Pilot Symbol Assisted QAM with Iterative Filtering and Turbo Decoding over Raleigh Flat-Fading Channels

Yingjiu Xu, Hsuan-Jung Su, and Evaggelos Geraniotis *
Department of Electrical Engineering
and Institute for Systems Research
University of Maryland
College Park, MD 20742, USA
yixu@eng.umd.edu, hjsu@eng.umd.edu, evagelos@eng.umd.edu

Abstract

In this paper, we investigate the application of iterative channel estimation and decoding for Turbo coded M-QAM signals transmitted over slow Rayleigh fading channels. Pilot symbols are inserted periodically in the encoded data stream for the estimation of the time-variant channel state information(CSI). The decoder uses this estimated CSI, together with the received data sequence, to generate soft decisions for each data symbol. These soft decisions and the known pilot symbols are then filtered again to obtain improved estimation of the CSI. Two different algorithms, *algorithm A* and *algorithm B*, have been studied for the iterative filtering process and their performances are evaluated by simulation. At a BER of 10^{-3}, both algorithms are within 1.3 dB of the ideal system with perfect CSI. Compared to the exponentially growing complexity of the optimal joint channel estimation and decoding system, the proposed system has linear complexity when *algorithm A* is used and polynomial complexity when *algorithm B* is used, thus providing low-complexity alternatives while maintaining an effective performance improvement.

1 Introduction

Motivated by development in mobile and wireless communications, the problem of system design for reliable, high-speed data transmission over fading channels receives considerable current interest. Various newly developed techniques have been combined together to combat fading and improve the system performance. In [1], pilot symbol assisted modulation (PSAM) [2], turbo codes [3], and joint iterative filtering and decoding, have been applied together for the demodulation of BPSK signals transmitted over Rayleigh fading channel. However, due to the inherent drawback of the BPSK signal, the above system has limited power and bandwidth efficiency.

Given the emerging interest in higher order modulation for speech and data transmission for mobile systems, we concentrate on the system design of transmitting QAM signals over slow Rayleigh fading channels. The bandwidth efficient turbo coding scheme using Gray mapping proposed in [4] is adopted due to its advantages in simplicity, applicability to a wide range of constellations and code rates, and compatibility to binary turbo codes. In PSAM, to avoid the loss of throughput caused by the "worst case" filter design, adaptive filters are used in both pilot symbol filtering and subsequent iteratively filtering.

The outline of this papers is as follows. In section 2, a description of the channel model is presented. A detailed presentation of the proposed PSAM system with iterative filtering and decoding of the QAM signals are given in section 3, where two algorithms are proposed for iterative filtering. Simulation results are graphed and discussed in section 4. We conclude simulation results in section 5.

2 Raleigh Fading Channel and PSAM

We assume the transmitted signal $s(t)$ has a complex envelope given by

$$s(t) = A \sum_{k=-\infty}^{\infty} b(k)p(t-kT)$$

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where T is the symbol duration, $b(k)$ is the kth symbol for 16-QAM. A is the amplitude factor and p(t) is a unit energy pulse:

$$\int_{-\infty}^{\infty} |p(t)|^2 \, dt = 1$$

At the receiver, the fading channel output $r_c(t)$ is given by

$$r_c(t) = c(t)s(t) + n_c(t)$$

in which $n_c(t)$ is AWGN with power spectral density N_0 in both real and imaginary components. The channel's complex gain $c(t)$ incorporates both fading and frequency offset:

$$c(t) = \exp(j2\pi f_0 t)g(t)$$

where f_0 is the residual frequency offset, and $g(t)$ is the complex Gaussian fading process with variance σ_g^2 and Doppler spread f_D. Its autocorrelation function can be written as:

$$R_c(\tau) = \sigma_g^2 \tilde{R}_c(\tau)$$

where $\tilde{R}_c(\tau)$ is the unit power equivalent. According to Jake [5],

$$\tilde{R}_c(\tau) = exp(j2\pi f_0 \tau) J_0(2\pi f_D \tau)$$

where $J_0(\cdot)$ is the zeroth order Bessel function of the first kind.

The receiver detects the pulses using a matched filter with impulse response $p^*(-t)/\sqrt{N_0}$ [2]. The symbol-spaced samples $r(k)$ of matched filter output are given by

$$r(k) = u(k)s(k) + n(k)$$

where $u(k) = \frac{Ac(k)}{\sqrt{N_0}}$ and the Gaussian noise samples $n(k)$ are white with unit variance.

In PSAM, known symbols are inserted periodically into the data symbol sequence once per $(M-1)$ symbols. At the receive, these symbols are filtered and interpolated to estimate the channel gain associated with each data symbols. The optimum filter under a given signal-to-noise-ratio and Doppler spread is Wiener filter [2]. The receiver then scales and rotates a reference decision grid for QAM constellation with the estimate, and feeds the modified decision boundaries to the data branch. This modified decision boundaries are then used to generate logarithm likelihood ratios of four bits associated with each received 16-QAM data symbol, which will be discussed in detail in the next section. To reduce the loss of throughput, the pilot symbol filters are adapted in a way similar to *Algorithm A* in section 3. Here we don't suffer from the loss due to the non-Gaussian noise since all the pilots are of the same value.

3 Iterative Decoding and Filtering

3.1 Logarithm Likelihood Ratio(LLR) Calculation

For reasons stated in the introduction, a set of binary turbo encoder and decoder is used in the system. However, the symbols transmitted over the fading channel and received after matched filtering are 16-QAM symbols. This requires that an appropriate mapping from binary coded bits to 16-QAM symbols be used in the transmitter and that noise-corrupted data symbols be broken into their associated bit logarithm likelihood ratio at the receiver. As in [4], Gray mapping is used in the transmitter. Next we will deal with the calculation of bit logarithm likelihood ratio for 16-QAM signals.

Consider a sequence of information bits $b_1 b_2 b_3 b_4 ... b_{4k-3} b_{4k-2} b_{4k-1} b_{4k} ...$, a sequence of 16-QAM symbols $r(1), r(2), ..., r(k), ...$, and the following mapping.

$b_{4k-3}b_{4k-2}$	X_k	$b_{4k-1}b_{4k}$	Y_k
01	-3	01	-3
00	-1	00	-1
10	+1	10	+1
11	+3	11	+3

The bit logarithm likelihood ratio for 16-QAM signals is given by

$$\Lambda(b_{4k-i}) = Klog\frac{Pr\{b(4k-i) = 1|r(k)\}}{Pr\{b(4k-i) = 0|r(k)\}}$$

for $i = 0, 1, 2, 3$ and K is a constant.

In the case of additive white Gaussian channel with no fading, according to [4], a good approximation of the LLR can be achieved using the following expressions:

$$\Lambda(b_{4k-3}) = X_k$$
$$\Lambda(b_{4k-2}) = |X_k| - 2$$
$$\Lambda(b_{4k-1}) = Y_k$$
$$\Lambda(b_{4k}) = |Y_k| - 2$$

Let $r(k) = X(k) + jY(k)$. In the case of Rayleigh fading channel, we need to scale and rotate the reference frame according to the channel gain before calculating the LLR. It is easy to prove that the following expressions hold, where $u(k)$ is the complex channel gain associated with symbol (X_k, Y_k).

$$\Lambda(b_{4k-3}) = |u(k)|X_k^c$$
$$\Lambda(b_{4k-2}) = |u(k)|(|X_k^c| - 2)$$
$$\Lambda(b_{4k-1}) = |u(k)|Y_k^c$$
$$\Lambda(b_{4k}) = |u(k)|(|Y_k^c| - 2)$$

where

$$X_k^c + jY_k^c = (X_k + jY_k)/u(k)$$

3.2 Soft Decision for Information and Coded Bits

A standard binary turbo decoder generates the soft decisions only for the information bits, the extrinsic part of which are to be used in the next iteration as a priori information. In this design, the soft decisions for the coded bits as well as information bits are needed for the iterative filtering. Let u_k be the information bit associated with the transition from time $k-1$ to k. Let $x_{k,1}, ..., x_{k,\nu}$ be the coded bits associated with the branch transition from time $k-1$ to k. And let the trellis states at level $k-1$ and at level k be denoted by the integer s' and s respectively. Then for the information bit, the soft decision can be calculated by

$$L(u_k) = log\frac{\sum_{\{(s',s):u_k=1\}} \alpha_{k-1}(s')\gamma_k(s',s)\beta_k(s)}{\sum_{\{(s',s):u_k=0\}} \alpha_{k-1}(s')\gamma_k(s',s)\beta_k(s)}$$

where

$$\gamma_k(s',s) = Pr(s|s')p(\underline{y}_k|s',s)$$

$\alpha_k(s)$ is yielded by the forward recursion

$$\alpha_k(s) = \sum_{s'} \alpha_{k-1}(s')\gamma_k(s',s)$$

$\beta_k(s)$ is yielded by the backward recursion

$$\beta_{k-1}(s) = \sum_{s'} \gamma_k(s',s)\beta_k(s)$$

For the coded bits $x_{k,1}, ..., x_{k,\nu}$, their soft decisions are given by

$$L(x_{k,i}) = log\frac{\sum_{\{(s',s):x_{k,i}=1\}} \alpha_{k-1}(s')\gamma_k(s',s)\beta_k(s)}{\sum_{\{(s',s):x_{k,i}=0\}} \alpha_{k-1}(s')\gamma_k(s',s)\beta_k(s)}$$

for $i = 1, 2, ..., \nu$.

3.3 Iterative Filtering

With the help of soft decisions generated by the turbo decoder, the iterative filters use the received pilot symbols and the data symbols to obtain a more accurate estimation of the channel state information.

Assume perfect interleaving between the turbo encoder output and the 16-QAM mapping, the reliability value of the kth data symbol $s(k)$ can be calculated in the product form.

$$\Pr(s(k)|r_1^L) = \prod_{i=0}^{i=3} Pr(b(4k-i)|r_1^L)$$

where r_1^L is the entire received symbol sequence of length L, $s(k)$ is the QAM symbol corresponding to $b(4k-i), i = 0, 1, 2, 3$.

Of course, for pilot symbols, their reliability values are always equal to the assigned pilot values with probability 1.

To do iterative filtering, we have studied two approaches. One is to extend the algorithm described by Su in [1] from binary case to more general nonbinary QAM symbols through the use of rotation and scaling. The other is to use a threshold-controlled feedback to improve the channel estimation. For convenience, we label the former with *algorithm A* and the latter with *algorithm B*.

Algorithm A We first select for each specific symbol the most reliable point out of 16-QAM constellation, i.e.

$$\hat{s}(k) = max_{i=1,2,...,16} Pr\{s(k) = q_i\}$$

where $q_i, i = 1, 2, ..., 16$ are 16-QAM constellation points. We then have

$$r(k) \approx u(k)\hat{s}(k) + n(k)$$

Divide both sides by $\hat{s}(k)$, we have

$$r'(k) \approx u(k) + n'(k)$$

where $r'(k) = r(k)/\hat{s}(k)$ and $n'(k) = n(k)/\hat{s}(k)$. Since $\hat{s}(k) = |\hat{s}(k)|e^{jarg(\hat{s}(k))}$, this process is equivalent to a scaling of the amplitude of $r(k)$ by $|\frac{1}{\hat{s}(k)}|$ and a rotation of the phase of $r(k)$ by $-arg(\hat{s}(k))$

Although the new noise term $n'(k)$ are no longer white Gaussian noise, we treat it approximately as Gaussian noise in the subsequent processing. To get the channel estimation of a specific symbol, we use the all symbol filter

$$\hat{u}(k) = \underline{h}_a^H \underline{r}_a(k)$$

where

$$\underline{r}_a(k) = \begin{bmatrix} r'(k-\frac{K}{2}) \\ \vdots \\ r'(k-1) \\ r'(k+1) \\ \vdots \\ r'(k+\frac{K}{2}) \end{bmatrix}$$

$\underline{h}_a$ is a vector of all symbol filter coefficients. H stands for the matrix transpose and complex conjugate.

With this updated version of channel estimation, the estimation error is given by

$$e_n(k) = r'(k) - \hat{u}(k)$$

where n is the index of iteration. The noise power can be estimated by

$$\sigma_{ae}^2 = E\{e_n e_n^*\}$$

Then we can update the coefficients of all symbol filter using the gradient method. The updated filter coefficients are to be used in the next iteration.

$$\underline{h}_{a;n+1} = \underline{h}_{a;n} + \mu_a E\{\underline{r}_{a;n} e_n^*\}$$

Algorithm B In this algorithm, a threshold controlled feedback are used to obtain a better estimation of the channel gain under the help of decoded symbols with reliabilities higher than a certain level. The soft value of maximum a posteriori probability generated by the turbo decoder reflects the extent that a particular bit can be trusted. The higher the value, the more reliable its corresponding hard decision. The less the value, the less reliable its hard decision. The purpose of the threshold controller is to decimate the bits with a confidential level lower than the threshold and pass only the bits with a higher confidential level. Those bits will then be turned into hard decisions and form the 16-QAM symbols to be used to enhance the channel estimation. Note that a symbol can only be formed when all four bits pass the threshold controller. Otherwise, no symbol will be formed in that symbol position.

We start with calculating the autocorrelation matrix of the received symbol $r(k)$ and the cross-correlation vector for the calculation of all symbol filter coefficients based on the estimated symbol value.

$$\begin{aligned}
R(k,l) &= \frac{1}{2}E[r(k)r^*(l)] \\
&= \frac{1}{2}E[u(k)\hat{s}(k)\hat{s}^*(l)u^*(l)] + \frac{1}{2}E[n(k)n^*(l)] \\
&= \frac{1}{2}E[u(k)u^*(l)]\hat{s}(k)\hat{s}^*(l) + \delta_{k,l} \\
&= \frac{A^2}{2N_0}E[c(kT)c^*(lT)]\hat{s}(k)\hat{s}^*(l) + \delta_{k,l} \\
&= \frac{A^2}{N_0}\sigma_g^2 J_0(2\pi f_D T(k-l))\hat{s}(k)\hat{s}^*(l) + \delta_{k,l} \\
w_k(l) &= \frac{1}{2}E[r(l)u^*(k)] \\
&= \frac{1}{2}E[u(l)u^*(k)]\hat{s}(l)
\end{aligned}$$

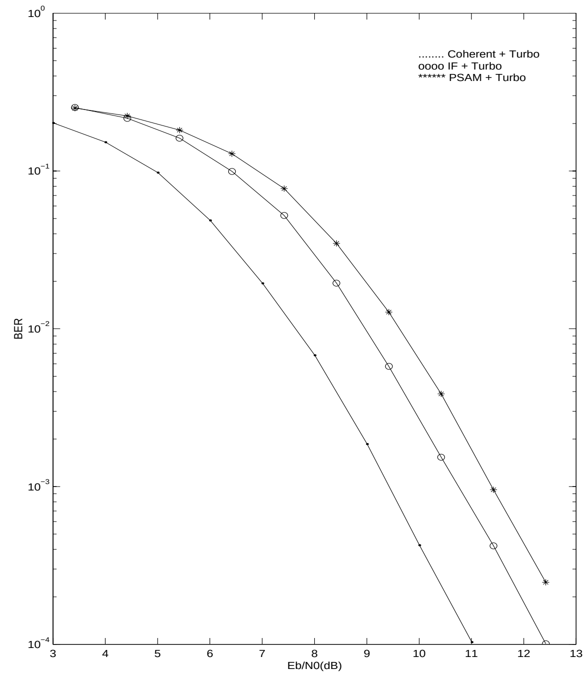

Figure 1: BER of Algorithm A for Rayleigh Fading

$$\begin{aligned}
&= \frac{A^2}{2N_0}E[c(lT)c^*(kT)]\hat{s}(l) \\
&= \frac{A^2}{N_0}\sigma_g^2 J_0(2\pi f_D T(l-k))\hat{s}(l)
\end{aligned}$$

Where $\hat{s}(k), \hat{s}(l)$ are the data symbols formed in the feedback path. At this stage, we can use calculate the Wiener filter coefficients directly by

$$\underline{h}(k) = R^{-1}\underline{w}(k)$$

4 Simulation Results

4.1 Performance of *algorithm A* and *algorithm B* over Rayleigh Fading Channels

We show in Figure 1 the simulation results of *algorithm A* at normalized fading rate, $f_D T$, of 0.01. For comparison, we also include the simulation results of 16-QAM systems with perfect channel state information and the results with pilot symbol assisted demodulation but without iterative filtering. The turbo interleaver is a random interleaver of block size 1036. In all cases, the proposed system using *algorithm A* outperforms the PSAM system without iterative filtering but is inferior to the ideal system with perfect CSI. At 10^{-3} BER, there is only a 1.3 dB difference

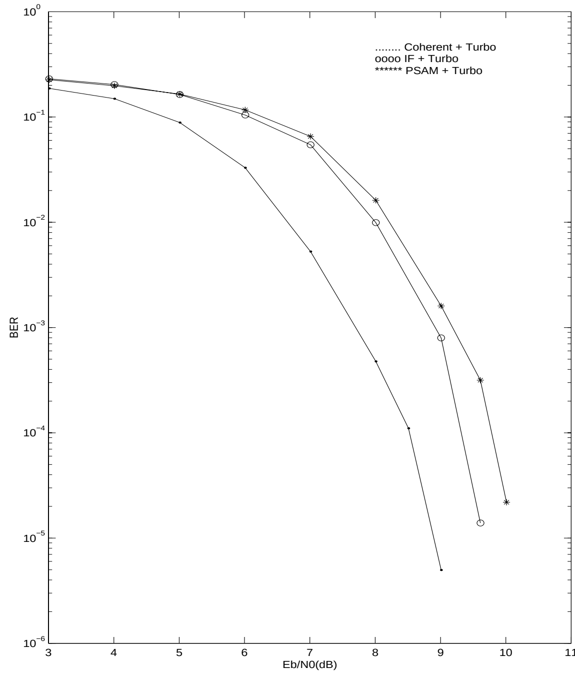

Figure 2: BER of Algorithm B for Rayleigh Fading

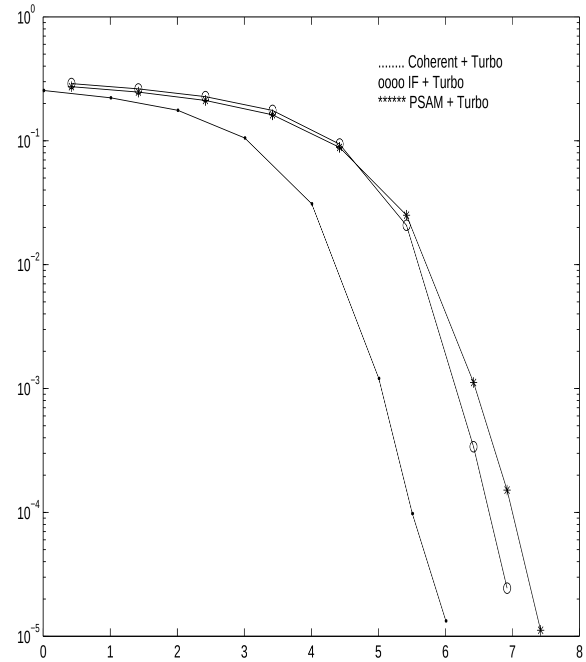

Figure 3: BER of Algorithm A for Ricean Fading (k = 10)

in signal to noise ratio per information bit between the proposed system with *algorithm A* and the system with perfect CSI. *Algorithm A* outperforms the system with only pilot symbol assisted demodulation by 0.5 dB at 10^{-3} BER.

Shown in Figure 2 are the simulation results of *algorithm B* at normalized fading rate, $f_D T$, of 0.01. Also included in this figure are the performance results of 16-QAM systems with perfect channel state information and the results with only pilot symbol assisted demodulation. The turbo interleaver is a random interleaver of block size 4096. This results in a performance gain in curves of Figure 2 over those in Figure 1. Again, the proposed system using *algorithm B* outperforms the PSAM system without iterative filtering in all cases but is inferior to the ideal system with perfect CSI. At 10^{-3} BER, the proposed system with *algorithm B* is only 1.2 dB inferior to that of the coherent receiver with perfect CSI but outperforms the system with only pilot symbol assisted demodulation.

From above it can be clearly seen that both algorithms with iteratively filtering effectively improve the system performance on the base of pilot symbol assisted demodulation. Although there is hardly any difference between the performances of *algorithm A* and *algorithm B*, the linear complexity of *algorithm A* is superior to the polynomial complexity of *algorithm B*.

4.2 Performance of *algorithm A* over Ricean Fading Channels

To further study the performance of *algorithm A*, we also simulate *algorithm A* over Ricean fading channel. The results of $K = 10$ and $K = 30$ are shown in Figure 3 and Figure 4. Other parameters are the same as those used in Figure 1. It can be seen that as K value goes higher($K = 0$ in the case of Rayleigh, $K = \infty$ in the case of AWGN), the performance advantage gained by the iterative filtering over PSAM becomes smaller. This is because that in the case of Ricean fading with high K value, the channel state information estimated from pilot symbols are closer to the ideal channel state information. From Figure 1 to Figure 3 and to Figure 4, the difference between the PSAM system and the system with perfect CSI drops from about 2 dB to 1 dB. On the other hand, the performance gain of iterative filtering is slightly dragged behind by the estimation error caused by the incorrectly decoded data symbols. As a result, in the case of $K = 30$ in Figure 4, the two curves of PSAM and iterative filtering almost overlap at large SNR. Although not shown here, similar results were generated using convolutional codes.

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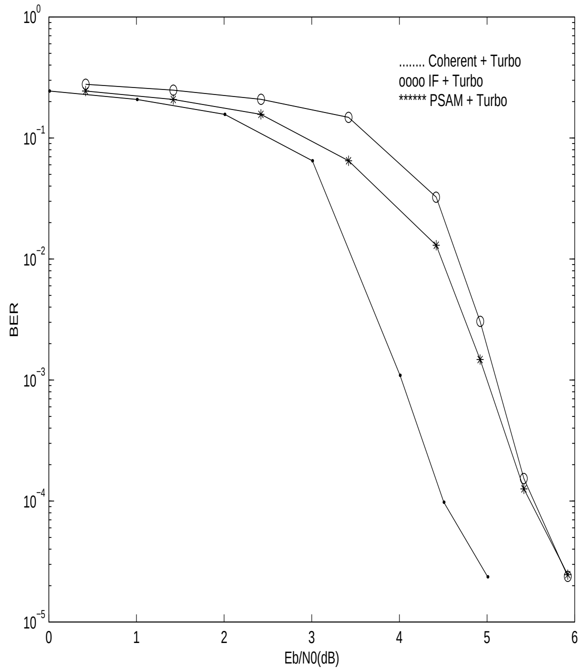

Figure 4: BER of Algorithm A for Ricean Fading (k = 30)

5 Conclusion

In this paper, we have proposed a joint iterative filtering and decoding system for Turbo coded pilot symbol assisted QAM systems over slow Rayleigh fading channels. The information exchanging between channel estimation and decoding alternately improves the estimated CSI and strengthens the soft decisions of the decoder. Two different algorithms, *algorithm A* and *algorithm B*, have been studied for the iterative filtering process and their simulation results are compared. Compared to the exponentially growing complexity of the optimal joint channel estimation and decoding system, the proposed system has a linear complexity in the case of using *algorithm A*, and a polynomial complexity in the case of using *algorithm B*. Simulation results show that both algorithms are within 1.3 dB of the ideal system with perfect CSI at a BER of 10^{-3}. Compared to the system of simple concatenated channel estimation and decoding, both algorithms exhibit a clear performance advantage with added complexity due to the iterative filtering. *Algorithm A* is more suitable for practical purpose than *algorithm B* in that it uses a single filter and does not need to do matrix inversion. We also observed that in the case of Ricean fading, as K value goes higher, the performance advantage gained by the iterative filtering over PSAM becomes smaller. While 16-QAM modulation is used in the simulation, this system applies generally to other high spectral efficiency modulation systems as well. Finally, while all simulation results were generated for Turbo codes the approach of the paper is also applicable to convolutionally coded M-QAM systems.

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