Performance Analysis of Non-Coherent BFSK Using First, Second, and Third Order Selection Combining in a Nakagami Fading Channel

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Abstract The performance of a non-coherent Binary Frequency Shift Keying (BFSK) receiver using first, second, and third order selection combining methods over a frequency non-selective and slowly Nakagami-m fading channel is investigated. Analytical and numerical results for first order selection combining are compared to second and third order selection combining. Probability of error plots are provided for a range of the Nakagami-m variable.

Technical Subject Area: Diversity and interference suppression, Nakagami-m channel, selection combining.

1 Introduction
Channel diversity is a technique that uses $L$ fading replicas of the transmitted signal which are received over $L$ statistically independent channels. The received information from the diversity branches can be combined to decrease the bit error rate. There are several methods of diversity combining. We will investigate first, second, and third order (SC, SC-2, and SC-3) selection combining. Order 1 (SC-1 or SC) uses the signal branches with largest amplitudes. Order 2 (SC-2) combines the two dominant signal branches. Order 3 (SC-3) combines the three dominant signal branches. The communication channel is assumed to be a frequency non-selective, slowly fading Nakagami-m channel, with additive white Gaussian noise.

The Nakagami-m distribution is a generalized formula that can model different fading environments using different values of its parameters. If $m = 1/2$ it models a one-sided Gaussian fading channel, if $m = 1$ the channel has Rayleigh fading characteristics, while for large $m$ the channel becomes non-fading [1].

A block diagram of $L$ parallel diversity branches feeding the BFSK receivers is shown in Fig. 1. BFSK is used, which allows non-coherent detection.

2 Selection Combining
Selection combining (SC) is a diversity combining technique where the signal with the largest amplitude, or largest signal-to-noise ratio, in $L$ diversity branches is selected. Thus, the decision variable for the selection combining technique is defined as
\[
\gamma = \max \{\gamma_1, \gamma_2, \ldots, \gamma_L\}
\]
where the signal-to-noise ratios per diversity channel $\gamma_k (k = 1, 2, \ldots, L)$ are independent, identically distributed random variables with identical mean $\overline{\gamma}_c$.

The pdf of $\gamma_k$ is derived from (1) as [1]
\[
f_{\overline{\gamma}_k}(\gamma) = \frac{\gamma_k^{m(m-1)} e^{-\gamma_k \overline{\gamma}_B}}{\Gamma(m) \overline{\gamma}_B^m \gamma_k^{m-1}}
\]
where $\overline{\gamma}_B = L\overline{\gamma}_c = $ average SNR per bit. The probability of error is given by
\[
P_B = \int_0^\infty P_B(\gamma) f_{\overline{\gamma}_k}(\gamma) d\gamma
\]
where the bit error rate of BFSK for a fading channel, conditioned on the signal-to-noise ratio $\gamma$, is given by

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The pdf of \( \gamma \) is given by
\[
 f_\gamma(\gamma) = \frac{L^{m+1}m^m}{[\Gamma(m)]^L \frac{\pi B}{7 B}} \gamma^{m-1} \exp \left( -\frac{m \gamma L}{7 B} \right) \left[ g \left( m, \frac{m \gamma L}{7 B} \right) \right]^{L-1},
\]
where
\[
g \left( m, \frac{m \gamma L}{7 B} \right) = \int_0^{(m \gamma L)/7 B} \exp(-t) t^{m-1} dt.
\]
Equation (6) can be reduced to
\[
P_B = \frac{L^{m+1}m^m}{2 [\Gamma(m)]^L \frac{\pi B}{7 B}} \int_0^{\infty} \gamma^{m-1} \exp \left[ -\frac{m \gamma}{7 B} \right] \left[ g \left( m, \frac{m \gamma L}{7 B} \right) \right]^{L-1} d\gamma.
\]

### 3 Second Order Selection Combining (SC-2)

Second order selection combining (SC-2) uses the two largest amplitudes as decision variables from the \( L \) diversity branches. The two decision variables for SC-2 are defined as
\[
 V_1 = \max \{ \gamma_1, \gamma_2, \ldots, \gamma_L \}
\]
\[
 V_2 = \text{second max} \{ \gamma_1, \gamma_2, \ldots, \gamma_L \},
\]
where the signal-to-noise ratios per diversity channel \( \gamma_k (k = 1, 2, \ldots, L) \) are independent, identically distributed random variables with the pdf given in (5). The output \( \gamma \) is given by
\[
 \gamma = v_1 + v_2.
\]
The probability of error is given by (6), where [2]
\[
P_B(\gamma) = \frac{1}{8} \left[ \exp \left( -\frac{\gamma}{2} \right) \right]^{4 + \gamma/2}.
\]
The pdf for \( \gamma \), after some manipulations, is obtained as
\[
f_\gamma(\gamma) = (L-1) \frac{m^{2m}L^{2m+1}}{[\Gamma(m)]^L \frac{\pi B}{7 B}} \exp \left( -\frac{m \gamma L}{7 B} \right) \int_0^{\gamma/2} (v_2(\gamma - v_2))^{m-1}
\times \left[ g \left( m, \frac{mn v L}{7 B} \right) \right]^{L-2} dv_2.
\]

### 4 Third Order Selection Combining (SC-3)

Third order selection combining (SC-3) uses the three largest amplitudes as decision variables from the \( L \) diversity branches. From \( L \) independent, identically distributed random variables, the three decision variables for SC-3 are defined as
\[
 V_1 = \max \{ \gamma_1, \gamma_2, \ldots, \gamma_L \}
\]
\[
 V_2 = \text{second max} \{ \gamma_1, \gamma_2, \ldots, \gamma_L \}
\]
\[
 V_3 = \text{third max} \{ \gamma_1, \gamma_2, \ldots, \gamma_L \},
\]
where \( \gamma_k (k = 1, 2, \ldots, L) \) are independent, identically distributed random variables with the probability density function in (5). The output \( \gamma \) is given by
\[
 \gamma = v_1 + v_2 + v_3.
\]
The probability of error is given by (6) where [2]
\[
P_B(\gamma) = \frac{1}{32} \left[ \exp \left( -\frac{\gamma}{2} \right) \right]^{16 + 3 \gamma + \gamma/8}.
\]
The pdf for \( \gamma \) is given by
\[
f_\gamma(\gamma) = (L-1)(L-2) \frac{m^{3m}L^{3m+1}}{[\Gamma(m)]^L \frac{\pi B}{7 B}} 
\times \exp \left( -\frac{m \gamma L}{7 B} \right) \int_0^{\gamma/3} g \left( m, \frac{mn v L}{7 B} \right) \int_0^{(\gamma-v_3)/2} (v_2(\gamma - v_3))^{m-1}
\times \left[ g \left( m, \frac{mn v L}{7 B} \right) \right]^{L-2} dv_2 dv_3.
\]
Using (18) and (19) in (6) we obtain the probability of error as
\[
P_B = (L-1)(L-2) \frac{m^{3m}L^{3m+1}}{32 [\Gamma(m)]^L \frac{\pi B}{7 B}}.
where $g(\cdot, \cdot)$ is defined in (9).

5 Results

The error rate is evaluated using the derived equations for signal-to-noise ratio per bit $\bar{\gamma}_B$ from 6 to 20 dB. Plots are shown in Figs. 2–8. Figures 2–5 allow comparison of SC, SC-2, and SC-3 for values of $m$ of 0.5, 1, 2, and 3. The diversity $L$ is kept fixed at 5. Figures 6–8 demonstrate the performance increase for $m = 1/2, 3/4, 1, 1.5, 2,$ and 3 for values of diversity of 2, 3, and 4. For a fixed $L$ (i.e., 5), Figs. 2–8, SC-3 outperforms SC-2 which in turn outperforms SC. Figures 6-8 show performance improvements as $L$ increases from 2 to 3 to 4. The performance also increases as the Nakagami parameter $m$ increases above some SNR threshold value. Some figures (Figs. 4 and 5) show a non-coherent recombining loss at low SNR levels.

6 Conclusion

The SC techniques are in general simple techniques because they can give satisfactory performance without an $L$ dependency. This is something we desire in order to construct simpler receivers. On the other hand, these techniques are not optimal techniques since they do not use all the available diversity branches at the same time. But, if the diversity order $L$ varies as a function of location or time, it is desirable that the receiver have an $L$ independency.

It is shown that system with a higher diversity order $L$ performs better than a system with a smaller $L$. As far as the comparison between the three techniques is concerned, we conclude that as the order increases the selection combining techniques perform better. Finally, regarding the Nakagami fading channel, it is shown that as the factor $m$ increases the system performs better.

References


Figure 3: Receiver performance of SC, SC-2, SC-3 over a Nakagami fading channel with $m = 1$ for diversity order of $L = 5$.

Figure 4: Receiver performance of SC, SC-2, SC-3 over a Nakagami fading channel with $m = 2$ for diversity order of $L = 5$.

Figure 5: Receiver performance of SC, SC-2, SC-3 over a Nakagami fading channel with $m = 3$ for diversity order of $L = 5$.

Figure 6: Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 0.5$, 0.75, 1, 1.5, 2, and 3 using first order selection combining (SC) for diversity order of $L = 2$. 
Figure 7: Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 0.5, 0.75, 1, 1.5, 2,$ and $3$ using second order selection combining (SC-2) for diversity order of $L = 3$.

Figure 8: Performance of the non-coherent BFSK receiver over a Nakagami fading channel with $m = 0.5, 0.75, 1, 1.5, 2,$ and $3$ using third order selection combining (SC-3) for diversity order of $L = 4$. 