

PERFORMANCE EVALUATION OF BLIND CHANNEL IDENTIFICATION METHODS BASED ON OVERSAMPLING

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ABSTRACT

This paper addresses the performance evaluation of two methods that marked the beginning of a brand new direction in blind channel identification using only second-order statistics: TXK and Subspace methods. Computer simulations are conducted to investigate the following features of the algorithms: sensitivity to SNR, rate of convergence, computational complexity, sensitivity to noise correlation, robustness to error in model order estimates.

1. INTRODUCTION

Blind channel equalizers eliminate the need for training signal in digital communication systems when the transmission of a training sequence is impractical or very costly. Some of the recent blind methods first identify the channel before equalizing it and use only second-order statistics. These techniques significantly outperform several previously developed higher-order statistics based methods, specially for short data sequences. Two methods marked the beginning of a brand new direction in blind channel identification using only second-order statistics: TXK and Subspace methods. In despite of many promising features, the performance of these methods has not been evaluated in depth. The main purpose of the paper is elaborate a comparative performance study of TXK and Subspace methods. Computer simulations are conducted to investigate some factors that can limit the performance of these methods.

This paper is organized as follows. Following the problem statement in Section 2, we briefly summarize in Section 3 the TXK and Subspace description. In Section 4, we outline the results of computer simulations. Finally, in Section 5, we present our conclusions.

As a general notational convention, symbols for matrices (in capital letters) and vectors are in boldface. The notations $(.)^H$ and $(.)^T$ stand for Hermitian and transpose, respectively. $E\{.\}$ denotes expectation and $\mathbf{R}_y(k)$ denotes correlation matrix with lag k of the process represented by vector \mathbf{y} .

2. PROBLEM STATEMENT

The oversampled received signal can be expressed as

$$x(m) = \sum_{k=-\infty}^{\infty} s_k h(m - kP) + n(m) \quad (1)$$

where $\{s_k\}$ is a zero mean and wide sense stationary (WSS) sequence of identically distributed complex random variables; $h(.)$ is the discrete-time "composite" channel impulse response that includes the transmitting filter, the channel and the receiving filter; P is the oversampling factor; and $n(.)$ is the Gaussian additive noise, correlated by receiving filter. It may be shown that $x(m)$ is a wide sense cyclostationary (WSCS) process[2].

The objective of blind channel identification is to estimate $h(.)$ given only the received signal $x(.)$. We assume a FIR channel model of order L and that the source stochastic model is known. Once such identification is achieved, various equalization techniques can be applied to obtain the estimation of the symbols.

The oversampled channel can be represented as a Single Input-Multiple Output (SIMO) model[1]. This model consists of P virtual channels (subchannels) fed from a common input $\{s_k\}$, as depicted in Fig. 2.1. Each virtual channel has the same time support and a noise contribution of its own.

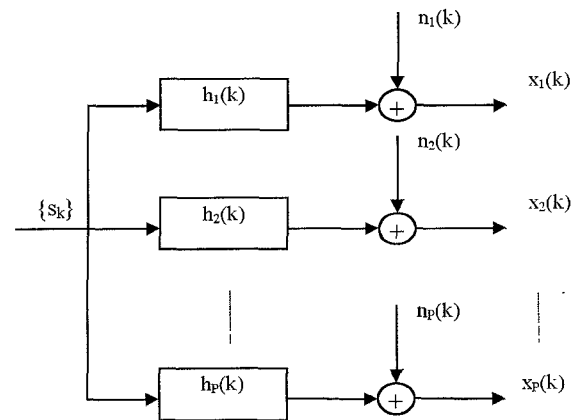


Figure. 2.1. Oversampled channel represented as a SIMO model.

As shown in Fig. 2.1, $x_i(\cdot)$ denotes the output from the i th virtual channel with FIR channel impulse response $h_i(j)$, $j=0, \dots, L$. Hence, $x_i(\cdot)$ is related to $\{s_k\}$ and $h_i(j)$ by

$$x_i(k) = \sum_{j=0}^L h_i(j) s_{k-j} + n_i(k), i=1, \dots, P. \quad (2)$$

Let $\mathbf{X}(k)$ denotes a received signal observed in a finite time interval of duration MP sampling periods. It is not hard to obtain the following matrix representation of the oversampled channel

$$\mathbf{X}(k) = \mathbf{H} \mathbf{s}(k) + \mathbf{N}(k). \quad (3)$$

where $\mathbf{s}(k) = [s_k \ s_{k-1} \ \dots \ s_{k-M-L+1}]^t$, \mathbf{H} is a MP -by- $(M+L)$ matrix associated to channel and $\mathbf{N}(k)$ is a MP vector associated to noise. In the next Section, we will can distinguish two different forms of $\mathbf{X}(k)$, \mathbf{H} e $\mathbf{N}(k)$ to describe the methods under consideration.

3. DESCRIPTION OF THE METHODS

3.1 - TXK Method

In TXK method, $\mathbf{X}(k)$, \mathbf{H} e $\mathbf{N}(k)$ are defined as

$$\mathbf{X}(k) = [x_P(k) \ x_1(k) \ x_P(k-M+1) \ \dots \ x_1(k-M+1)]^t \quad (4.a)$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}(0) & \mathbf{h}(1) & \dots & \mathbf{h}(L) & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{h}(0) & \dots & \mathbf{h}(L-1) & \mathbf{h}(L) & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & & & & & \vdots & \ddots & \vdots \\ \mathbf{0} & \dots & \dots & \mathbf{0} & \mathbf{h}(0) & \mathbf{h}(1) & \dots & \mathbf{h}(L) \end{bmatrix} \quad (4.b)$$

$$\mathbf{h}(j) = [h_P(j) \ h_{P-1}(j) \ \dots \ h_1(j)]^t, j=0, \dots, L, \quad (4.c)$$

$$\mathbf{N}(k) = [n_P(k) \ n_1(k) \ \dots \ n_P(k-M+1) \ \dots \ n_1(k-M+1)]^t. \quad (4.d)$$

The block-Toeplitz matrix \mathbf{H} is called a Sylvester resultant matrix[1] and satisfies[2]

$$\mathbf{H} = \mathbf{U}_s \mathbf{\Sigma} \mathbf{V}, \quad (5)$$

where \mathbf{U}_s and $\mathbf{\Sigma}$ are matrices obtained from singular value decomposition (SVD) of $\mathbf{R}_0 = \mathbf{R}_x(0) - \mathbf{R}_n(0)$ and \mathbf{V} can be univocally determined from $\mathbf{R}_1 = \mathbf{R}_x(1) - \mathbf{R}_n(1)$.

The SVD of \mathbf{R}_0 have the following form

$$\mathbf{U}^H \mathbf{R}_0 \mathbf{U} = \text{diag}(\lambda_1^2, \lambda_2^2, \dots, \lambda_d^2, 0, \dots, 0), \quad (6)$$

where \mathbf{U} is a unitary matrix and d is the signal subspace dimension. Let \mathbf{u}_i denote the i th column of \mathbf{U} , then $\mathbf{U}_s = [\mathbf{u}_1 \ \mathbf{u}_2 \ \dots \ \mathbf{u}_d]$ and $\mathbf{\Sigma} = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_d)$.

On the other side, \mathbf{V} can be obtained from solution of the following linear system

$$\mathbf{R} \mathbf{v}_k = \begin{cases} \mathbf{v}_{k+1}, k = 1, \dots, d-1 \\ \mathbf{0}, k = d \end{cases}, \quad (7)$$

where $\mathbf{R} = \mathbf{\Sigma}^{-1} \mathbf{U}_s^H \mathbf{R}_1 \mathbf{U}_s \mathbf{\Sigma}^{-1}$ and \mathbf{v}_k , $k=1, \dots, d$, are the columns of \mathbf{V} .

3.2 - Subspace Method

In Subspace method, $\mathbf{X}(k)$, \mathbf{H} e $\mathbf{N}(k)$ are defined as

$$\mathbf{X}(k) = [x_P(k) \ \dots \ x_P(k-M+1) \ \dots \ x_1(k) \ \dots \ x_1(k-M+1)]^t, \quad (8.a)$$

$$\mathbf{H} = [\mathbf{H}_P \ \mathbf{H}_{P-1} \ \dots \ \mathbf{H}_1]^t, \quad (8.b)$$

$$\mathbf{H}_i = \begin{bmatrix} h_i(0) & \dots & h_i(L) & 0 & \dots & \dots & 0 \\ 0 & h_i(0) & \dots & h_i(L) & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & 0 & h_i(0) & \dots & h_i(L) \end{bmatrix} \quad (8.c)$$

$$\mathbf{N}(k) = [n_P(k) \ \dots \ n_P(k-M+1) \ \dots \ n_1(k) \ \dots \ n_1(k-M+1)]^t \quad (8.d)$$

The Toeplitz matrix \mathbf{H} is called a *filtering matrix*. By orthogonality between the noise subspace and the signal subspace, the columns of \mathbf{H} are orthogonal to any vector \mathbf{G}_i in the noise subspace $MP-d$ dimensional[3]. Hence, we have

$$\mathbf{G}_i^H \mathbf{H} = \mathbf{0}, 1 \leq i \leq MP-d. \quad (9)$$

The vectors \mathbf{G}_i , $1 \leq i \leq MP-d$, are obtained from SVD of $\mathbf{R}_x(0)$ which has the following form

$$\mathbf{R}_x(0) = \begin{bmatrix} \mathbf{S} \mathbf{0} \\ \mathbf{0} \mathbf{G} \end{bmatrix} \begin{bmatrix} \mathbf{\Sigma}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{\Sigma}_n \end{bmatrix} \begin{bmatrix} \mathbf{S}^H & \mathbf{0} \\ \mathbf{0} & \mathbf{G}^H \end{bmatrix}. \quad (10)$$

The columns of \mathbf{S} span the signal subspace (dimension $M+L$), while the columns of \mathbf{G} span its orthogonal complement, the noise subspace.

3.3 - Channel Identifiability

The channel impulse response is identifiable if and only if \mathbf{H} has full column rank. An equivalent identifiability condition on the channel transfer function is that it does not have uniformly $(2\pi/P)$ -spaced zeros[1].

4. COMPUTER SIMULATIONS

Computer simulations were conducted to investigate the following features of the algorithms: *sensitivity to SNR*, *rate of convergence*, *computational complexity*, *sensitivity to noise correlation*, *robustness to error in model order estimates*.

The source symbols were drawn from a 16-QAM signal constellation. The key simulations parameters are summarized in Table 1, where N_t represents the number of symbol intervals used to produce each channel estimate and NE_{st} represents the number of channel estimates produced.

| N_t | P | SNR(dB) | NE_{st} |
|-------|---|---------|-----------|
| 100 | 4 | 15~40 | 250 |

Table 1: Key simulations parameters

Three computer experiments were conducted. As a measure of channel estimator performance we used the normalized root-mean-square error (NRMSE) defined by

$$\text{NRMSE} = \sqrt{\frac{1}{NE_{st}} \sum_{i=1}^M \frac{\|\mathbf{h} - \hat{\mathbf{h}}_i\|^2}{\|\mathbf{h}\|^2}} = \sqrt{\varepsilon_{MQN}} \quad (11)$$

where $\hat{\mathbf{h}}_i$ is the i th estimate.

4.1 - Computer Experiment I

In this experiment, the channel impulse response has the zero location shown in Fig. 4.1.

We can see that exist a subset of zeros close uniformly

spaced at $\pi/2$ rad, which is an unfavorable situation to both methods.

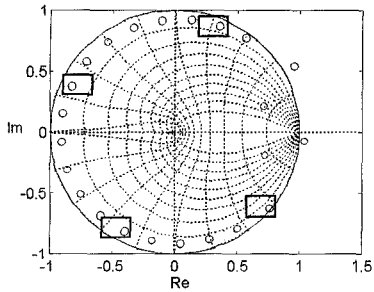


Figure 4.1. The zero location in experiment I.

NRMSE as a function of SNR, for known variance white noise, is shown in Fig. 4.2. We assumed that L is known.

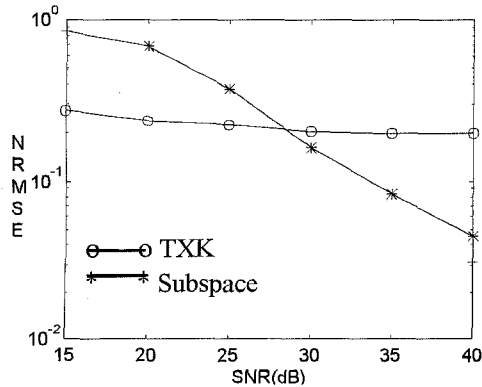


Figure 4.2. NRMSE vs. SNR for known variance white noise.

Clearly, we can see that the subspace method performs better than TXK method when SNR is higher 28 dB. However, an decrease of SNR leads to higher performance of TXK method.

NRMSE as a function of SNR, for known variance correlated noise, is shown in Fig. 4.3. In the implementation of the Subspace method, we used the solution that consists in whitening the signal by the inverse of the Hermitian square root of $R_n(0)$ [3]. We assumed that the channel order L is known.

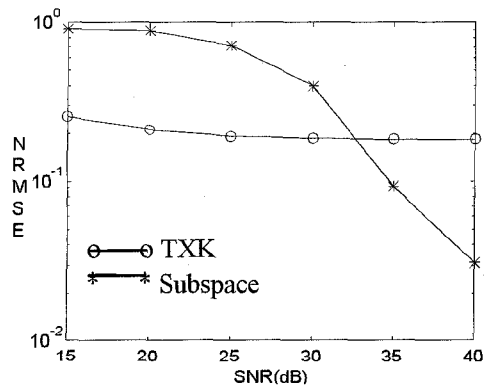


Figure 4.3. NRMSE vs. SNR, for known variance

correlated noise.

Comparing Figures 4.3 and 4.2, we can see that the Subspace method performance is more sensitive to noise correlation.

The NRMSE as a function of model order estimates, for known variance white noise and SNR=40 dB, is shown in Fig. 4.4. We assumed that the noise variance is known and the model order is overestimated from 5 (correct order) to 11.

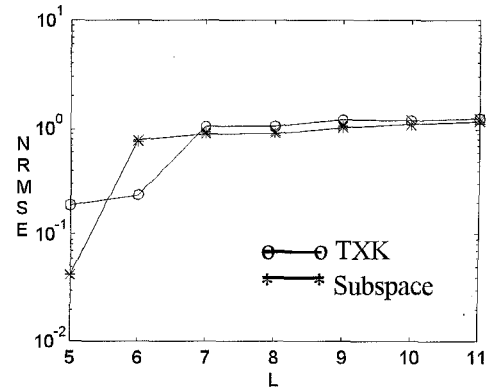


Figure 4.4. NRMSE vs. model order estimates, for known variance white noise.

Clearly, both methods have a same bad performance when the estimate $\hat{L} > 6$. However, for $\hat{L} = 6$ the TXK method performs better than the Subspace method.

The NRMSE as a function of N_t , for known variance white noise and SNR=15 dB, is shown in Fig. 4.5. We assumed that N_t is varied from 100 to 1000 symbol intervals and L is known.

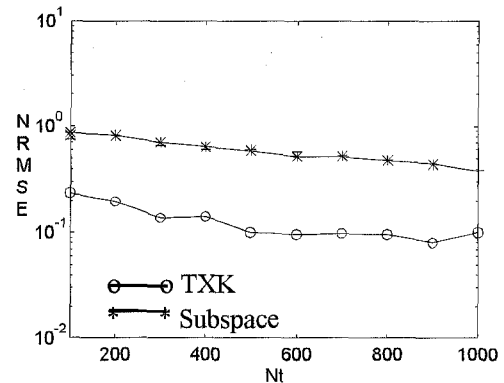


Figure 4.5. NRMSE vs. N_t .

Finally, analyzing the computational complexity we obtained that the TXK method requires 200800 arithmetics operations, while the Subspace method requires 359488 arithmetics operations, i.e., the TXK method is less computationally demanding than Subspace method. Both methods use two SVD operations to produce one estimate of the channel impulse response.

4.2 - Computer Experiment II

In this experiment, the same tests were conducted. The channel impulse response here considered has the zero location shown in Fig. 4.6, which is more favorable than that of the Fig. 4.1 for blind channel identification.

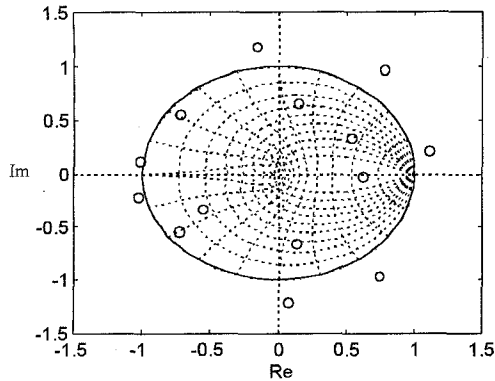


Figure 4.6. The zero location in experiment II.

NRMSE as a function of SNR, for known variance white noise, is shown in Fig. 4.7. We assumed that L is known. Clearly, we can see that the Subspace method performs better than the TXK method.

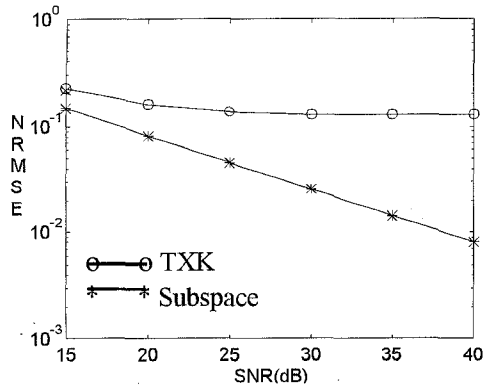


Figure 4.7. NRMSE vs. SNR for known variance white noise.

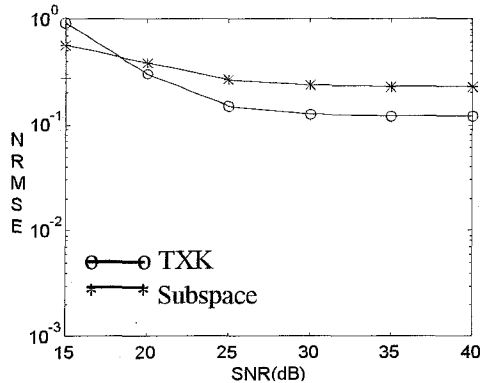


Figure 4.8. NRMSE vs. SNR, for known variance correlated noise.

NRMSE as a function of SNR, for known variance correlated noise, is shown in Fig. 4.8. We assumed that the channel order L is known.

Comparing Figures 4.7 and 4.8, we can see that the Subspace method performance is more sensitive to noise correlation.

The NRMSE as a function of model order estimates, for known variance white noise and SNR=40 dB, is shown in Fig. 4.9. We assumed that the noise variance is known and the model order is overestimated from 3 (correct order) to 9.

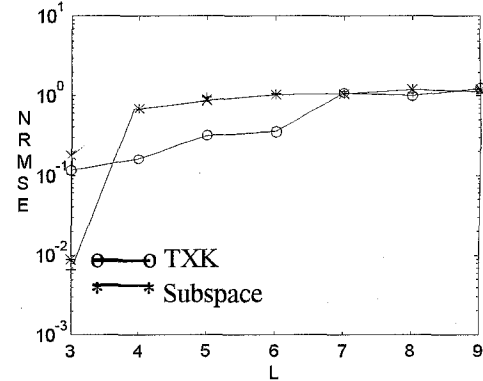


Figure 4.9. NRMSE vs. model order estimates, for known variance white noise.

Clearly, both methods have a bad performance when the estimate $\hat{L} > 6$. However, for $4 \leq \hat{L} \leq 6$ the TXK method performs better than the Subspace method.

The NRMSE as a function of N_t , for known variance white noise and SNR=15 dB, is shown in Fig. 4.10. We assumed that N_t is varied from 100 to 1000 symbol intervals and L is known.

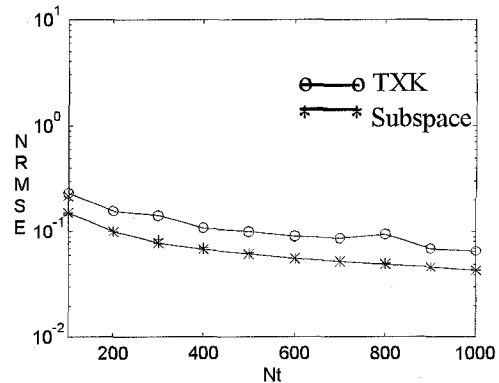


Figure 4.10. NRMSE vs. N_t .

Finally, analyzing the computational complexity we obtained that the TXK method requires 190112 arithmetics operations, while the Subspace method requires 215872 arithmetics operations, i.e., the TXK method is again less computationally demanding than Subspace method.

4.3 - Computer Experiment III

In this experiment, we generated 250 snapshots of a multipath channel impulse response using a reference statistical model for frequency selective slow fading

channels[4]. We defined the normalized root-mean-square deviation (NRMSD) as

$$\text{NRMSD} = \sqrt{\frac{1}{N_{\text{Est}}} \sum_{i=1}^M \frac{\|h_i - \hat{h}_i\|^2}{\|h_1\|^2}} \quad (12)$$

NRMSD as a function of SNR, for known variance white noise, is shown in Fig. 4.11. We assumed that L is known.

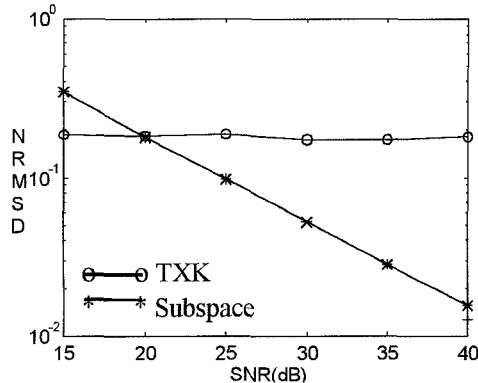


Figure 4.11. NRMSD vs. SNR for known variance white noise.

The NRMSD as a function of N_t , for known variance white noise and SNR=15 dB, is shown in Fig. 4.10. We assumed that N_t is varied from 100 to 1000 symbol intervals, and the model order is known.

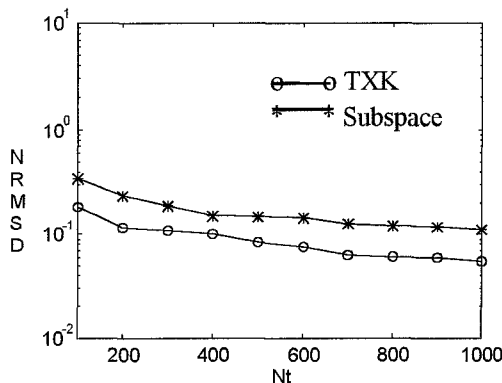


Figure 4.12. NRMSD vs. N_t .

We also conducted some tests regarding bias and verified that both methods seems to produce unbiased channel estimates.

5. CONCLUSIONS

This paper has presented a comparative performance evaluation of TXK and Subspace methods for oversampling based blind channel identification.

We investigated through computer simulation the sensitivity of the methods to several factors such as SNR, noise correlation, zero location of transfer function and errors in the estimation of channel model order. The results obtained showed that the performance of the methods can be

seriously affected by those factors.

In experiments I and II, we verified that their performances are particularly sensitive to zero location. In experiment III, we considered many snapshots produced by a reference slow fading model and verified that the NRMSD is close to the mean of the NRMSE obtained in experiments I and II.

Regarding performance comparison, we verified that: Subspace method performs better than TXK method for white or correlated noise if SNR was high. otherwise, TXK method generally performs better. The curves of NRMSE as a function of model order estimates showed that TXK method is more robust than Subspace. The simulations demonstrated that both methods are rapidly converging.

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