ABSTRACT

In this paper, we consider signal reception in multicarrier code-division multiple-access (MC-CDMA) systems. A blind adaptive algorithm is proposed to determine a weight vector which optimally combines the desired signal contributions from different carriers as well as suppresses noises and interference. No knowledge of the channel conditions (fading coefficients, signature sequences and timing of interferers, statistics of other noises, etc.), nor any training sequence is required. The performance is examined under Rayleigh fading channels. Results show that the proposed algorithm performs well and is robust to the near-far problem.

I. INTRODUCTION

Recently, multicarrier code-division multiple-access (MC-CDMA) systems with direct-sequence spreading have been proposed [1] to derive the benefits of both multicarrier systems and direct-sequence systems. Together with the possibility of efficient fast Fourier Transform (FFT) implementations [2], MC-CDMA systems can be of practical interest. In order to effectively combine the desired signal contributions from different carriers, the maximal ratio combiner (MRC) is employed in [1]. However, the use of the MRC requires explicit estimation of the fading coefficients associated with the carriers. When interference and noises are also present, not only does it become more difficult to estimate the fading coefficients, but the statistics of the interference and the noises also have to be estimated for optimal reception. Explicit estimation of these quantities might be inefficient and might not be able to adapt quickly to changes in channel conditions.

This material is based on work supported in part by the U. S. Army Research Office under grant number DAAH04-95-1-0246 and in part by the Chinese University of Hong Kong Research Committee Funding.

In this paper, we propose a blind adaptive algorithm to determine a weight vector which optimally combines the contributions from different carriers. The desired signal contributions are constructively combined while noises and interference are suppressed. No knowledge of the fading coefficients, the signature sequences and the timing of the interferers, the statistics of other noises, etc., is assumed, and no training sequence is required. The proposed algorithm yields a weight vector that maximizes the signal-to-noise ratio (SNR), and, in general, performs better than the MRC. In direct-sequence systems, the near-far problem is of major concern. With multicarrier modulation, while the MRC fails to do so, the proposed algorithm can exploit the correlation between the received signals across different carriers to cancel multiple-access interference (MAI), and, hence, alleviate the near-far problem.

II. SYSTEM MODEL

In this section, we describe the model of the MC-CDMA system. We assume that there are K simultaneous users in the system, and each user uses the same M carriers.

The k-th user, for 1 ≤ k ≤ K, is provided with a random signature sequence \( a^{(k)} \) given by

\[
a^{(k)} = (\ldots, a_0^{(k)}, a_1^{(k)}, \ldots, a_{N-1}^{(k)}, \ldots)
\]

where the elements \( a_i^{(k)} \) are modeled as independent and identically distributed (iid) random variables such that \( \Pr(a_i^{(k)} = -1) = \Pr(a_i^{(k)} = 1) = 1/2 \). The same signature sequence \( a^{(k)} \) is used to modulate each of the M carriers of the k-th user.

The k-th user, for 1 ≤ k ≤ K, generates a stream of data symbols \( b^{(k)} \), given by

\[
b^{(k)} = (\ldots, b_0^{(k)}, b_1^{(k)}, b_2^{(k)}, \ldots).
\]
Each data symbol is multiplied by $N$ chips of the signature sequence.

The transmitted signal of the $k$-th user can be expressed as the real part of the following complex signal:

$$
\sum_{m=1}^{M} \sqrt{2P_k c_m^{(k)}} \left\{ \sum_{i=-\infty}^{\infty} b_{i/N}^{(k)} a_i^{(k)} \psi(t-iT_c) \right\} e^{j\omega_m t}.
$$

The parameter $P_k$ is the power for each carrier of the $k$-th user signal, $\omega_m$ is the frequency of the $m$-th carrier, and the parameters $c_m^{(k)}$ can be used as a code across the $M$ carriers of the $k$-th user, or can be chosen to condition the peak-to-average power ratio of the $k$-th user's transmitted signal [5].

For simplicity, we assume that $|c_m^{(k)}|^2 = 1$. We assume that the chip waveform $\psi(t)$ is bandlimited and the carrier frequencies are well separated so that adjacent frequency bands do not interfere with each other. We also assume that $\psi(t)$ is normalized so that $\int_{-\infty}^{\infty} |\psi(t)|^2 dt = T_c$. The parameter $T_c$ is the delay between consecutive chips.

Without loss of generality, we consider the signal from the first user as the desired signal and the signals from all other users as interfering signals throughout the paper.

We now describe the channel model. We assume that the channel is a frequency selective fading channel. By suitably choosing $M$ and the bandwidth of $\psi(t)$ [1], we can assume that each carrier undergoes independent frequency non-selective slow Rayleigh fading. We also assume the presence of additive white Gaussian noise (AWGN) with power spectral density $N_0$.

The received signal in complex analytic representation is given by

$$
r(t) = \sum_{k=1}^{K} \sum_{m=1}^{M} \sqrt{2P_k c_m^{(k)}} \left\{ \sum_{i=-\infty}^{\infty} b_{i/N}^{(k)} a_i^{(k)} \right\} \psi(t-T_k-iT_c) e^{j\omega_m (t-T_k)} \alpha_{k,m} + n(t),
$$

where $\alpha_{k,m}$ accounts for the overall effects of phase shift and fading for the $m$-th carrier of the $k$-th user, $T_k$ represents the delay of the $k$-th user signal, and $n(t)$ represents AWGN. We model $\alpha_{k,m}$, for $k = 1, \ldots, K$ and $m = 1, \ldots, M$, as iid zero-mean complex Gaussian random variables. We assume that synchronization has been achieved with the first user signal. Therefore, the delay of the first user signal $T_1$ can be taken to be zero.

We consider the receiver shown in Fig. 1. It consists of $M$ branches. Each branch consists of a correlator and an appropriate weight, and is responsible for the demodulation of one carrier. The correlator on the $m$-th branch consists of a chip-matched filter and a combiner that combines the contributions from different chips according to the signature sequence of the first user. We assume that the chip waveform and the chip-matched filter are chosen to satisfy the Nyquist criterion so that there is no inter-chip interference. The weight vector $w = [w_1, w_2, \ldots, w_M]^T$, whose value will be determined in the following section, is an $M$-dimensional vector that optimally combines the contributions from the $M$ branches to give the decision statistic.

### III. WEIGHT VECTOR DETERMINATION

We consider symbol-by-symbol detection. Without loss of generality, we consider the detection of the symbol $b_0^{(1)}$. The output of the correlator on the $m$-th branch due to the first user signal is given by

$$
s_m = b_0^{(1)} \sqrt{2P_1 N T_c} c_m^{(1)} \alpha_{1,m}.
$$

We define an $M$-dimensional vector $s = [s_1, s_2, \ldots, s_M]^T$. The output of the correlator on the $m$-th branch due to the $k$-th user signal, for $k \geq 1$, is given by

$$
i_{k,m} = \sqrt{2P_k c_m^{(k)}} e^{-j\omega_m T_k} \alpha_{k,m} \sum_{\ell=1}^{N-1} \sum_{\lambda=-\infty}^{\infty} b_{\ell/N}^{(k)} a_{\lambda}^{(k)} \psi((i-\lambda)T_c - T_k),
$$

where the function $\psi(\cdot)$ is the output of the chip waveform through the chip-matched filter, i.e., $\psi(t) = \int_{-\infty}^{\infty} \psi(s) \psi^*(s-t) ds$. We also define $M$-dimensional vectors $i_k = [i_{k,1}, i_{k,2}, \ldots, i_{k,M}]^T$. We denote the output of the correlator on the $m$-th branch due to AWGN by $n_m$, and similarly define an $M$-dimensional vector $n = [n_1, n_2, \ldots, n_M]^T$. The overall output of the correlators, in vector form, is given by

$$
z = s + n + \sum_{k=1}^{K} i_k.
$$

Clearly, $n$ is uncorrelated with $s$ and $i_k$ for all $k$. Moreover, it is easy to see that the vectors $i_k$ are uncorrelated for different $k$ and are uncorrelated with $s$. Therefore, the noise and
interference correlation matrix is given by

$$R_{ni} = E_{\alpha} \left[ nn^H + \sum_{k=2}^{K} ik_k^H \right],$$  \hspace{1cm} (8)

and the overall output correlation matrix is given by

$$R_z = E_{\alpha}[zz^H] = E_{\alpha}[ss^H] + R_{ni},$$  \hspace{1cm} (9)

where $E_{\alpha}[]$ denotes the conditional expectation given $\alpha_{k,m}$, for $k = 1, \ldots, K$ and $m = 1, \ldots, M$.

The decision statistic for the symbol $\ell_{0}^{(1)}$ is given by $Z = w^H z$. We assume that the channel coefficients $\alpha_{k,m}$ and $T_k$ vary slowly so that they effectively remain constant within the time interval used to determine an appropriate weight vector.

One way to determine a weight vector is the approach of maximal ratio combining [1]. Each component of the weight vector is determined separately by

$$w_m = \frac{E_{\alpha} [\ell_{0}^{(1)} z_m] \sqrt{2P_1 NT_c \alpha_{1,m}}}{\text{var}_{\alpha}[\ell_{0}^{(1)} z_m]},$$  \hspace{1cm} (10)

where $z_m$ is the $m$-th component of $z$ and $[R_{ni}]_{mm}$ is the $m$-th diagonal component of the correlation matrix $R_{ni}$. We note that the constant gain $\sqrt{2P_1 NT_c}$ can be omitted without affecting the performance of the MRC. The MRC is optimal in the sense of maximizing the signal-to-noise ratio (or minimizing the probability of error) when noises and interference across different carriers are uncorrelated, for example, when only AWGN is present. In order to obtain the MRC, the channel coefficients $\alpha_{k,m}$ have to be estimated. The estimation of these coefficients becomes more difficult when multiple-access interference (MAI) is also present. Actually, since MAI across different carriers are correlated, the MRC is not optimal.

In this paper, we determine the optimal weight vector that maximizes the SNR, defined by

$$\text{SNR} = \frac{E_{\alpha} [w^H s]^2}{E_{\alpha} [w^H (n + \sum_{k=2}^{K} ik_k)]^2}. \hspace{1cm} (11)$$

Equivalently, we find the weight vector that maximizes

$$\text{SNR} + 1 = \frac{E_{\alpha} [w^H z]^2}{E_{\alpha} [w^H (n + \sum_{k=2}^{K} ik_k)]^2} = \frac{w^H R_z w}{w^H R_{ni} w}. \hspace{1cm} (12)$$

It can be shown [6] that the optimal weight vector that maximizes the last expression in (12), and hence the SNR, is given by the generalized eigenvector associated with the largest eigenvalue of the matrix pencil $(R_z, R_{ni})$. The matrix $R_{ni}$ can be easily estimated from the outputs $z$ of the correlators. It remains to estimate $R_z$.

Due to the special structure of the spread spectrum signals, it is possible to remove the desired signal component while maintaining the same statistics of the noises and interference. To estimate $R_{ni}$, we pass the outputs of the chip-matched filter on each branch through another combiner as shown in Fig. 2. On the $m$-th branch, the sequence $(\hat{a}_0^{(1)}, \hat{a}_1^{(1)}, \ldots, \hat{a}_{N-1}^{(1)})$ is chosen to be orthogonal to the sequence $(a_0^{(1)}, a_1^{(1)}, \ldots, a_{N-1}^{(1)})$. For example, $N$ is chosen to be even, and

$$\hat{a}_i^{(1)} = \begin{cases} a_i^{(1)} & \text{for } 0 \leq i < \frac{N}{2}, \\ -a_i^{(1)} & \text{for } \frac{N}{2} \leq i < N - 1. \end{cases} \hspace{1cm} (13)$$

Then, at the output of the second combiner on each branch, the first user signal component is nullified. On the other hand, it is straightforward to check that the statistics of the output due to other user signals and noises are the same as those of the first combiner. In particular, if the overall output from these second combiners is denoted by $\hat{z}$, then its correlation matrix is given by

$$R_{\hat{z}} = E_{\alpha} [\hat{z} \hat{z}^H] = R_{ni}. \hspace{1cm} (14)$$

Therefore, $R_{ni}$ can be estimated from $\hat{z}$ and, hence, the optimal weight vector can be determined.

In [4], it is shown that the generalized eigenvector associated with the largest eigenvalue of a matrix pencil of the form taken by $(R_z, R_{ni})$ can also be found by a constrained stochastic gradient algorithm similar to the least mean squares (LMS) algorithm but without a training sequence. The computational complexity of this blind adaptive algorithm is of the order $M$ per iteration, which is much smaller than the calculation of the generalized eigenvectors. In the notation of this paper, the algorithm can be expressed as follows. For each symbol interval, we obtain the vectors $z$ and $\hat{z}$ at the outputs of the $2M$ combiners. For the $j$-th symbol interval, we denote them by $z(j)$ and $\hat{z}(j)$, respectively. We denote the weight vector in the $j$-th symbol interval by $w(j)$ and update $w(j)$ for each $j$ according to the following rule:
For $j > 0$, get $\mathbf{w}(j)$ by

$$\mathbf{w}(j) = C(j)\left\{\mathbf{w}(j-1) + \mu [\mathbf{z}(j)^H \mathbf{w}(j-1)] \right\}, \quad (15)$$

$$\cdot [(\hat{\mathbf{z}}(j)^H \hat{\mathbf{z}}(j))\mathbf{z}(j) - (\hat{\mathbf{z}}(j)^H \mathbf{z}(j))\hat{\mathbf{z}}(j)]\},$$

where $C(j)$ is chosen to stabilize the algorithm. Details concerning the stabilization constant $C(j)$ and the initialization of the algorithm can be found in [3] and [4]. This blind adaptive algorithm has been shown to be effective for direct-sequence systems.

**IV. PERFORMANCE**

In this section, we consider the performance of the proposed receiver both theoretically and through simulations. For convenience, we define, for $k = 1, \ldots, K$, the vectors

$$\mathbf{d}_k = [d_{k,1}, d_{k,2}, \ldots, d_{k,M}]^T,$$

by

$$d_{k,m} = c_m^{(k)} e^{-j\omega_0 T_k \alpha_{k,m}}, \quad (16)$$

for $m = 1, \ldots, M$. It is shown in [6] that the generalized eigenvector associated with the largest generalized eigenvalue of the matrix pencil $(\mathbf{R}_\sigma, \mathbf{R}_\rho)$, i.e., the optimal weight vector maximizing the SNR defined in (11), is given by

$$\mathbf{w}_* = \mathbf{R}_{ni}^{-1} \mathbf{d}_1. \quad (17)$$

For the simulations, the signal to thermal noise ratio (STNR) is fixed at 15dB. Moderate values of $N = 32$ and $M = 8$ are used. All curves shown are the results of averaging over 500 realizations.

First, we consider the case with only AWGN where

$$\mathbf{R}_{ni} = N_0 N T_c \mathbf{I}.$$ 

The optimal weight vector is given by

$$\mathbf{w}_* = \frac{1}{N_0 N T_c} \mathbf{d}_1, \quad (18)$$

which is the MRC obtained by (10). The proposed algorithm, actually, provides a way to estimate the fading coefficients $\alpha_{1,m}$. Simulation results are shown in Fig. 3. As expected, the SNR obtained is about 15 dB. Without any knowledge of the fading coefficients, both the eigen-analysis method and the blind adaptive algorithm can give SNR’s very close to the 15dB limit. In this simple case, we can choose the adaptation rate of the blind adaptive algorithm to be faster than the eigen-method. It can be seen that the weight vector adapts quickly and the SNR climbs to within 0.5dB of the optimal value in less than 20 symbol intervals.

We now consider the case with MAI. When the noise and interference components across the $M$ carriers are uncorrelated, the MRC gives the optimal weight vector. However, the noise and interference components are not always uncorrelated, especially when MAI is present. We examine the performance of the MC-CDMA system with the proposed algorithm in situations with the near-far problem. It is instructive to consider the case of $K=2$, i.e., when there is only a single strong interferer together with the desired user. Then, the noise and interference correlation matrix is given by

$$\mathbf{R}_{ni} = N_0 N T_c \mathbf{I} + \beta_2 \mathbf{d}_2 \mathbf{d}_2^H \quad (19)$$

where

$$\beta_k = 2 P_k N \sum_{\lambda=-\infty}^{\infty} |\hat{\psi}(\lambda T_c - T_k)|^2. \quad (20)$$

The optimal weight vector is given by

$$\mathbf{w}_* = \frac{1}{N_0 N T_c} \left( \mathbf{d}_1 - \frac{\beta_2 \mathbf{d}_2^H \mathbf{d}_1}{N_0 N T_c + \beta_2 |\mathbf{d}_2|^2} \mathbf{d}_2 \right), \quad (21)$$

and the resulting SNR is given by

$$\text{SNR}_* = \frac{2 P_1 N T_c}{N_0} \left( |\mathbf{d}_1|^2 - \frac{\beta_2 |\mathbf{d}_2|^2 |\mathbf{d}_1|^2}{N_0 N T_c + \beta_2 |\mathbf{d}_2|^2} \right). \quad (22)$$

We note that if $\mathbf{d}_1$ and $\mathbf{d}_2$ are orthogonal, then the optimal weight vector annihilates the contribution of the interferer without any penalty in SNR. Geometrically, the contributions of the users are represented by the vectors $\mathbf{d}_1$ and $\mathbf{d}_2$ in an $M$-dimensional space. If the vectors are perpendicular, then the optimal weight vector is $\mathbf{d}_1$. It collects energies from all carriers of the first user while rejecting energies from the second user by destructive interference. In general, $\mathbf{d}_1$ and $\mathbf{d}_2$ may not be perpendicular. However, in high dimensions, it is likely that they are close to perpendicular. In a Rayleigh fading channel, $d_{k,m}$, for $k = 1, 2$, and $m = 1, \ldots, M$, are iid complex Gaussian random variables with
zero mean and unit variance. We define an outage probability as the probability that the SNR of the first user suffers more than 3dB loss due to the interference from the second user. From (22), it is given by

\[ P_o = \Pr \left( \frac{|d_1|^2}{2} \leq \frac{\beta_2 |d_1^H d_2|^2}{N_0 NT_c + \beta_2 |d_2|^2} \right). \] (23)

It can be shown [7] that

\[ P_o \leq \frac{1}{2M-1}. \] (24)

The outage probability drops at least exponentially with the number of carriers. For example, with 8 carriers, the outage probability is bounded by \(7.81 \times 10^{-3}\). Simulation results are shown in Fig. 4. A strong interferer with power 40dB stronger than that of the desired user shares the channel with the desired user. We see that the optimal SNR is slightly less than 15dB. This implies that the interferer signal can be effectively cancelled. However, the SNR obtained by the MRC is about 2dB, showing that it is far from optimal. The eigen-method can attain a SNR very close to the optimal value. The SNR obtained by the blind adaptive algorithm is 3dB below the optimal value. The loss in performance by the blind adaptive algorithm is due to its stochastic nature, and can be traded off with its convergence speed.

V. CONCLUSIONS

We have considered a signal reception scheme for MC-CDMA systems. We have shown that by suitably combining the received signals on all the carriers, the proposed receiver can combine the desired signal contributions from different carriers constructively, as well as cancel noises and multiple-access interference, under fading channels. Moreover, we have proposed a blind adaptive algorithm to determine an optimal weight vector, which is used to combine the received signals from different carriers optimally. Simulations show that the proposed algorithm performs very well under different channel conditions including situations with the near-far problem.

REFERENCES