A Comparison between the Impact of Some Signals on an MSK Receiver

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***Abstract* - Transmitters and electrical equipment co-located to a radio system can, due to its radiated electromagnetic interference, cause serious degradation on the system performance. In real system design when predicting the loss of performance, with a set of interferences of various signal types, there is a need of simple approximations that simultaneously describe the interferences in a proper manner. The focus of this paper is to show how different interfering signals will affect the performance of a radio system. Furthermore, the consequences of approximating these interference signals as additive white Gaussian noise are investigated.**

I. INTRODUCTION

When a radio system is located in the vicinity of other transmitters or interfering electric equipment, the radio system will suffer a performance degradation, due to the radiated electromagnetic interference. With an increased use of electrical interfering equipment in the vicinity of the receiver, it is necessary to take this interference into account. Also when a system is subjected to deliberately interfering signals, such as jammers, we need to be able to predict the consequences.

Traditionally, these interfering signals have often been either neglected, if their power at the radio receiver is lower than a certain level, or approximated as additive white Gaussian noise (AWGN) when the performance of the system has been analysed. In other cases, rather complex calculations are adopted when the performance is determined. In real system design, with a set of interferences of various signal types, these methods are precarious and simple approximations are desirable. The consequences of approximating interfering signals as Gaussian processes are not very well documented, why there is a need to investigate this approximation for some relevant interfering signal types. The focus of this paper is to show how different interfering signals will affect the performance of a radio system when thermal noise is also present. Furthermore, the consequences of approximating these interference signals as additive white Gaussian noise are investigated.

This approximation is investigated for a radio system using minimum shift keying (MSK) modulation on a Gaussian channel, without using any error correction. This modulation scheme is of practical interest, since it is commonly used in military applications. The analyses are performed on a parallel MSK-type receiver. Comparisons are then made between the bit error probability (BEP), due to (i) AWGN, (ii) an interfering MSK signal, (iii) a continuous sine wave, (iv) a pulse modulated continuous sine wave and (v) pulse modulated AWGN, all with equal average power.

The performance of modulation schemes in AWGN are well investigated in the literature and the expressions for the BEP are, in general, quite simple to use. An MSK modulated signal as an interferer is thoroughly investigated in [1] and these results are used here. This case applies to the situation when several radio transmitters, using MSK modulation, are operating in a limited area in the same frequency band. The other interferences analysed, occur frequently when we consider electrical equipment in the vicinity of the radio receiver. For example, a periodic signal can, after an RF filter, appear to the receiver as one or a few sine waves. The pulse modulated noise is applicable as a model for a unit using repetitive signals. The impact of a continuous sine wave on an MSK receiver has been studied in [2] and [3], but these articles consider a slightly different receiver and do not consider differentially encoded MSK. The exposure of a continuous sine wave and a pulse modulated noise on this kind of receiver have not been published earlier and will be investigated. A comparison is made between the performance degradation due to the proposed interfering signals. These signals are relevant both in the jamming case and when the system is subjected to an adjacent interfering electric equipment.

The most fundamental information about the modulation scheme and the receiver analysed are given in section II. For more information the reader is referred to the references given. In section III, the performance degradation of the system is calculated and comparisons between the cases with different interfering signals and the Gaussian approximation are shown. It is shown that pulse modulated noise with a short duty cycle has a large impact on the receiver performance.

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Pulse modulated AWGN is therefore further investigated in section IV from a jammer's point of view. Finally, the conclusions are drawn in section V. The overall conclusion is that the performance degradation is highly dependent on the signal type of the interfering signal and that it is therefore misleading, especially for pulse modulated noise, to only consider the interference power at the input of the radio receiver, when predicting the communication quality.

II. PRELIMINARIES

The minimum shift keying signal is defined as [4]:

$$s(t,\overline{\alpha}) = \sqrt{\frac{2E_b}{T}} \cos[2\pi f_c t + \phi(t,\overline{\alpha})] \ , \qquad (1)$$

where

- f_c is the carrier frequency [Hz],
- E_b is the bit energy [Ws],
- T the bit duration [s],
- $\phi(t,\overline{\alpha})$ is the signal phase, and
- $\overline{\alpha}$ is the data sequence of statistically independent symbols taking the values with equal probability.

The continuously changing phase is dependent on the time and previous data bits and can be expressed as

$$\phi(t,\overline{\alpha}) = \frac{\pi}{2}\sum_{i=-\infty}^{n-1}\alpha_i + \pi\alpha_n q(t-nT) \ , \qquad (2)$$

where

$$q(t) = \begin{cases} 0 & t \le 0 \\ t/(2T) & 0 \le t \le T \\ 1/2 & t \ge T \end{cases} \ . \qquad (3)$$

In (2), it is obvious that the phase depends on previous symbols and that the phase either increases or decreases by $\pi/2$ every bit interval, depending on the present data bit.

The receiver is a parallel MSK-type receiver, see Fig. 1, which is an optimum receiver, with respect to bit error rate performance. The filters $a(t)$ are normalized signal matched filters. Before the decision logic the detector makes phase decisions whether the phase is 0 or π every $t = 2nT$ in the upper path, and whether the phase is $\pi/2$ or $-\pi/2$ every $t = (2n+1)T$ in the lower path. The decision logic then differentially determines the information bits out of the phase decisions every bit interval.

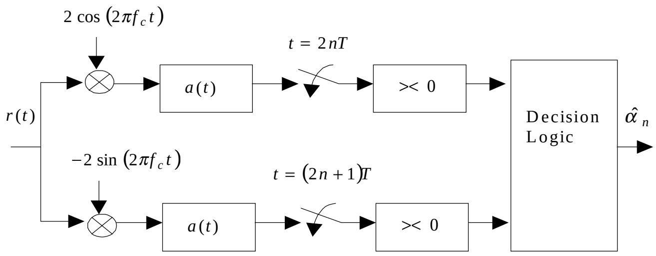

Fig. 1: A parallel MSK-type receiver, [5].

III. THE IMPACT OF INTERFERING SIGNALS

A. An MSK receiver subjected to AWGN

When the received signal consists of the transmitted MSK signal and thermal noise, modelled as additive white Gaussian noise with double-sided power spectral density of $N_0/2$, the probability of making an incorrect phase decision is obtained as

$$P_{phase} = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right) \ , \qquad (4)$$

where $erfc(x)$ is the complementary error function. The phase decision is an ordinary BPSK bit decision. To make a correct bit decision, two successive correct phase decisions or two successive wrong phase decisions are required, [6], which yields the total bit error probability

$$P_b = 2P_{phase}(1 - P_{phase}) \ . \qquad (5)$$

B. An MSK Receiver subjected to another MSK signal

In order to investigate the impact of an interfering MSK signal, the derived expression of the BEP in [1] is used.

C. An MSK Receiver subjected to a continuous sine wave

The interfering continuous sine wave is described as

$$i(t) = \sqrt{\frac{2E_b}{T}}\gamma \cos[2\pi(f_c + \Delta f)T + \varphi] \ , \qquad (6)$$

where

- φ is a random variable uniformly distributed over $[0, 2\pi]$ and
- Δf is the difference between the receiver center frequency and the carrier frequency of the continuous wave.

By using the notations in the definition of an MSK signal, see (1), the ratio between the signal power and the interference power, SIR, before the detector becomes SIR=$1/\gamma^2$. To obtain the error probability, when the receiver is subjected to a continuous sine wave and thermal noise in form of AWGN, we study the decision variable for the phase decisions. Considering the random phase φ fixed, the decision variable will be a normally distributed with mean $\sqrt{E_b} + \lambda(\Delta f, \varphi)$ and variance $N_0/2$, where $\lambda(\Delta f, \varphi)$ is the contribution from the interfering signal. It is quite straight forward to derive the decision variable of the interfering signal $\lambda(\Delta f, \varphi)$. The phase error probability is then averaged over φ :

$$
\begin{aligned}
P_{phase} &= E\left[\frac{1}{2} erfc\left(\frac{\sqrt{E_b} + \lambda(\Delta f, \varphi)}{\sqrt{N_0}}\right)\right] \\
&= \frac{1}{4\pi}\int_0^{2\pi} erfc\left(\sqrt{\mathrm{SNR}}\left\{1 + \frac{4\cos\varphi\cos(2\pi\Delta f T)}{\pi\sqrt{\mathrm{SIR}}\left[1-(4\Delta f)^2\right]}\right\}\right) d\varphi
\end{aligned} \quad , \quad (7)
$$

where SNR is defined as the signal-to-noise ratio per bit, E_b/N_0 and SIR as $1/\gamma^2$. In a real application, φ is not known. Here, the phase error probability, averaged over φ , is used as the evaluation parameter of interest. In (7), we use the assumption that the phase error probability is independent of the information sequence $\overline{\alpha}$, i.e. $P_{phase} = E\{P(\text{phase error}|\overline{\alpha})\}$ = $P(\text{phase error}|\overline{\alpha})$, due to the symmetry of MSK signals, [5].

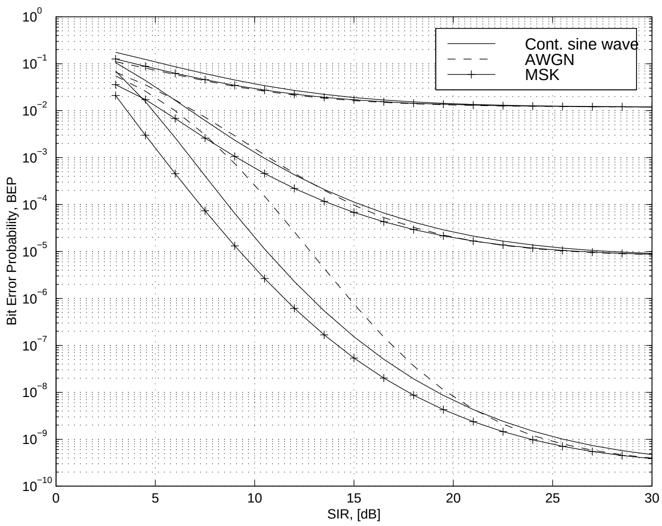

Fig. 2: The calculated BEP for different interfering signals.

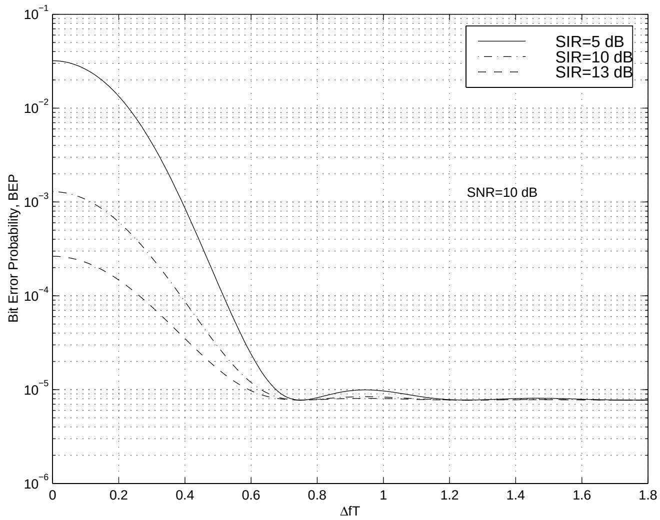

Fig. 3: Calculated BEP as a function of the product ΔfT for different SIR.

In order to compare how a radio system is affected by these interfering signals above discussed, we study the error probability. In Fig. 2, the bit error probability is calculated as a function of signal-to-interference ratio when an MSK signal is exposed to an interfering signal of additive white Gaussian noise, a continuous sine wave and an MSK signal, respectively. The interfering signals are located at the carrier frequency with an additional thermal noise present, yielding a constant signal-to-noise ratio of 5, 10 and 13 dB.

We can see that an interfering MSK signal always results in a lower error probability than a continuous sine wave and AWGN with equal power. The sine wave and the AWGN alternate in causing the worst performance. An AWGN approximation of an MSK or a continuous wave interferer often causes a larger BEP. For those SIR where it does not, it is only slightly lower than the real value or conditions are too bad, resulting in too high BEP to use for radio communication. When the SIR becomes large for a certain SNR, all curves tend to coincide, since the interference becomes negligible.

In Fig. 3, the BEP is shown as a function of the product ΔfT . Here we can see how the BEP is decreasing when the continuous wave is moved from the center frequency. Quite natural, from a jammer's point of view, it is most effective to locate a tone jammer as close as possible to the center frequency of the radio system. For $\Delta fT = 3/4, 5/4, \ldots$ the BEP assumes its minimum, due to the fact that the decision variable of the interfering signal becomes zero.

D. An MSK receiver subjected to a pulse modulated sine wave

Another commonly appearing and therefore relevant interference wave form is the pulse modulated signal. In this case we study a signal which periodically off and on transmits a sine wave. The fraction of time when the interference is present is defined as ρ and the probability that this will occur is therefore also ρ. If we make the assumption that the interfering signal is either present an entire bit interval or not present at all, we get the expression of the error probability as

$$P_b = \rho P_b(\text{interference and thermal noise}) + (1-\rho)P_b(\text{thermal noise}) \quad , \qquad (8)$$

where $P_b(\text{interference and thermal noise})$ is the BEP caused by AWGN with the single-sided power spectral density N_0 and an interfering signal. Consequently, $P_b(\text{thermal noise})$ is the BEP when only the thermal noise is present. When we compare the error probability for different interfering signals, it is important that the interfering signals have equal average power in order to make a relevant comparison. The signal-to-interference ratio is therefore defined as the average value of SIR and not as the instant value.

In Fig. 4, the BEP is calculated, when the receiver is subjected to a pulse modulated sine wave, a continuous sine wave and AWGN, respectively, by using (4), (7) and (8). It shows that a pulse modulated sine wave with the power concentrated to as short intervals as possible gives the worst performance degradation for SIR larger than a certain value. The sine wave is assumed to use the same frequency as the carrier frequency of the desired MSK signal. Furthermore, we can see that the BEP approaches $\rho/2$ when SIR is becoming small and the thermal noise is neglected. This is due to the fact that the BEP is approximately $\rho P_b(\text{dominating interference})$, when SIR is small. The figure also shows the BEP when the radio system is degraded by an interfering signal consisting of AWGN besides the thermal noise. The difference between impact of a continuous sine wave, $\rho = 1$, and AWGN is not especially large. It is mainly the fact that the interference is pulse modulated rather than what wave form the interference has, that affects the BEP. We can see that it would be incorrect to assume a noise source to be AWGN when we are dealing with pulse modulated noise. For small values of ρ it can result in a much lower BEP than the real value. This conclusion is based on the fact that the pulse width of the interfering signal is at least T, which means that the whole interference power is

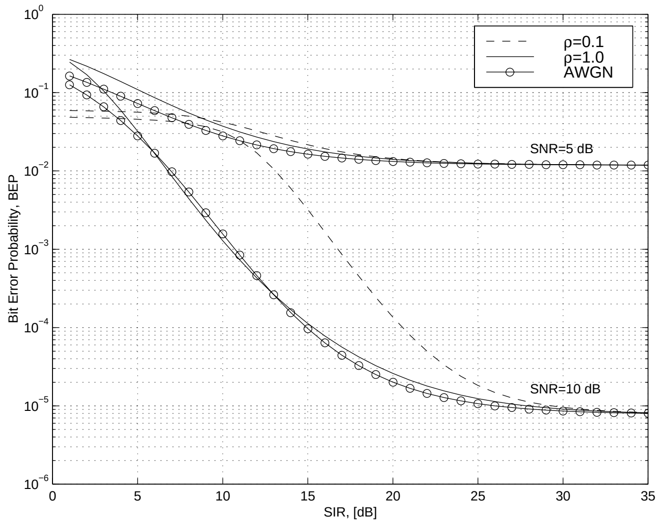

Fig. 4: Calculated BEP for a continuous wave, a pulse modulated continuous wave and AWGN with the same average power.

within bandwidth of the matched filters. Electrical units in the neighbourhood of the receiver are often of a pulse modulated nature. Therefore it is of great importance to be able to estimate how these units work in time, i.e. ρ, and not to approximate the noise sources with AWGN out of its average power.

E. An MSK receiver subjected to pulse modulated additive white Gaussian noise

The BEP when the receiver is subjected to pulse modulated AWGN and pulse modulated continuous wave are very similar and the degradation is highly dependent of the pulse duty factor. Therefore, it is inappropriate to approximate pulse modulated AWGN with AWGN with the same average power without having knowledge of the duty factor.

IV. Optimum Duty Factor in a Jammer's Aspect

A jammer could take advantage of the fact that the receiver for certain ranges of SIR is very vulnerable to pulse modulated noise. By an intelligent choice of the pulse duty factor ρ of a pulse modulated AWGN a severe degradation of the performance can be accomplished.

If we neglect the thermal noise, the optimum ρ that achieves worst performance degradation becomes

$$\rho = \begin{cases} \dfrac{0.82}{E_b/N} & E_b/N > 0.82 \\ 1 & E_b/N \le 0.82 \end{cases} \quad . \qquad (9)$$

We have made the assumption that the interfering signal is either present an entire bit interval, with the instant single-sided power spectral density N/ρ, or not present at all. The optimum value of ρ is found by maximizing the BEP. Since the BEP contains several erfc-function expressions, the relation between ρ and E_b/N is calculated numerically.

When a jammer dominates in signal power at the receiver, it accomplishes worse degradation by using a continuous interferer instead of a pulse modulated one, as shown in (9). By choosing the optimum value of the pulse duty factor ρ to achieve maximum performance degradation, the error probability becomes

$$P_b = \begin{cases} \dfrac{0.15}{E_b/N} & E_b/N > 0.82 \\ erfc\left(\sqrt{\dfrac{E_b}{N}}\right)\left[1-\dfrac{1}{2}erfc\left(\sqrt{\dfrac{E_b}{N}}\right)\right] & E_b/N \le 0.82 \end{cases}, \quad (10)$$

According to (10), the maximum bit error probability is obtained by the continuous AWGN for $E_b/N \le 0.82$, while a pulse modulated noise causes worst degradation when $E_b/N > 0.82$, where the BEP becomes a linear function of SIR expressed in decibel.

If the thermal noise is taken into consideration also SNR will affect the optimal duty factor. We get a relation between the ρ and SNR and E_b/N as

$$\rho \approx \begin{cases} \dfrac{0.8 \cdot 10^{1/\mathrm{SNR}}}{E_b/N} & E_b/N > 0.8 \cdot 10^{1/\mathrm{SNR}} \text{ and SNR} \ge 2 \\ 1 & E_b/N \le 0.8 \cdot 10^{1/\mathrm{SNR}} \text{ or SNR} < 2 \end{cases}, \quad (11)$$

Equation (11) is derived numerically, by the same reasoning as in (9). In (11), we have a rather simple expression for how the error rate can be maximized when a jammer uses a certain ρ. However, a jammer is seldom in possession of the exact values of SNR and E_b/N. Instead, this relation gives a hint of interference situations that can bring a serious impact on a radio receiver.

V. Conclusions

We show that the performance degradation is highly dependent on the signal type of the interfering signal and that it is therefore misleading to only consider the interference power at the input of the radio receiver, when predicting the communication quality. The largest differences in BEP are obtained when we compare a system disturbed by pulse modulated noise, with short pulse duty factor, and AWGN with the same average power. These differences are smaller when a system is subjected to either an interfering MSK signal or a continuous sine wave, with the same frequency as the receiver center frequency. An interfering signal of AWGN is in most cases causing a higher BEP than a continuous sine wave and always causing a higher BEP than an interfering MSK signal, although the difference is not very large.

To sum up, it is in certain cases appropriate to approximate an interfering MSK signal and an interfering continuous sine wave with AWGN. When evaluating pulse modulated noise, on the contrary, the approximation does not work, except for very large pulse duty factors of the interfering signal. That is, only information about the average power of an interfering signal is not sufficient when predicting the error probability.

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