

ON SEQUENCE SHARING FOR MULTI-CODE DS/CDMA SYSTEMS

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ABSTRACT

This paper focuses on sequence sharing, a technique of sequence reuse for multi-code DS/SSMA systems. Each user in the system owns a subset of orthogonal sequences, whereby the different subsets of sequences are not disjoint—a given sequence may be shared by more than one user. Thus, the system needs fewer orthogonal sequences, compared to a system which does not employ sequence sharing. Since, in our system, sequence collision is an inevitable problem, we investigate branch-, user- and system-collision-free probabilities for each sequence assignment. In order to achieve a collision-free design at the system level, a low-complexity protocol for coordinating users' start times is put forward. With this protocol, codes (sequences) and times-of-arrival are used in combination to distinguish the signals of different users. Numerical results and comparisons are provided to illustrate the performance of the scheme, and our analytical results are supported by simulations.

INTRODUCTION

Multi-code direct sequence (DS) code division multiple access (DS/CDMA) is an effective technique to achieve high data rates and possible support for multimedia wireless communication [1]. In [2], we generalized the concept of multi-code systems and unified various existing multi-code systems with what we referred to as a multidimensional DS spread spectrum (SS) system. In a multidimensional DS/CDMA system, typically many more orthogonal sequences are needed compared to traditional DS/CDMA systems. "Code allocation" [1] is a technique for a multi-code system to achieve a high data rate without dramatically increasing the number of employed sequences, where each user has sole use of as many sequences as it needs based on some dynamic sequence-coordinating criterion. In this paper we focus

on an alternative technique, namely, sequence sharing [2] [3]. In this sequence-sharing system, one sequence can be simultaneously used by more than one user.

The system model is shown in Fig.1, where the baseband part of the transmitter can be regarded as consisting of H -parallel branches. Each branch has its own spread spectrum sequence and thus generates a branch SS signal, and all branch signals are summed up and then transmitted; the $V_i(t)$'s are time-continuous SS sequences with chip duration T_C and period $T = NT_C$, and all H sequences employed by a given user are orthogonal to one-another. The number of sequences simultaneously transmitted by a single user is $w (\leq H)$, and in the following, we will assume that $w = H$. With this sequence reuse technique, we need a procedure to assign the sequences among the users, and we also have to deal with the problem of sequence collision. If the sequence collisions are intolerable, a protocol for coordinating the start times of all users for a collision-free scenario can be used. Toward this end, the orthogonality of the sequences and the difference of the times-of-arrival will be jointly used for distinguishing the users in the system.

Our study is done for an asynchronous DS/CDMA, and either multipath is not considered, or we assume the receiver is locked on to just one path for each user.

SEQUENCE ASSIGNMENT

Assume there are K users in the system. We use $V_h^{(k)}$ to represent the sequence for the h th branch of the k th user. Due to sequence sharing, the same sequence may be labelled differently for different users, e.g., $V_h^{(k)}$ and $V_{h'}^{(k')}$ could refer to the same sequence. Define a $K \times H$ matrix $[V]$ as

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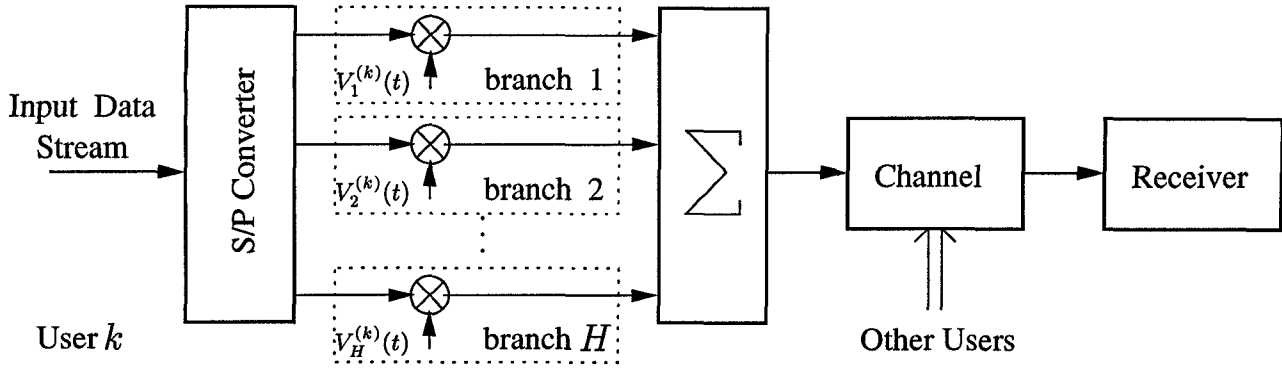


Figure 1: System model

$$[V] = \begin{pmatrix} V_1^{(0)} & V_2^{(0)} & V_3^{(0)} & \dots & V_H^{(0)} \\ V_1^{(1)} & V_2^{(1)} & V_3^{(1)} & \dots & V_H^{(1)} \\ V_1^{(2)} & V_2^{(2)} & V_3^{(2)} & \dots & V_H^{(2)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ V_1^{(K-1)} & V_2^{(K-1)} & V_3^{(K-1)} & \dots & V_H^{(K-1)} \end{pmatrix}, \quad (1)$$

where each row corresponds to the H orthogonal sequences employed by one user, and an entry at the h th column corresponds to the h th branch of the user.

Now assume there are J distinct orthogonal sequences U_j , $0 \leq j \leq J-1$, $J < K \cdot H$. The sequence assignment is to map the U_j 's to the $V_h^{(k)}$'s. Thus, based on an appropriate mapping between the U_j 's and the $V_h^{(k)}$'s, we form a $K \times H$ matrix $[U]$ by replacing all $V_h^{(k)}$'s with U_j 's in the matrix $[V]$. In this way, the problem of sequence assignment becomes one of designing the matrix $[U]$. For case of notation, the entries of the matrix $[U]$ are denoted by the index j , the subscript of U_j , for $0 \leq j \leq J-1$.

We define a parameter $\bar{\eta}$, the average sharing efficiency per sequence, to reflect how efficiently the sequences are reused, where $\bar{\eta}$ is given by

$$\bar{\eta} = \frac{K \cdot H}{J}. \quad (2)$$

Another parameter, λ , is defined as the maximum number of overlapping entries between any two rows in $[U]$. Suppose $K = LG$, where L and G are positive integers. The sequence assignment is defined as follows: For $(g-1)L \leq k \leq gL-1$ and

$1 \leq g \leq L$, $(V_1^{(k)}, V_2^{(k)}, V_3^{(k)}, \dots, V_H^{(k)})$ is taken as $(U_{(g-1)H}, U_{(g-1)H+1}, U_{(g-1)H+2}, \dots, U_{gH-1})$.

Example: $G = 3$, $K = J = 12$, $H = 4$, $\bar{\eta} = 4$, $\lambda = 4$.

$$[U] = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \\ \dots & \dots & \dots & \dots \\ 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 \\ \dots & \dots & \dots & \dots \\ 8 & 9 & 10 & 11 \\ 8 & 9 & 10 & 11 \\ 8 & 9 & 10 & 11 \\ 8 & 9 & 10 & 11 \end{pmatrix},$$

In this *Example*, the entries 0, 1, 2, ..., 11 refer to the sequences $U_0, U_1, U_2, \dots, U_{11}$, the rows in $[U]$ are divided into three groups of four rows, and all rows in each group overlap, but a row does not overlap with another row if they belong to different groups. Generally, the matrix $[U]$ can consist of $G \geq 1$ groups, corresponding to G independent groups of users. In this paper, we assume all of the groups have the same size.

COLLISION-FREE PROBABILITIES

In this section, we will derive the collision-free probabilities for the system.

Definition 1: When two signals containing the same sequence arrive at a receiver with a time-of-arrival difference $|\Delta\tau| < \alpha T_C$, we say these two signals collide with one-another, where α is a positive constant, and

αT_C is the interval needed to reliably distinguish the users.

Definitions 2–4: For a given user k , assume sequence U_j is used for the h th branch, and the user k shares U_j with at least one other user. (1) If the signal from user k does not collide with any of the signals that contain U_j , we say the h th branch of the given user is collision-free. (2) If all branches of user k are collision-free, we say user k is collision-free. (3) If all users in the system are collision-free, we say the system is collision-free.

Consider that a sequence U_j is shared by the h th branch of user k and by $\eta - 1$ other users. For simplicity, we omit the indices h and k . At the receiver, let S_η be the signal transmitted from a given user (user k), and let $S_{\eta-1}, S_{\eta-2}, \dots, S_1$ be the signals transmitted from the other $\eta - 1$ users that employ the same sequence, U_j . For $g \leq \eta$, denote an event CF_g by

$$CF_g = \bigcap_{i=1}^{g-1} \{S_g \text{ is collision-free with } S_i\}. \quad (3)$$

Then, the event that the given branch of the user is collision-free can be expressed as

$$\begin{aligned} & \{ S_\eta \text{ is collision-free with } S_{\eta-1}, S_{\eta-2}, \dots, S_1 \} \\ &= \bigcap_{i=1}^{\eta-1} \{S_\eta \text{ is collision-free with } S_i\} \\ &= CF_\eta. \end{aligned} \quad (4)$$

Define another relevant event, called the mutually collision-free event, as

$$MCF_g = \{ \text{any pair of } S_{g-1}, S_{g-2}, \dots, S_1 \text{ are collision-free} \}. \quad (5)$$

Then, MCF_g can be rewritten in the terms of $CF_g, CF_{g-1}, \dots, CF_2$ as

$$\begin{aligned} MCF_g &= CF_g \cap MCF_{g-1} \\ &= CF_g \cap CF_{g-1} \cap CF_{g-2} \cap \dots \cap CF_2. \end{aligned} \quad (6)$$

Without loss of generality, in the following we assume the times-of-arrival are defined on $[0, T)$. We further assume all times-of-arrival are uniformly distributed over $[0, T)$. Under the above conditions, the probability that two signals with respect to the same sequence are collision-free, defined as $Pr(CF_2)$, is given by

$$Pr(CF_2) = \frac{T-2\alpha T_C}{T} = 1 - 2\frac{\alpha}{N}, \quad 2\alpha T_C < T. \quad (7)$$

From Equations (4) and (7), the branch-collision-free probability is given by

$$P_B = Pr(CF_\eta) = (1 - 2\frac{\alpha}{N})^{\eta-1}, \quad 2\alpha T_C < T, \quad (8)$$

where η is the number of users that share the same sequence U_j . The user-collision-free probability is the same as the branch-collision-free probability, i.e.,

$$P_U = P_B = (1 - 2\frac{\alpha}{N})^{K/G-1}, \quad 2\alpha T_C < T, \quad (9)$$

where K/G is the number of the users belonging to one group.

Since only those users within the same group can collide with one another, and any two collision events for different groups are independent of one another, the system-collision-free probability is given by

$$P_S = [Pr(MCF_{K/G})]^G. \quad (10)$$

By using Equation (6), $Pr(MCF_\eta)$ can be expressed as

$$\begin{aligned} & Pr(MCF_\eta) \\ &= Pr(CF_\eta \cap MCF_{\eta-1}) \\ &= Pr(CF_\eta | MCF_{\eta-1}) Pr(MCF_{\eta-1}) \\ &= Pr(CF_\eta | MCF_{\eta-1}) Pr(CF_{\eta-1} \cap MCF_{\eta-2}) \\ &= \dots \\ &= Pr(CF_\eta | MCF_{\eta-1}) Pr(CF_{\eta-1} | MCF_{\eta-2}) \\ & \quad Pr(CF_{\eta-2} | MCF_{\eta-3}) \\ & \quad \dots Pr(CF_3 | MCF_2) Pr(CF_2). \end{aligned} \quad (11)$$

We have not found any approach to calculate $Pr(CF_g | MCF_{g-1})$ and $Pr(MCF_\eta)$ exactly, so the following approximation is used.

Theorem (proved in [4]): If the times-of-arrival of all users in the system are uniformly and independently distributed on $[0, T)$, then for any $g \geq 3$ and $2\alpha T_C(g-1) < T$, $Pr(CF_g | MCF_{g-1})$ is lower bounded by

$$Pr(CF_g | MCF_{g-1}) \geq 1 - 2(g-1)\frac{\alpha}{N}. \quad (12)$$

From the above theorem, and noting Equation (7), we get a lower bound for P_S :

$$P_S \geq \left\{ \prod_{g=2}^{K/G} [1 - 2(g-1)\frac{\alpha}{N}] \right\}^G, \quad 2\alpha T_C(K/G-1) < T. \quad (13)$$

We will show that this bound is a very good approximation to P_S .

PROTOCOL FOR ACHIEVING SYSTEM-COLLISION-FREE OPERATION

When the processing gain is large compared to the number of the users, the collision-free probability may be sufficiently high, even without any effort to coordinate the users. However, since a sequence collision is more likely to result in a detection error, a control mechanism to avoid the collisions is probably necessary. For that purpose, we propose here a protocol which is suitable for centralized wireless networks.

Protocol

Assume the system has both an adjustment period for arranging the start times, and a transmission period for data transmission. The use of the adjustment period is to guarantee a collision-free state. One approach for implementing the adjustment period is as described below:

- a) the central station signals all users to send their test headers;
- b) If user k , $0 \leq k \leq K - 1$, receives the command at time t_{0k} , it continuously transmits its distinct test header with a period of T (say, its first sequence $V_1^{(k)}$, $0 \leq k \leq K - 1$, when $J \geq K$) at a random start time t_k to the central station, where t_k is uniformly distributed over $[t_{0k}, t_{0k} + T)$;
- c) the central station measures the relative times-of-arrival of all received signals; if any of them are within αT_C seconds, it goes to step a; otherwise, it goes to step d;
- d) the central station informs all users to start data transmission (without changing their relative start times).

Average Number of Runs

Here a run is defined as either a successful realization of the steps a to d, or a false realization of the steps a to b to c and back to a. Note that, in spite of use of the protocol, the assumption that all times-of-arrival in the system are uniformly and independently distributed on $[0, T)$ is still valid. The average number of the runs in an adjustment period is given by

$$\bar{n} = \sum_{n=1}^{\infty} n(1 - P_S)^{n-1} P_S$$

$$= 1/P_S. \quad (14)$$

NUMERICAL RESULTS AND DISCUSSION

Shown in Fig.2 is the system-collision-free probability vs. the parameter α/N based on both the theoretical lower bound and a Monte-Carlo simulation. It is seen that the lower bound agrees very well with the simulation results. Collision-free probabilities for the *Example* described above are presented in Table 2, with some relevant parameter values given in Table 1. In Figures 3, 4 and 5, we show the effect of parameters α/N and K/G on the average numbers of runs and on $\bar{n} = 1/P_S$ for different values of G .

Note that α is a key parameter in our system. It is desired to use an extremely small α to achieve high collision-free probabilities. But, in practice, a decrease of α is limited by several factors, such as how accurate the estimation of the times-of-arrival is, how large is the radius the cell, how fast the users move, and how frequently the adjustment period is inserted.

References

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Table 1. Some parameters of the systems

G	K	J	$\bar{\eta}$	λ	H
3	12	12	4.0000	4	4

Table 2. Collision-free probabilities

$\alpha/N = 0.001$			$\alpha/N = 0.006$		
P_B	P_U	P_S	P_B	P_U	P_S
0.9940	0.9940	0.9646	0.9644	0.9644	0.8033

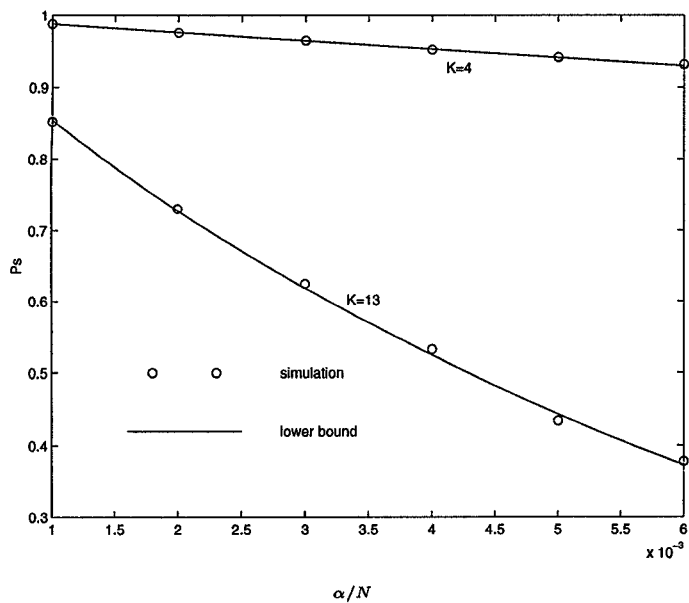


Figure 2: P_S vs. α/N for $G = 1$

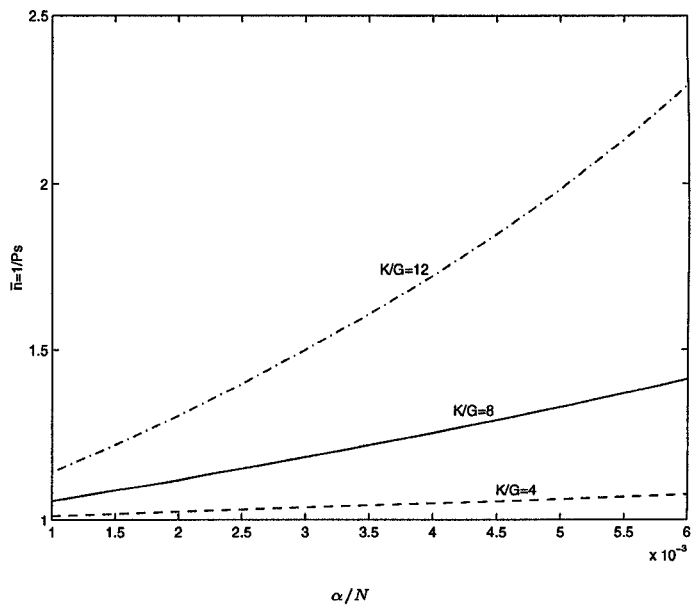


Figure 3: \bar{n} vs. α/N and K/G for $G = 1$

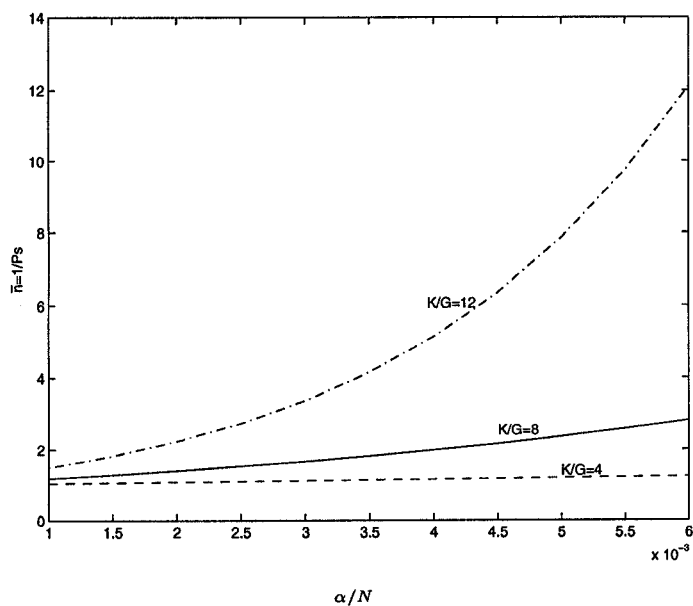


Figure 4: \bar{n} vs. α/N and K/G for $G = 2$

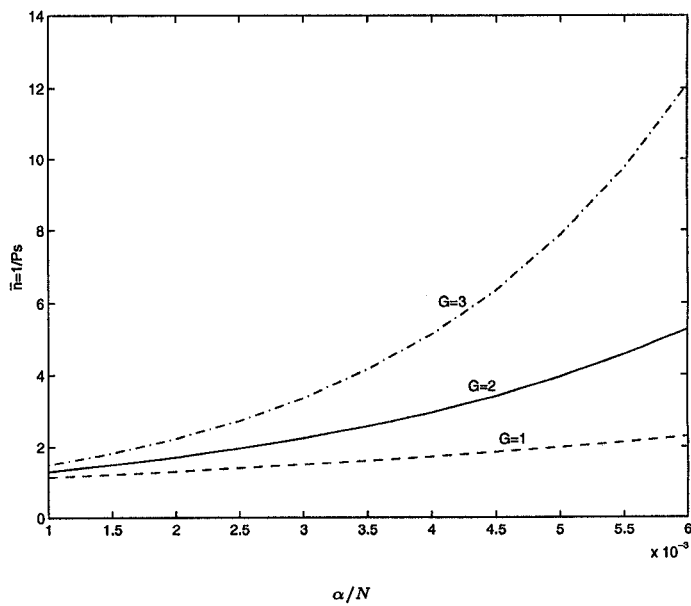


Figure 5: \bar{n} vs. α/N and K/G for $G = 3$