)$. The random number is compared with P_k; if $U \leq P_k$, an error is generated so $E = 1$, otherwise $E = 0$. This process is repeated for every symbol of the entire recorded data file. The resulting error sequence is then used in discrete level modeling.

The advantage of this method is that the noiseless system only needs to be simulated once. The error sequences for various E_b/N_o can be generated rapidly by using the QA technique described above or by the direct method.

The interference from both in-cell and out-of-cell of the forward link are modeled as Gaussian noise. Since the receiver is linear, noise can be injected at the decision point when the recorded data file is processed. In doing so, it speeds up the simulation dramatically. Instead of simulating the end-to-end analog system many times for a set of E_b/N_o values, with direct method or QA, one can simulate the system once and post-process the data for any specific E_b/N_o. We used the direct method to process data. The application of the QA method gave the same result.

Since bit error rate ratios are usually small, it is convenient to record intervals between consecutive errors rather than the sequence of zeros and ones. These error sequences are then used in the discrete level modeling by using HMMs as described in the following section.

V. DISCRETE CHANNEL CHARACTERIZATION

In this section, we present the results of fitting HMMs to the results of the waveform level simulation of the communication system described in Section 2.1.5. For the binary channel, an HMM is defined as $\{S, \boldsymbol{\pi}, \mathbf{A}, \mathbf{B}(e)\}$ where $S = \{s_1, s_2, \ldots, s_n\}$ is the set of the Markov chain states, $\boldsymbol{\pi}$ is a vector of state initial probabilities, $\mathbf{A} = [a_{ij}]_{n,n}$ is a matrix of state transition probabilities ($a_{ij} = Pr\{s_j | s_i\}$), and $\mathbf{B}(e) = diag\{b_j(e)\}$ is a diagonal matrix of the bit-error conditional probabilities ($b_j(e) = Pr\{e | s_j\}$, $e = 1$ for an error and $e = 0$ for the correct bit). Matrices $\mathbf{B}(e)$ are completely characterized by the conditional probability vector $\boldsymbol{\varepsilon} = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)$ where $\varepsilon_i = b_i(1)$.

If $n = 2$, the HMM is called the Elliott-Gilbert ([5], [6]) model which can be interpreted as a channel with two states: G (for "good") and B (for "bad"). The model state transition probability matrix has the form

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

In the good state, errors occur with small probability $b_1(1) = \varepsilon_1$ while in the bad state the bit error probability $b_2(1) = \varepsilon_2$ is larger. Thus we have

$$\mathbf{B}(0) = \begin{bmatrix} 1-\varepsilon_1 & 0 \\ 0 & 1-\varepsilon_2 \end{bmatrix}, \quad \mathbf{B}(1) = \begin{bmatrix} \varepsilon_1 & 0 \\ 0 & \varepsilon_2 \end{bmatrix}$$

These two diagonal matrices are completely characterized by the conditional probability vector $\boldsymbol{\varepsilon} = (\varepsilon_1, \varepsilon_2)$.

There are many methods for estimating the model parameters [9]. The most popular is the Baum-Welch algorithm [3] which is a special case of the EM algorithm [4].

The standard Baum-Welch algorithm can be applied only to short sequences, because the number of computations and memory requirement are proportional to the number of samples. However, in the case of the channel errors, we usually observe long error-free intervals. In this case, the algorithm can be modified to take advantage of the fast matrix exponentiation when there are long stretches of identical observations ([14], [11], [13]).

After applying the modified Baum-Welch algorithm to the results of the waveform level simulation we obtained the following models

TABLE I

TWO STATE HMM PARAMETERS

$\frac{E_b}{N_0}$	4dB	6dB
$\boldsymbol{\pi}$	0.985002 0.014998	0.991005 0.008995
$\mathbf{A}$	0.996330 0.003670 0.240474 0.759526	0.997357 0.002643 0.290565 0.709435
$\boldsymbol{\varepsilon}$	0.000000 0.217517	0.000000 0.220236

We have also estimated parameters of three and four state models. The parameters of the three state model are presented in the following table The number of states of the Markov chain estimation is a difficult task [10], [15]. The problem of selecting the number of states is a special case of the problem of parametric complexity of a statistical model. This problem is usually solved using the confidence limits.

TABLE II

THREE STATE HMM PARAMETERS

$\frac{E_b}{N_0}$	4dB	6dB
$\boldsymbol{\pi}$	0.987199 0.003649 0.009152	0.993116 0.002284 0.004600
$\mathbf{A}$	0.997031 0.001368 0.001601 0.334190 0.585774 0.080035 0.186705 0.018046 0.795249	0.998045 0.001047 0.000908 0.426803 0.521940 0.051257 0.212247 0.010233 0.777520
$\boldsymbol{\varepsilon}$	0.000206 0.352411 0.189159	0.000201 0.373519 0.207702

On the other hand, the number of states can be selected depending on the model application. The following examples demonstrate that a two-state Gilbert model is quite adequate for calculating probability distributions of the number of errors in a given interval.

The probability distribution $P(m,n)$ of the number of errors m in a block of length n is a matrix-binomial distribution ([12], pg. 76) which can be obtained recursively by the following forward algorithm:

$$\mathbf{p}_1(0) = \boldsymbol{\pi}\mathbf{P}(0), \quad \mathbf{p}_1(1) = \boldsymbol{\pi}\mathbf{P}(1)$$

$$\mathbf{p}_n(m) = 0, \quad \text{for} \quad m < 0 \quad \text{or} \quad m > n$$

$$\mathbf{p}_n(m) = \mathbf{p}_{n-1}(m)\mathbf{P}(0) + \mathbf{p}_{n-1}(m-1)\mathbf{P}(1)$$

$$P(m,n) = \mathbf{p}_n(m)\mathbf{1}$$

The cumulative distributions corresponding to this matrix-binomial distribution for the two-state ant three-state models are compared with the results of the waveform level simulation in Figures 1 and 2.

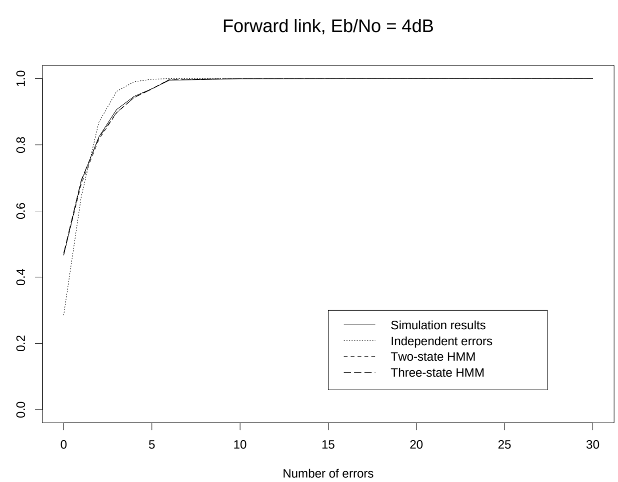

Fig. 1. Cumulative error number distribution (384 bit frame).

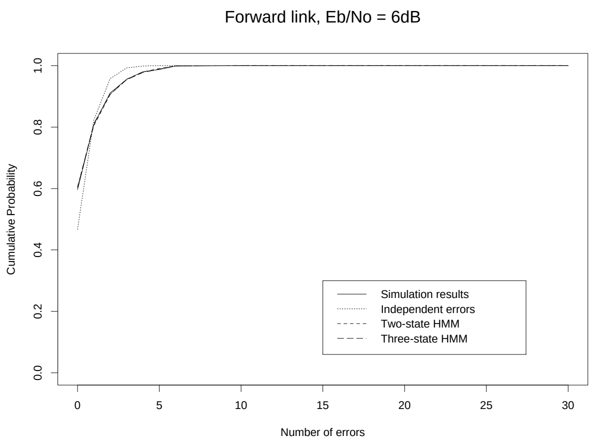

Fig. 2. Cumulative error number distribution (384 bit frame).

As we can see, the two-state Gilbert's model is quite adequate for the purpose of computing error number distribution in the block and, thus, can be used for evaluating block code error correction/detection performance.

VI. CONCLUSION

Waveform level simulation of a discrete channel is computationally expensive and is impossible to use for evaluating the performance of discrete channel devices (such as error correction/detection coders, scramblers, interleavers, etc). However, the waveform level simulation can be used to generate enough data for fitting a discrete channel model. Since HMMs are general enough to approximate a large variety of random processes, they are good candidates for modeling the discrete channel. In this paper, we have demonstrated that for the simulated forward link CDMA cellular system, the discrete channel can be modeled successfully with the Gilbert's model which is a special case of the HMM.

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