A METHOD AND METRIC FOR QUANTITATIVELY DEFINING
LOW PROBABILITY OF DETECTION

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ABSTRACT
The lack of a low probability of detection (LPD) waveform is often cited as one of the criteria against the use of Commercial Off-The-Shelf (COTS) communications systems in the tactical military environment. However, there is rarely any quantitative evidence given to support this understandable qualitative assertion. We propose a method for quantitatively defining LPD that is based on well known principles of communications theory and open sources of COTS system specifications. Such a method of quantitatively defining LPD allows researchers working in unclassified environments to consider the LPD characteristic of a communications system while in the design phase, fair comparisons in LPD characteristics between competing systems, and the potential for active participation of the electronic warfare (EW) community in the up front development of future tactical military communications systems.

INTRODUCTION
The lack of a low probability of detection (LPD) waveform is often cited as one of the criteria against the use of Commercial Off-The-Shelf (COTS) communications systems in the tactical military environment. However, there is rarely any quantitative evidence given to support this understandable qualitative assertion. The lack of quantitative evidence results in part from the necessary security considerations associated with the electronic warfare (EW) community. As the pressure to use COTS increases, there is a need for an unclassified quantitative method and metric for defining the LPD characteristic of a given system so that wise and prudent decisions can be made regarding the use of COTS communication systems.

We apply results from classical radiometer analysis combined with the geometry of friendly base stations and mobiles operating in a hostile environment to define a detectability distance for quantifying the LPD characteristics of a given COTS system. The detectability distance is a relative measure which exploits the behavior of the probability of detection curves when viewed as a function of the normalized distance from the transmitter. The detectability distance provides a consistent, useful and quantitative measure of covertness for wireless communication systems.

Using this method, we investigate the covertness of the following communication systems: GSM, IS-54, IS-95, and a generic wideband system which is representative of future wideband CDMA communication systems. The results quantify the tradeoff between the time an interceptor must wait for detection and the detectability distance. We found that if the CDMA COTS systems support multiple users, the base station will always be detected at a greater distance than the mobile user.

The remainder of the paper is organized as follows. The next section describes the system model and gives the baseline assumptions used in the simple radiometer and system analysis. Next, the methodology behind the analysis is discussed using classical communications theory. The final section provides the numerical results from the system comparisons, for both the single user and multiple user scenarios, including a discussion of the tradeoffs between the observation interval and the detectability distance. This section also discusses the covertness of a transmitting mobile user versus the covertness of a transmitting base station.

SYSTEM MODEL
All systems considered in this paper use a cellular architecture and are connection oriented. In these systems, the mobiles communicate with each other using a base station as a relay node. A hostile environment is assumed, where an intercepting receiver may be in the proximity of a transmitting base station. We assume throughout the paper that the hostile receiver is not set up to intercept and decode a transmission, rather, it is only used to determine the presence of a transmitting base station or mobile (detection) using a simple radiometer. (Note, however, that we use the term "interceptor" to describe this receiver). The position of this interceptor can be described with two parameters, using a polar coordinate system, where r is the distance from the base station to the interceptor, and $\alpha$ is the angle off of the line between the base station and the mobile as shown in Figure 1. From the figure, we see that $d$ is the distance from the interceptor to the mobile. All distances are normalized to $d_0$, the distance from the base station to the desired mobile.

The following baseline assumptions are used throughout the analysis. Initially we assume there is only one user in the system, meaning there are no friendly interferers or other friendly base stations. We neglect losses within the transmitter and receiver, and we assume that only thermal noise is present. We assume a mean path loss, $L_p$ in dB, to the
intended receiver. The antennas are unity gain and omni-
directional for both the transmitter and receiver. For this
paper, we assume that \( a \) in Figure 1 is zero, and the in-
teceptor is in line with the base station and the mobile. Also
we assume that all the receivers are ground based receivers
and we set the path loss exponent to be a constant value of 4
for all communications in a given area. We assume that the
systems have perfect power control and always operate at a
constant bit energy to noise spectral density ratio, \( E_b/N_0 \), of
10 dB. This 10 dB figure is reasonable since, without loss of
generality, we neglect system losses and circuit noise.

**METHODOLOGY**

Our development assumes the intercepting receiver is a
wideband radiometer. The major advantage of this type of
receiver is that it requires relatively little hardware, and no
additional hardware is needed to detect spread spectrum sys-
tems. It is the optimum receiver for detecting the prescence
of a signal in additive white Gaussian noise [1]. More sophis-
ticated radiometer configurations utilizing additional knowl-
dge about the target system could perform better, but we
confine the methodology presented in this paper to a simple
radiometer for generality.

The radiometer we use here consists of a filter of band-
width \( w \), a square law detector, and an integrator with in-
tegration time \( T \). The input to the radiometer is sampled
and processed with a fixed threshold level, \( l \). The prob-
abilities of detection, \( P_D \), and false alarm, \( P_{FA} \), are then
calculated.

The approach we use is to assume an allowable \( P_{FA} \) and
then find the corresponding threshold, \( l \). Once \( l \) is found, \( P_D \)
can be computed as a function of the post detection signal
to noise ratio.

Using detection theory and hypothesis testing [2], the rela-
tionship between \( P_{FA} \) and \( l \) are determined. A false alarm
occurs when the output of the integrator is greater than the
threshold in the absence of the signal. With only noise
present at the input, the output of the integrator has a cen-
tral Chi-square density function which is approximated by a
Gaussian distribution when the time-bandwidth, \( Tw \), product
is large. If the target system is transmitting a determinis-
tic signal, and \( n(t) \) is bandlimited white Gaussian noise with
two sided power spectral density equal to \( N_0/2 \), and where
\( E \) denotes the signal energy, then \( P_{FA} \), [1], [3] is given by

\[
P_{FA} = \frac{1}{\sqrt{2\pi N_0^2 Tw}} \int_{-\infty}^{\infty} \exp \left( -\frac{(v - N_0 Tw)^2}{2N_0^2 Tw} \right) dv \tag{1}
\]

\[
= \frac{1}{2} \text{erfc} \left( \frac{l - N_0 Tw}{\sqrt{2N_0^2 Tw}} \right), Tw \gg 1 \tag{2}
\]

Similarly, if the signal is present, \( P_D \), is given by

\[
P_D = \frac{1}{2} \text{erfc} \left( \frac{l - N_0 Tw - E}{\sqrt{2N_0^2 Tw}} \right), Tw \gg 1 \tag{3}
\]

The threshold level, \( l \), can be written as

\[
l = \sqrt{(2N_0^2 Tw) \text{erf}^{-1}(1 - 2P_{FA}) + N_0 Tw} \tag{4}
\]

where \( T \) is the interval of time that the intercepting receiver
observes the channel, and \( w \) is the radiometer filter band-
width.

We can assume that an intercepting receiver would have
complete knowledge of COTS system specifications, and
would therefore set the radiometer parameters accordingly.
For this reason, we also assume the following: The radiome-
ter bandwidth, \( w \), is set to the bandwidth of one frequency
band for the corresponding target system. In addition, the
intercepting receiver is synchronized with the intended re-
ceiver, meaning the observation interval begins exactly with
the start of a transmission burst. As will be shown later,
this assumption does not affect the results if the observation
interval is larger than one frame time.

The received power of the traffic channel\(^2\) is a function of
the transmitted power, path loss, antenna gains and the in-
ternal losses of the transmitter and receiver. Using the link
budget analysis for cellular systems in [4] and applying the
assumptions given above, \( E_t \) is found for a given (acceptable)
value of \( E_b/N_0 \) in dB, and \( P_D \) is calculated for a given \( P_{FA} \).
Using the intended receiver as a reference at a distance \( d_0 \),
and assuming that the interceptor will experience a similar
path loss exponent at a distance \( d \), \( E_b \) is found at the inter-
cepting receiver and \( P_D \) can be obtained as a function of the
ratio of distances between the two receivers.

This analysis can be used to detect both transmitting base
stations and mobiles. As an example, we apply these equa-
tions to an IS-95 system. Applying the assumptions given
above to this system, the filter bandwidth of the radiometer
is set to the bandwidth of the system, \( w = 1.25 \text{ MHz} \). Also,
assuming a constant bit rate, \( b = 9600 \text{ bps} \), the observation
interval is set to \( 1/b \). The interceptor must choose an ac-
ceptable \( P_{FA} \). We assume throughout that \( P_{FA} = 0.02 \). A
plot of \( P_D \) versus the distance to the intercepting receiver,
normalized to the distance between the base station and the
mobile is shown in Figure 2.

In Figure 2, we see that as the distance increases, the \( P_D \)
decreases and the curve for the \( P_D \) is relatively steep be-
 tween 0.2 and 0.9, and can be approximated by a step function.
Because of this step function like behavior, the information from
the curve can be represented by a single distance. Thus, as a
measure of covertness, we propose defining the detectability
distance as the distance where the \( P_D \) is some acceptable
value (e.g., 0.5). As the detectability distance gets smaller,
the interceptor must get closer to the transmitter to detect a
transmission and the system becomes more covert.

\(^2\)We neglect the IS-95 pilot signal in this analysis.
Although the detectability distance is a function of $P_{FA}$, there is a way to remove the dependence on $P_{FA}$ when comparing two systems. By considering a ratio of detectability distances of two systems at one $P_{FA}$ and comparing this ratio to the ratio of detectability distances at another $P_{FA}$, we found empirically that the ratios are the same. This provides a way to compare covertness of the two systems independent of the $P_{FA}$. For example, for IS-95 and IS-54, the detectability distance for IS-54 is 1.5929 times that of the detectability distance for IS-95 given a $P_{FA} = 0.2$. If the $P_{FA}$ is changed to $3 \times 10^{-17}$, the detectability distance for IS-54 is 1.5930 times that of the detectability distance for IS-95. The ratio of these two values is 0.9999. Therefore, the two systems exhibit the same covertness characteristics (i.e., the ratio of detectability distances), no matter what $P_{FA}$ is chosen. Comparisons of other systems yielded the same result.

In the previous example, $T$ was set to the bit time. It is interesting to view the detectability distance as a function of $T$. If the transmission burst length, $t$, is constant and $T$ increases beyond $t$, more noise energy is added while the amount of received signal energy remains constant, and thus $P_D$ decreases. However, if $t$ is changed such that the base station is transmitting continuously, $P_D$ increases with increasing $T$. The result of this relationship is that if $t \geq T$, then increasing the observation interval decreases the received signal energy required for detection and if $t < T$, an increase in the observation interval increases the received signal energy required for successful detection.

**SYSTEM COMPARISONS**

The following systems are compared in this section: IS-54, IS-95, GSM, and IS-95 derivatives (same as IS-95, but with larger bandwidths) representative of future wideband CDMA systems. We start by considering only one user per transmitting base station and assume that the observation interval is exactly equal to the burst interval (which is known from COTS system specifications) of the communication system to maximize $P_D$. Since the CDMA base stations transmit continuously rather than having a fixed burst interval, we assume that the base stations transmit for the same length of time as the IS-54 system. The parameters in Table I are used for the comparison of the COTS systems.

<table>
<thead>
<tr>
<th>System</th>
<th>$T = t$</th>
<th>$T = 576.9\mu s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wideband CDMA</td>
<td>1.15</td>
<td>0.85</td>
</tr>
<tr>
<td>GSM</td>
<td>1.26</td>
<td>1.26</td>
</tr>
<tr>
<td>IS-95</td>
<td>1.36</td>
<td>1.00</td>
</tr>
<tr>
<td>IS-54</td>
<td>2.17</td>
<td>1.60</td>
</tr>
</tbody>
</table>

**TABLE II**

Detectability distances for transmitting COTS base stations with one mobile user. In the first column, the observation interval is equal to the transmission burst interval for each system. In the second column, the observation interval is equal to the transmission burst interval for GSM.

Although it appears in this comparison that GSM is more covert than IS-95, the result is dependent on the choice of the observation interval. GSM has a small transmission burst interval, and is therefore more difficult to detect if the observation interval is equal to a frame time. The IS-95 base station transmits continuously and is therefore less covert despite the use of a spread spectrum (direct sequence) waveform, because an intercepting receiver can simply "listen" longer and increase the $P_D$. Reducing the observation interval for the CDMA systems to the transmission burst length of GSM reduces the detectability distances of the IS-95 system and its wideband derivatives. These detectability distances are...
given in the second column of Table II. The detectability distance for the wideband CDMA system has been reduced to 0.85. The interceptor must be closer to the base station than the intended user because the intended receiver is matched to the transmitted signal while the simple radiometer is not. One conclusion of our study is that the detectability distance is heavily dependent upon the observation interval — a more detailed discussion on the observation interval follows in the subsection entitled “Transmission Burst length & Observation Interval”.

A similar comparison was completed for several IS-95 like “wideband” systems. The systems considered have bandwidths of 5 MHz, 2.5 MHz, and 1.25 MHz. It is expected that as the bandwidth is increased, the amount of noise energy that is added increases while the signal energy remains constant and thus the system would become more covert. As the bandwidth of the system increases, the interceptor must move closer to the transmitter to detect the signal with significant probability. The detectability distances are given in Table III. For the wideband system, higher bandwidth implies a decrease in detectability distance or a corresponding increase in covertness.

<table>
<thead>
<tr>
<th>Bandwidth (MHz)</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.15</td>
</tr>
<tr>
<td>2.5</td>
<td>1.25</td>
</tr>
<tr>
<td>1.25</td>
<td>1.35</td>
</tr>
</tbody>
</table>

TABLE III
Detectability distances for wideband system base stations with one mobile user and \( T = 6.667 \text{ ms} \).

**Multiple-User Scenario**

The preceding comparisons consider only one mobile per base station channel. This is not necessarily a realistic scenario. On the battlefield, there may be enough active users to fill all available TDMA channels, and thus increase the transmission burst length for the base station. To compare the systems assuming this situation, we must recalculate the energy transmitted by the base station to include multiple users.

In the TDMA systems, GSM and IS-54, increasing the number of mobiles leads to a corresponding increase in the number of time slots in which the base station transmits. For GSM, there are up to eight connections allowed per frame in a frequency band, and therefore the transmission time is increased by a factor of eight. Similarly for IS-54, there are three slots per frame, thus increasing the transmission burst interval by a factor of three for a fully loaded frequency band.

For a fair comparison we assume the CDMA systems include eight users, since this is the maximum number of users supported by one GSM frequency band. In the CDMA systems, each additional user increases the total power in the system, and we adjust the energy received at the radiometer accordingly. As the number of users is increased, the systems are detectable at a greater distance, thus decreasing the covertness. The detectability distances are shown in Table IV.

<table>
<thead>
<tr>
<th>System</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wideband CDMA</td>
<td>1.29</td>
</tr>
<tr>
<td>IS-95</td>
<td>1.56</td>
</tr>
<tr>
<td>GSM</td>
<td>1.65</td>
</tr>
<tr>
<td>IS-54</td>
<td>2.47</td>
</tr>
</tbody>
</table>

TABLE IV
Detectability distances for transmitting base stations with multiple users.

**Transmission Burst Length & Observation Interval**

We have assumed that the transmission burst length is constant during communication. However, it is not likely that the mobile users will have constant transmissions. For this analysis, the observation interval is held constant, while the transmission burst length is varied. We assume that the transmission burst lengths are random and are uniformly distributed. For each value of the transmission time, the detectability distance is calculated and since we assumed the transmission times are a uniformly distributed random variable, the average distance to detection for each system can be obtained by summing the distances, and dividing by the number of observations.

Figure 4 shows a plot of 200 uniformly distributed transmission times and their respective detectability distances for each of the four systems. Moving up the plot, the detectability distance increases implying reduced covertness. The average distances to detection are given in Table V.

<table>
<thead>
<tr>
<th>System</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>IS-54</td>
<td>2.94</td>
</tr>
<tr>
<td>GSM</td>
<td>2.32</td>
</tr>
<tr>
<td>IS-95</td>
<td>1.85</td>
</tr>
<tr>
<td>Wideband</td>
<td>1.55</td>
</tr>
</tbody>
</table>

TABLE V
Average detectability distances for transmitting COTS mobiles with variable transmission burst lengths.

In the above discussion, the observation interval is held constant while the burst length is varied. We now assume continuous transmission. For this analysis, the observation
interval is varied and we assume that the transmitter is a base station with a transmission burst length long enough to transmit for the entire observation interval. Figure 5 shows that as the observation interval increases, the detectability distance increases as well. The sawtooth nature of the GSM and IS-54 curves is because of the duty cycle of the system. As the observation interval increases beyond the first time slot, no more signal energy is received until the next frame. The radiometer receives more noise energy without a corresponding increase in signal energy, so the interceptor must get closer to the transmitter to continue to detect the signal with the same \( P_D \).

Figure 6 considers a loaded system as outlined in previously. As users are added to the TDMA system to fill all the time slots, the transmission duty cycle of the base stations increases. This results in the curves being shifted up, as shown in Figure 6, indicating an increase in the detectability distance from the single user scenario. In this scenario, GSM is more covert than the other COTS systems. Intuitively, we expect the spread spectrum systems to be more covert, but as the observation interval increases, the radiometer receives more energy, resulting in increased detectability for the spread spectrum systems. In the single user case, GSM is more covert because of the reduced duty cycle when compared to the transmitted signal of a spread spectrum system base station. GSM maintains this covertness for a larger number of users because the spread spectrum system base station increases average transmitted power as the number of users increases.

**Mobile vs. Base Station**

Military communication systems based on the standard cellular architecture are dependent on base stations for communication on the battlefield. However, a base station transmits with more power than an individual user, and is therefore more likely to be detected by an intercepting receiver. If a base station is detected and destroyed, communication by soldiers served by that base station is impossible. Also, the detection of a base station alerts potential enemies that there are mobiles in the area as well. Therefore, for each COTS system it is important to know whether the mobile or the base station is detected first.

Assume that the intercepting receiver, base station, and mobile form a straight line. Also assume that for the TDMA systems, the base station has enough channels to support one user per channel. This assumption means that there is no need to fill all the slots in the TDMA frame, and the base station and mobile will transmit with roughly the same average power. In the CDMA systems, we again assume there is only one user “active” and the base station and mobile transmit with roughly the same power. However, a voice activity factor is applied as outlined in [4]. The voice activity factor takes into account the fact that the mobile user is active less than 100 percent of the time. The duty cycle of the mobile transmitter is controlled by the voice activity factor, but the base station duty cycle is not affected. We use a voice activity factor of 50% for the mobile, thus the transmission time for the base station will be twice that of the mobile. Figure 7 is a plot of the resulting \( P_D \) as the interceptor moves along the \( d \) axis for GSM. The detectability distances with one user for all the COTS systems, both base stations and mobiles, are given in Table VI.

**Figure 7. Probability of detection vs. distance for both GSM base station and mobile with only one user.**

Figure 7 shows that the intercepting receiver will always detect the mobile before the base station in this scenario (note — when the interceptor moves closer to the base station, the interceptor is moving away from the mobile and the
System	Distance
---
GSM	base station 0.97
	mobile 1.97
Wideband CDMA	base station 1.31
	mobile 2.06
IS-95	base station 1.56
	mobile 2.26
IS-54	base station 1.89
	mobile 2.89

**TABLE VI**
Boresight detection distances for COTS systems with one user.

**Fig. 8.** Probability of detection vs. distance for both IS-95 base station and mobile with multiple users.

probability of detecting the mobile decreases). However, if the number of users served by the base station are increased, the same results do not necessarily hold. To increase the number of users in the TDMA systems, the time slots must be filled as before, and the transmitted power adjusted accordingly. Similarly the CDMA system base stations increase the total transmitted power to account for the extra users. Figure 8 shows that the intercepting receiver will detect the base station before the mobile in the IS-95 system. The detectability distances for all of the COTS systems with multiple users are given in Table VII.

<table>
<thead>
<tr>
<th>System</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSM</td>
<td>base station 1.63</td>
</tr>
<tr>
<td></td>
<td>mobile 1.97</td>
</tr>
<tr>
<td>Wideband CDMA</td>
<td>base station 2.21</td>
</tr>
<tr>
<td></td>
<td>mobile 2.06</td>
</tr>
<tr>
<td>IS-95</td>
<td>base station 2.62</td>
</tr>
<tr>
<td></td>
<td>mobile 2.25</td>
</tr>
<tr>
<td>IS-54</td>
<td>base station 2.49</td>
</tr>
<tr>
<td></td>
<td>mobile 2.89</td>
</tr>
</tbody>
</table>

**TABLE VII**
Boresight detection distances for COTS systems with multiple users.

**CONCLUSION**

Using classical radiometer analysis and communication theory, we have developed a methodology to quantify the LPD characteristics of a wireless communication system using a *detectability distance*. This detectability distance is obtained by exploiting the step like function behavior of the $P_D$ curve for a given system. We have investigated the tradeoff between the observation time of the interceptor and the detectability distance. The detectability distance is also largely dependent upon the initial $P_{FA}$. However, we show empirically that a relative comparison between two systems does not depend on the initial $P_{FA}$. We have shown for the wide-band systems that higher bandwidth implies a decrease in detectability distance or a corresponding increase in covertness. We have also shown that, under certain conditions, if CDMA COTS systems are loaded to support multiple users, the interceptor will detect the base station at a greater distance than the mobile user. Using the simple radiometer interceptor, certain comparisons of these COTS systems also suggest that a GSM system is more covert than the spread spectrum systems examined. This is due to the bursty nature of a TDMA transmission, where as a spread spectrum system base station transmits continuously.

**REFERENCES**