ABSTRACT

This paper considers the detection performance of a serial thresholding receiver for pre-coded binary GMSK signal with $BT=0.5$. The conditional symbol (bit) error probability of the GMSK signal with carrier phase and timing errors is derived and compared to that of unfiltered BPSK. It is shown that the GMSK signal has practically the same performance as the ideal BPSK signal in the absence of sync errors and that the GMSK signal is more sensitive to carrier phase error but much more robust to symbol timing error than is the BPSK signal.

INTRODUCTION

Gaussian filtered minimum shift keying is a member of the class of continuous phase modulation signal with a constant signal envelope and a spectrum that can be made compact with the appropriate choice of the signal $BT$ product. The constant envelope GMSK signals can be amplified by saturated amplifiers without the concomitant spectral re-growth of filtered PSK signals. These properties make GMSK an attractive signal to use in frequency division multiple access communication systems where the transmitters are operated at maximum power output efficiency.

This paper assesses the detection performance of a GMSK signal with $BT=0.5$ for a serial receiver operating with or without symbol timing or carrier phase errors. The performance analysis is based on a Pulse Amplitude Modulation (PAM) representation of the complex envelope of the GMSK signal attributed to Laurent [1], as well as a simple precoding algorithm for absolute phase encoding described in the next section. The paper is organized into five sections. After describing the GMSK communication system and its components in the next section, we derive in the third section the symbol error probability performance of the receiver. The fourth section presents numerical performance results of the receiver and compares these performance to those of unfiltered BPSK. The fifth section concludes the paper.

SYSTEM DESCRIPTION

Figure 1 shows a diagram of the GMSK communication system. The system consists of a data source, a data precoder, a GMSK modulator, a communication channel, a receiver, and a data sink. Descriptions of the GMSK modulator, data precoder, and receiver follow.
The complex symbols defined by (2) and (3) are called the pseudo data symbols in the Laurent expansion of the complex envelope for the GMSK BT = 0.5 signal. These symbols are seen to depend on all of the past and present data symbols applied to the Gaussian filter at the modulator. From the received pseudo data symbols \( \hat{s}_0 \), \( \hat{s}_{-1} \), the transmitted data symbols can be differentially demodulated as

\[
\alpha_n = -j\hat{a}_{0,n}\hat{a}_n^{*}-1
\]

However, there will be a loss in demod performance due to differential detection. Fortunately, this loss can be eliminated by a data precoding algorithm applied

to the source NRZ data symbols prior to GMSK modulation. The precoding algorithm is as follows

Let \( d_k, d_k \in \{ \pm 1 \}, k \geq 0 \), denote the equally probable source data symbols at time \( t = kT \), the output of the precoder, \( \alpha_k \), which is the input to the GMSK modulator, is formed by

\[
\alpha_k = (-1)^k d_k d_{k-1} \quad \text{with} \quad d_{-1} = 1.
\]

Since the symbols \( d_k \) and \( \alpha_k \) have identical statistics, both the precoded and un-precoded GMSK signals have the same power spectrum. The computed power spectrum of the BT = 0.5 GMSK signal is shown in Figure 4 along with the power spectra of the MSK and unfiltered BPSK signals.

\[
\begin{align*}
\frac{g_0(t)}{g_0(2)-g_0(0)} & \left\{ \begin{array}{ll}
\cos\left(\frac{\pi}{2}g(t)\right), & 0 \leq t \leq 2T \\
0, & \text{elsewhere}
\end{array} \right.
\end{align*}
\]

\[
g(t) = \frac{g_0(t) - g_0(0)}{g_0(2) - g_0(0)} u(t)
\]
where $\phi_c$ is an unknown carrier phase and $n(i)$ is a zero mean complex white Gaussian noise process with 2-sided power spectral density $2N_0$.

With precoding applied to the source data symbols $d_k$, the pseudo data symbols become

$$a_{0,n} = J(n)d_n$$

$$a_{n,n} = j(-1)^n J(n-2)d_{n-1}d_{n-2}$$

Rewriting (9) yields

$$r(t) = \frac{2E_b}{T} e^{-j\phi_c} \sum_{k=0}^{\infty} \{J(n)d_nh_0(t-nT) + j(-1)^n J(n-2)d_{n-1}d_{n-2} h_1(t-nT)\} + n(t)$$

In the presence of symbol sync error $\tau$, $|\tau| \leq 1/2$, and carrier phase estimation error $\theta$, $|\theta| \leq \pi$, the sampled output of the matched filter at time $t = (m+\tau)T$ is:

$$r_m = \sum_{n=m}^{\infty} \{J(n)d_nh_0(n-m-\tau) + j(-1)^n J(n-2)d_{n-1}d_{n-2} R_0(n-m-\tau) + G_0(0, R_0(0) N_0 T) + n(t)\}$$

where

$$R_0(s) = \frac{1}{T} \int_0^T h_0(t)h_0(t-s)dt, \quad i = 0,1$$

and $G_0(0, R_0(0) N_0 T)$ and $G_1(0, R_0(0) N_0 T)$ are independent Gaussian variables with variance $R_0(0) N_0 T$.

The auto-correlation function $R_{0,0}(s)$ and cross-correlation function $R_{0,1}(s)$ are plotted in Figure 6.

![Figure 6. Correlation Functions of Laurent Pulses](image)

Since $R_{0,0}(s) = 0$, $|s| \geq 3$ and $R_{0,1}(s) = 0$, $s \leq -1$, or $s \geq 3$ the decision variable is found to be:

$$z_m = \begin{cases} 
\text{Re}(r_m), & m \text{ odd} \\
\text{Im}(r_m), & m \text{ even} 
\end{cases}$$

where

$$\mu(\tau, \theta, d_m, d) = \begin{cases} 
\{d_m R_0(0,2) + d_{m-1} R_0(2) \} \cos(\theta) & m \text{ odd} \\
\{d_m R_0(0,2) + d_{m-1} R_0(2) \} \sin(\theta), & m \text{ even} 
\end{cases}$$

The conditional symbol decision error probability, conditioned on the synchronization errors $\tau$, $\theta$, the data sequence $d$ and the transmitted data symbol $d_m = 1$, is

$$\Pr\{d_m = 1, \tau, \theta, d\} = \Pr\{z_m \leq 0|d_m = 1, \tau, \theta, d\} = \Pr\left\{\frac{2E_b}{R_0(0)} \nu(\tau, \theta, 1, d) + G(0, R_0(0) N_0 T) \leq 0\right\}$$

Averaging (15) over the 64 possible 6-symbol random data sequences then yields the conditional symbol error probability

$$P_{\text{GMSK}}(\gamma_b, \theta, \tau) = \frac{1}{64} \sum_{d=0}^{63} \{2\gamma_b R_0(0) \nu(\tau, \theta, 1, d)\}$$
where \( d_j \) is the \( j \)-th 6-symbol data pattern and \( \gamma_b = \frac{E_b}{N_0} \).

It should be noted that although (15) depends on the symbol time index \( m \) for a given random data pattern, averaging over all data sequences removes this dependency in (16). Thus, in the evaluation of (16) the symbol time index \( m \) can be taken as even (or odd) without a loss of generality.

In the case of no synchronization errors, i.e., \( \tau = 0 \) and \( \theta = 0 \), (16) reduces to the ideal performance of the GMSK receiver

\[
G_{\text{MSK, ideal}}(\gamma_b) = \frac{1}{4} Q(\sqrt{2\gamma_b R_{0,0}(0)}[1 + \overline{R}_{0,1}(1)]) + \frac{1}{4} Q(\sqrt{2\gamma_b R_{0,0}(0)}[1 - \overline{R}_{0,1}(1)]) + \frac{1}{8} Q(\sqrt{2\gamma_b R_{0,0}(0)}[1 + 2\overline{R}_{0,0}(2)]) + \frac{1}{8} Q(\sqrt{2\gamma_b R_{0,0}(0)}[1 - 2\overline{R}_{0,0}(2) - \overline{R}_{0,1}(1)])
\]

(17)

where

\[
R_{0,0}(0) = 0.99973
\]
\[
\overline{R}_{0,0}(2) = 0.013022
\]
\[
\overline{R}_{0,1}(1) = 0.013025
\]

ERROR PROBABILITY OF BPSK RECEIVER

When synchronization errors are present in a matched-filter receiver with a unfiltered BPSK signal, it can be shown that bit error probability, conditioned on the carrier phase and symbol sync errors, is

\[
P_{\text{BPSK}}(\gamma_b, \theta, \tau) = \frac{1}{2} \left( Q(\sqrt{2\gamma_b \cos(\theta)}) + Q(\sqrt{2\gamma_b \cos(\theta)(1 - 2|\tau|)}) \right)
\]

(18)

and the ideal performance of the BPSK receiver is

\[
B_{\text{PSK, ideal}}(\gamma_b) = P_{\text{BPSK}}(\gamma_b | \theta = 0, \tau = 0) = Q(\sqrt{2\gamma_b})
\]

(19)

Numerical results of (16), (17), (18) and (19) are presented in the next section.

NUMERICAL RESULTS

Figure 7. presents the bit error rate (BER) performance of the GMSK and BPSK receivers.

In the absence of synchronization errors, we see that the GMSK receiver performs practically as well as the BPSK receiver over a signal-to-noise ratio of 0 to 10 dB. The BER performance of the BPSK and GMSK receivers for the case with carrier phase error and no symbol timing error and the case with symbol timing error and no phase error, given by (16) and (18), are shown in Figures 8 and 9, respectively. We see that the GMSK receiver is more sensitive to carrier phase errors than the BPSK receiver whereas the BPSK receiver is much more sensitive to symbol timing error than the GMSK receiver. Finally, Figure 10 compares the BER performance of BPSK and GMSK in the presence of both carrier phase and symbol timing errors for the cases. (5 deg, 0.057), (10 deg, 0.17) and (10 deg, 0.27). In each of these cases GMSK is found to perform superior to BPSK signal and the poor performance of the BPSK signal is seen due practically entirely to the symbol timing error. For a coded communication system requiring a channel error rate of 0.01, we see that GMSK can outperform BPSK by 1 to 2 dB for these synchronization errors.

CONCLUSIONS

We have presented in this paper a data preceding technique and a simple thresholding receiver for the detection of the GMSK BT = 0.5 signal. The performances of the receiver with synchronization errors were derived and compared to those of the unfiltered BPSK signal. In the case of no synchronization errors, results indicate that the BER performance of the GMSK receiver is practically indistinguishable from ideal BPSK performance. It was also shown that the GMSK receiver is far more robust to symbol timing error than is the BPSK receiver. These results indicate that the BT=0.5 GMSK signal could be a very promising modulation scheme to use in a bandwidth and power efficient FDMA communication system.
REFERENCES

