

# Asymmetric Modulation and Multistage Coding for Multicasting with Multi-Level Reception over Fading Channels

Mohamed M. Khairy, and Evaggelos Geraniotis \*  
Department of Electrical Engineering  
and Institute for Systems Research  
University of Maryland  
College Park, MD 20742  
mkhairy, evaggelos@eng.umd.edu

## Abstract

*In this paper we investigate the application of joint iterative decoding and channel estimation for the transmission of multilevel coded signals using asymmetric QPSK modulation over flat Rayleigh fading channels. The goal is the transmission of multiresolution data streams to receivers with multi-level reception capabilities. We consider two types of receivers: the receivers in the first class can decode only the most significant bit (MSB) of the transmitted information; the receivers in the second class decode both the MSB and the LSB (least significant bit).*

*Symbol aided demodulation (SAD) is used according to which known symbols are inserted in the encoded data stream to assist in the channel estimation process. Furthermore, an iterative algorithm is proposed that uses soft decoding information in addition to the information contained in the known symbols to improve the estimation of the fading channel. Simulation results show significant performance gains of the new scheme over both differential demodulation and conventional SAD.*

## 1 Introduction

One of the major disturbances that affect the transmission of digital information over land-mobile and mobile-satellite links is fading. The need for reliable digital transmission in mobile and SATCOM applications has motivated a great deal of work in developing techniques that are resistive to fading.

In this paper multilevel codes and asymmetric modulation are used so as to provide reliable multi-level reception. A number of broadcast-only wireless systems (SATCOM and terrestrial) require the capability of providing reliable transmission to different users which may employ various types of equipment and may be located at distinct sites.

The different reception capabilities of the intended receivers (e.g., antenna gain, constraints on weight and volume on hardware that limit the complexity of signal processing algorithms used) as well as the varying severity of fading channel conditions of the individual users, necessitate the use of transmission techniques that can guarantee reliable reception at a minimum transmission rate to “disadvantaged” users while other users can receive reliably at higher transmission rates. As an example consider a system that is required to deliver a data stream to two different sets of users (having different receiver capabilities). An example of this is a system transmitting image signals such that all users are able to receive the image

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with low resolution while some of the users are also able to receive the image with higher resolution.

Receiver capabilities may vary by several dBs (in received signal power). There are a number of ways to achieve different QoS for different receivers. Multilevel coding is the perfect match for applications with different BER requirements. A block diagram for a multilevel coding scheme with 2 stages is shown in Fig. 1. In this scheme, 2 data streams  $d_0$  and  $d_1$  enter 2 different encoders, which may have different rates. The outputs of the encoders enter the mapper where a signal constellation point is chosen.

Here comes the role of asymmetric modulation. This method is the most efficient method to achieve unequal error Protection for uncoded systems, where depending on the difference between the users, the constellation points can be placed asymmetrically to achieve the required performance [2]. Fig. 2 shows the signal constellation for asymmetric QPSK signal which protects the coded stream  $x^0$  more than  $x^1$ . The stream  $x^1$  may or may not be more protected by coding than stream  $x^0$  depending on the application.

Channel estimation is one of the most important tasks for systems operating over fading channels. The reason is that coherent demodulation reception always preferable over differential or non-coherent demodulation techniques but requires a good quality signal reference; this reference can be provided by channel estimation. One of the best solutions for this problem is the use of pilot Symbol Aided Demodulation (SAD) technique described in [1] and [3]. It was shown that the gap between coherent demodulation and differential demodulation is intolerable for some applications where the difference can be more than 3 dBs for high order modulation. The SAD scheme was shown to compensate for most of the difference.

In the SAD scheme, known symbols are inserted in the data stream and being known to the receiver, they can be filtered to obtain good estimates of the channel at the unknown data symbols. Detailed description and numerical results are given in [3]. The use of joint iterative filtering and decoding can reduce the required insertion

rate of pilot symbols and thus the throughput loss associated with SAD.

As in our previous work [4] we propose the use of the unknown symbols together with the known symbols to estimate the channel parameters and incorporate the iterative decoding in this process.

It is known that the optimum receiver should perform a joint maximum likelihood decoding/demodulation of the received signal, but this is very difficult and computationally complex to implement. The best solution to this problem is to use Soft Input Soft Output (SISO) decoding modules and iterative decoding.

The interest in iterative decoding was ignited after the proposal of Turbo codes [5]. Turbo codes exploit the iterative decoding principle, with improvement in performance still noticed up to 20 iterations or more. Iterative decoding can also be applied without using Turbo codes. For example, multilevel coding shows improvement in its performance if iterative decoding is used with no improvement after two or three iterations if convolutional codes (not Turbo codes) form the constituent codes.

## 2 System Model

The transmitted signal for the user under consideration, is given by

$$s(t) = A \sum_{k=-\infty}^{\infty} s_k p(t - kT) \quad (1)$$

where  $T$  is the symbol duration;  $s_k = |s_k|e^{j\phi_k}$  is the complex encoded sequence or the known symbols with average energy=1; The received signal is

$$Y(t) = c(t)s(t) + n(t) \quad (2)$$

where  $n(t)$  is AWGN with power spectral density  $N_0$  in the real and imaginary parts; and  $c(t)$  represents the flat frequency nonselective Rayleigh fading channel complex gain, with autocorrelation function:

$$R_c(\tau) = \sigma_g^2 J_0(2\pi f_D \tau) \quad (3)$$

where  $J_0$  is the zero order modified Bessel function of the first type;  $f_D$  is the fading bandwidth;  $\tau$  is the timing difference.

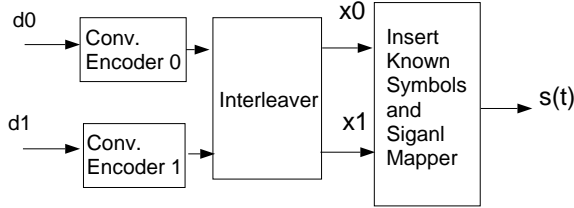


Figure 1. Encoder.

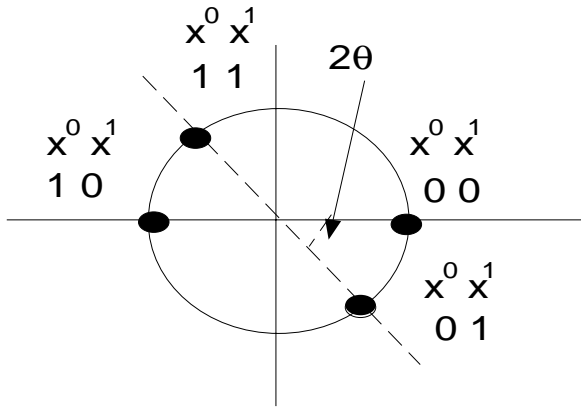


Figure 2. Asymmetric QPSK constellation.

The output of the matched filter, with impulse response  $p^*(-t)/\sqrt{N_0}$ , is given by.

$$Y(m) = u(m)s(m) + n(m) \quad (4)$$

where the complex Gaussian noise samples are white with unit variance, and the symbol gain  $u(k)$  has mean

$$E[u(m)] = 0$$

and variance

$$\sigma_u^2 = \gamma_s \frac{J-1}{J}$$

where

$$\gamma_s = \frac{E_s}{N_0}$$

and  $E_s$  is given by

$$E_s = \frac{(J-1)E_b r_c}{J} \log_2 M \quad (5)$$

where the modulation is assumed M-ary, and  $r_c$  is the error control coding rate. Each known symbol is followed by  $J-1$  encoded data symbol.  $E_b$  is the average energy per received data bit and  $E_s$  is the average energy per received coded symbol.

### 3 Soft Decoding

The channel estimate  $\hat{c}(m)$  at time  $m$  is available to the decoder from the channel estimator, described in the next section. Now define

$$p_j^l(m) = p(x^l(m) = j), \quad l, j \in \{0, 1\}$$

Those probabilities are calculated from the output of the decoder. Before the first iteration, the first decoder uses  $p_0^1(m) = p_1^1(m) = 0.5$ . After each half iteration the new probabilities are updated and used in the next step using the reliability information of the decoder  $L(m)$ .

$$p(x(m) = 1) = e^{L(m)} / (1 + e^{L(m)})$$

#### 3.1 First Decoder

Fig. 3 shows a block diagram of the decoder/channel estimator. In order to perform iterative decoding, the log-likelihood ratio of the received bits should be calculated. As an example we show it for the QPSK constellation shown in Fig. 2.

$$L^0(m) = \ln \frac{p(Y(m)|x^0(m) = +1, x^1(m))}{p(Y(m)|x^0(m) = 0, x^1(m))} \quad (6)$$

where

$$\begin{aligned} p(Y(m)|x^0(m) = +1, x^1(m)) &= p_1^1(m) e^{-|Y(m) - e^{i(\pi-2\theta)} \hat{c}(m)|^2 E_s/N_0} \\ &\quad + p_0^1(m) e^{-|Y(m) + \hat{c}(m)|^2 E_s/N_0} \\ p(Y(m)|x^0(m) = 0, x^1(m)) &= p_1^1(m) e^{-|Y(m) - e^{-i2\theta} \hat{c}(m)|^2 E_s/N_0} \\ &\quad + p_0^1(m) e^{-|Y(m) - \hat{c}(m)|^2 E_s/N_0} \end{aligned}$$

where  $\hat{c}(m)$  is the channel estimate. The symbols are assumed independent because of the interleaving which scrambles successive fading samples.

The information  $L^0(m)$  enters the decoder, either a MAP or SOVA decoder and the first bits  $x^0$  are decoded. The output of the decoder contains reliability information about the data bits and the code bits which can be used by the next decoder

### 3.2 Second Decoder

The second decoder calculates the probabilities of the bits  $x^0$  (information and code) from the output of the first decoder

$$p_1^0 = p(x^0 = 1) = e^{x^0} / (1 + e^{x^0})$$

$$p_0^0 = p(x^0 = 0)$$

now

$$L^1(m) = \ln \frac{p(Y(m)|x^1(m) = +1, x^0(m))}{p(Y(m)|x^1(m) = 0, x^0(m))}$$

where

$$p(Y(m)|x^1(m) = +1, x^0(m)) = p_0^0(m)e^{-|Y(m) - e^{-i2\theta}\hat{c}(m)|^2 E_s/N_0} + p_1^0(m)e^{-|Y(m) - e^{i(\pi-2\theta)}\hat{c}(m)|^2 E_s/N_0}$$

$$p(Y(m)|x^1(m) = 0, x^0(m)) = p_0^0(m)e^{-|Y(m) - \hat{c}(m)|^2 E_s/N_0} + p_1^0(m)e^{-|Y(m) + \hat{c}(m)|^2 E_s/N_0}$$

after the decoding of the second decoder, the first decoder can iterate the decoding with the new probabilities of  $x^1$ .

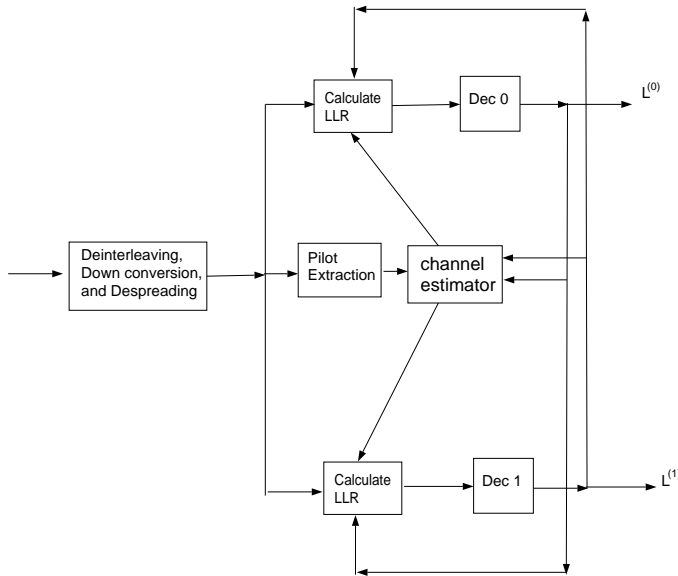


Figure 3. Proposed Decoder/Estimator.

## 4 Channel Estimation Schemes

In this section we describe the two channel estimation schemes used in this paper. The first one

is the “conventional” SAD scheme, where only the known symbols are used to estimate the channel. The second scheme is an adaptation of the first one where also unknown data symbols are used to assist channel estimation.

### 4.1 SAD for channel estimation

In this scheme, only the KNOWN symbols are used to estimate the channel. However, a different estimation filter, depending on the symbol position within the frame is used. The procedure for obtaining the filter coefficients is described in [1] and [3]. In this case the channel estimates are calculated prior to the first decoding step and it does not change with consequent iterations.

### 4.2 Joint Iterative decoding and SAD for Recursive Channel Estimation

In this scheme, the first iteration is performed like in the SAD scheme. The same filters used in the first scheme can be used in the first Iteration. Following the first iteration, which is identical to the first scheme, the soft outputs of the two decoders are used to determine the probabilities of different symbols. Now define

$$p_0 = p_0^0 p_1^1, \quad p_1 = p_0^0 p_0^1, \quad p_2 = p_1^0 p_1^1, \quad p_3 = p_1^0 p_0^1$$

now

$$\hat{c}(m) = \sum_m h(m) P(m) r(m)$$

where

$$P = p_1 - p_3 + p_0 e^{j2\theta} + p_2 e^{j(-\pi + 2\theta)}$$

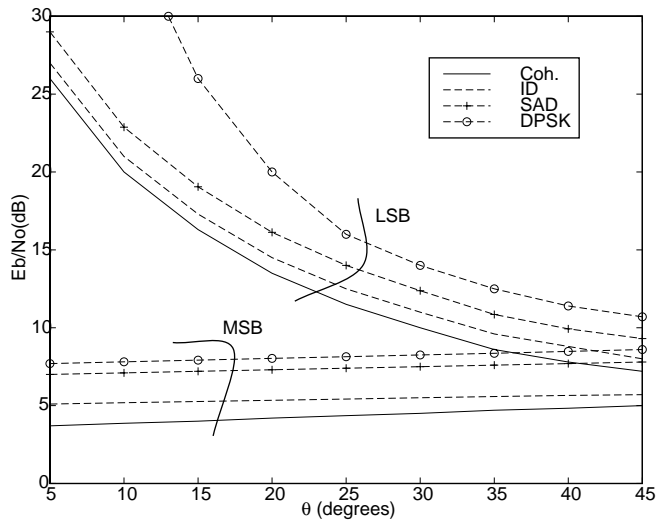
For the known symbols  $P = 1$  assuming the known symbols are such that their  $p_0 = 1$ .

After obtaining the estimate of the complex fading gain, decoding is performed for the next iteration using this modified channel estimate. It is worth mentioning that the estimation filter used in this scheme does not have equally spaced samples.

## 5 Numerical Results and Conclusions

Fig. 4 shows the  $E_b/N_0$  required by the different schemes to achieve a BER of  $10^{-3}$  for the MSB and

LSB. The codes used are convolutional codes with rates 1/2 and 1/2 for both data streams. The constraint length of both codes is 3. MAP decoding [6] is used for both codes with 3 iterations, where more iterations doesn't show improvement. The schemes considered are the coherent, Differential PSK, SAD, and SAD-iterative decoding schemes. The SAD schemes uses the nearest 6 known symbols to estimate the channel. SAD-ID scheme uses in addition to the 6 known symbols, the nearest 12 unknown symbols.  $[R_s/(2R_f)] = 20$ , where  $R_s$  is the symbol rate and  $R_f$  is the fading rate.



**Figure 4. Required  $E_b/N_0$  vs.  $\theta$  for different schemes to achieve  $BER = 10^{-3}$  for MSB and LSB**

It should be noticed that the DPSK scheme can not achieve  $10^{-3}$  for the LSB for  $\theta = 5$  degrees. This shows the clear disadvantage of this scheme. The SAD-ID scheme is much better than the SAD scheme and very close to the coherent scheme. The important thing to note about the SAD-ID scheme is that the improvement with decreasing  $\theta$  is less than other schemes for the MSB. For example the gain for the MSB in the coherent scheme for  $\theta$  going from 45 to 5 degrees is 1.3 dB. For the SAD-ID scheme it is only 0.6 dB while for SAD it is 0.9 dB. The small improvement for the SAD-ID scheme is

due to the fact that by decreasing  $\theta$ , the decoder for the LSB is not able to give very reliable information to the decoder of the MSB and to the channel estimator, which decreases the improvement of the MSB decoder with iterations, unlike the coherent case where the channel estimation is assumed perfect or even the SAD case where the channel estimation is independent of the unknown data. This is offset by the large improvement obtained by the SAD-ID scheme.

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