

# Multicarrier Demultiplexing and Demodulation for an MDR Frequency Hopped Satcom System : Simulation Analysis

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## Abstract

This paper presents a Multi Carrier Demodulator (MCD) for FDM systems, which is based on Fast Fourier Transform (FFT) technique. We show that, without pulse shaping, user's synchronization errors and doppler shifts imply the use of weighting functions or guard time on the received symbols. Both techniques, evaluated through computer simulations, lead to a loss in SNR and spectral efficiency. The next step is to study a possible coherent QPSK mode in order to oppose these losses. Because of the small number of symbols per hop, coherent demodulation is not an easy task. We present a demodulation technique, based on a feedback principle, that shows little loss compared to theoretical QPSK performances and resists high enough doppler values.

## 1 Basic Multi Carrier demodulator

The concern is about *On Board Processing (OBP)* for a frequency hopping system. The MDR standard of the future EHF military satellite communication system faces the requirement to make efficient use of the available spectrum. A Multi Carrier Demodulator (MCD) have been designed to increase this efficiency by demodulating several FDM users working at different data rates by means of variable rate digital accumulators. Symetric Differential Binary Phase Shift Keying (SDBPSK) is the currently considered modulation. A hop can handle one of the 4 SDBPSK modes listed in table 1. For example, in the second mode, a hop begins with 1 pilot symbol for SDBPSK phase reference, followed by 8 useful symbols. Thus, the symbol rate is  $9f_H$ , where  $f_H$  is the hopping frequency. Several users, working at different data rates, may be multiplexed with a minimum frequency spacing of  $1/T_s$ , where  $T_s$  is the symbol duration of the fastest modulation mode (i.e. mode 4 + 32). This minimum spacing ensures orthogonality properties between users.

## 2 Doppler and timing error mitigations

FDM demultiplexing is carried out using Fast Fourier Transform (FFT) over the symbol duration of the fast-

Modulation	$n_p$ pilots	$n_u$ symbols	$T/T_s$
SDBPSK	1	3	9
SDBPSK	1	8	4
SDBPSK	2	16	2
SDBPSK	4	32	1
QPSK	4	32	1

Table 1: List of MDR modes.

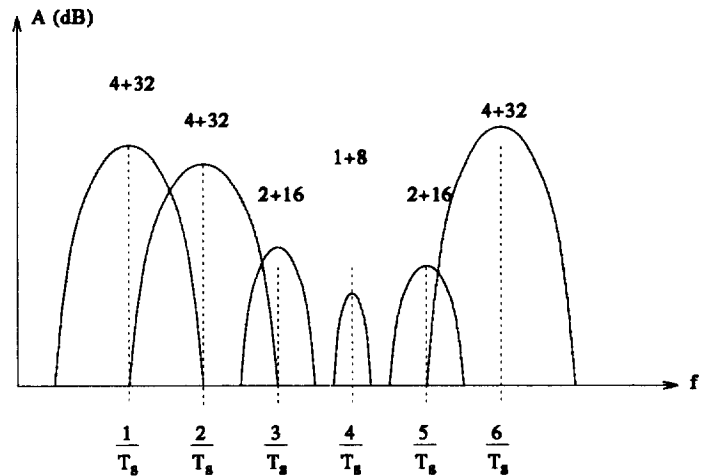


Figure 1: FDM spectrum with several users working at different symbol rates. The  $1/T_s$  spacing ensures orthogonality between users.

est modulation mode. At the time of the writing,  $N = 32$  signal samples feed the FFT device every  $T_s$ , allowing a maximum of 16 users in a MCD. The symbol sequence for each user is recovered by accumulating the FFT output samples at the users frequency slot. For example, in the SDBPSK 1 + 8 mode, 4 samples will be accumulated to create the symbol that will feed the SDBPSK demodulator. Thus a MCD can easily handle different symbol rates. The technique is however very sensitive to timing errors, synchronization errors and doppler shift of the different users because of Inter Channel Interference (ICI) and Inter Symbol Interference (ISI) :

**Doppler shift** The ambiguity function of the rectangular pulse (figure 2) shows than one can expect two deleterious effects : a decrease of the signal strength

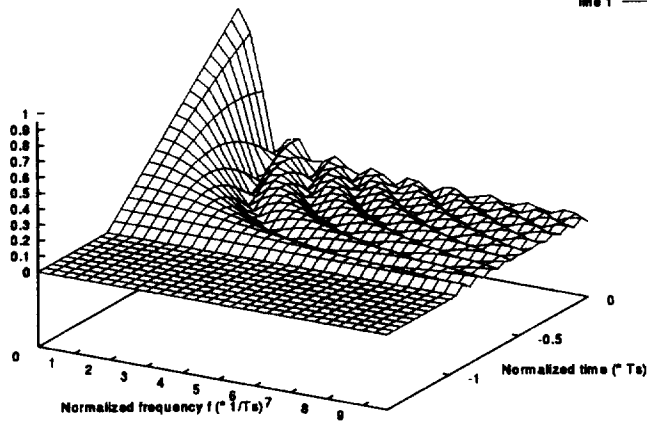


Figure 2: Ambiguity function of the rectangular pulse.

at the doppler shifted user's FFT slot and ICI (which is the worst effect) because of the loss of orthogonality with other carriers. [Moose, 1994] have shown that for an OFDM system :

$$SNR > \frac{E_c}{N_0} \frac{\text{sinc}^2(\pi\epsilon)}{1 + 0.5947 \sin^2(\pi\epsilon) E_c/N_0}$$

where  $\epsilon$  is the normalized frequency offset and  $E_c/N_0$  is the carrier to noise ratio of the individual carriers. Calculations indicate that  $\epsilon$  should remain under 0.04 to maintain good performances in OFDM systems. This value is further reduced in our case because the different carriers will have different amplitudes due to, for example, different propagation path attenuations<sup>1</sup> or different user terminals.

**Synchronization and timing** The synchronization reference is provided by the satellite to each user which has to track this reference. If a user gets desynchronized, by a value that is more than  $T_s/N$ , two compromising effects will be observed. First, a reduction in signal power at the output of the FFT. Additionally, the user will contaminate its neighbours because of ISI.

Since we wish to keep the constant envelope property of the digital phase modulations, no pulse shaping will be introduced at the emission. Thus, to maintain acceptable performances in the absence of filtering, we propose two methods. The first consists in applying a weighting function on each received symbol. The second is derived from guard time techniques employed in OFDM systems [LeFloch et al., 1989].

<sup>1</sup>In EHF systems, rain attenuation can reach 10 to 20 dB.

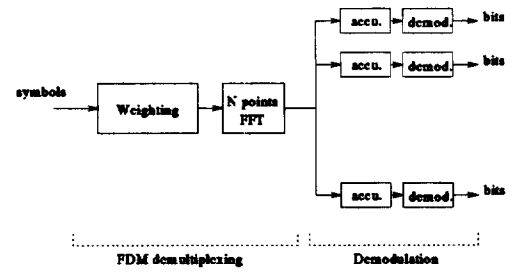


Figure 3: FFT demultiplexing of FDM users at different binary rates

## 2.1 Weighting functions

The role of the weighting function is twofold. First, it must restrict the bandwidth occupied by the signal. This means that the secondary lobes at frequency multiples of  $1/T_s$ , must decrease rapidly. Second, it has to reduce the importance of the samples at the beginning and at the end of the array passed to the FFT in order to limit the ISI caused by synchronization errors. However, to recover orthogonality conditions, the spacing between carriers must be increased, at least to  $2/T_s$ , depending on the choice of the window. It is also important to note that the receiving filter widens and that the  $E_b/N_0$  ratio is reduced. This behaviour is summarized in table 2 for several weighting functions.

Table 2:  $E_b/N_0$  loss and isolation between carriers for the most commonly used weighting functions.

Window	Loss	$2/T_s$ Isol.	$3/T_s$ Isol.
Hanning	1.6 dB	>40 dB	>40 dB
Hamming	1.5 dB	40 dB	>40 dB
Bartlett	1.45 dB	>40 dB	27 dB
Tuckey	1.82 dB	>40 dB	>40 dB
Kaiser ( $\beta = 2$ )	0.28 dB	30 dB	38 dB
Blackman	2.56 dB	18.5 dB	>40 dB

In our choice of a window we wish to have at least 20 dB of isolation between a carrier and its neighbours at  $2/T_s$ . An additional factor of 20 dB, due to propagation conditions and ground station differences, must also be considered. Therefore, for a  $2/T_s$  spacing, only the Hamming, Bartlett and Tuckey windows meet the minimum isolation of 40 dB. Additionally, because of its triangular shape the Bartlett window is too sensitive to synchronization errors. Finally, we choose the Hanning window for its smaller loss.

## 2.2 Guard time

An alternative technique involving guard times, may be used mainly to improve performances against synchronization errors. The method, illustrated in figure 4, involves a sampling interval  $\gamma T_s$  ( $\gamma < 1$ ) that is the symbol

duration reduced by twice the maximum synchronisation error. With an accurate value of  $\gamma$ , ICI and ISI are eliminated. To maintain orthogonality conditions, the frequency spacing between carriers must be increased to  $(1 + 2\gamma)/T_s$ .

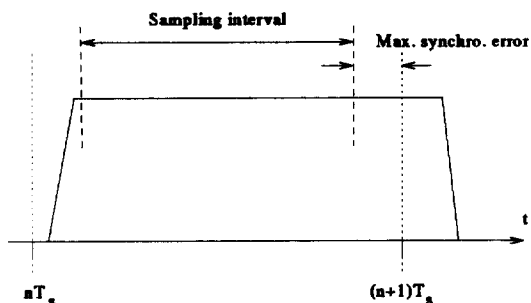


Figure 4: Technique of guard intervals to compensate for synchronization errors. The sampling interval with guard times is shown between dashed lines. The solid line represents a symbol pulse shape with maximum synchronization error (i.e. the guard time).

### 2.3 Discussion

A MCD simulation has been performed under *Ptolemy* to evaluate the performances of both methods. Simulation results are presented on figure 5. It shows the loss in  $E_b/N_0$ , for a BER of  $10^{-3}$  as a function of  $\sigma_T/T_s$ , the normalized standard deviation of the synchronization error of the disturbing neighbour. The latter is  $2/T_s$  away and 12 dB above the considered carrier.

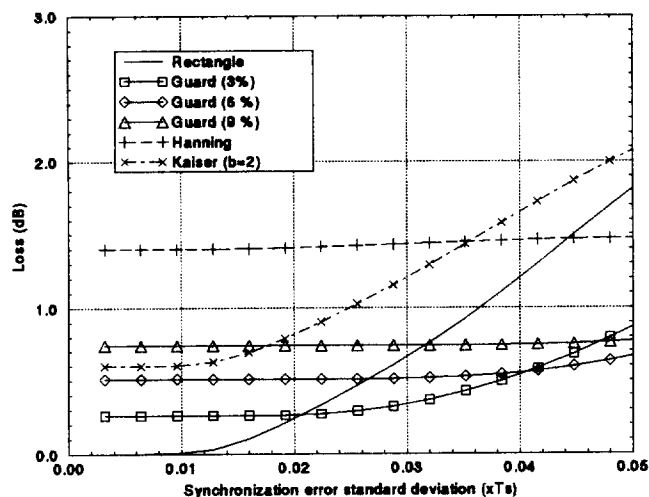


Figure 5:  $E_b/N_0$  loss (dB) for a BER of  $10^{-3}$  as a function of  $\sigma_T$ . This disturbing carrier is  $2/T_s$  away and is 12 dB above considered carrier.

It is obvious that, for small values of  $\sigma_T/T_s$ , the guard time technique provides the best performances. Moreover, the guard time may be sized to the synchronization precision of the modem, so that spectrum

efficiency remains high if  $\gamma \ll 1$ . On the other hand, the Hanning window gives very stable performances over a wide range of values. As a conclusion, it is interesting to choose the guard time technique if  $\sigma_T/T_s$  is small. However, this conclusion is only valid in the absence of doppler shift on the disturbing carrier, because the frequency localisation of the rectangular pulse with guard time remains that of the simple rectangular pulse. Hence, with this choice, the demodulation remains very sensitive to doppler shift.

## 3 FH QPSK demodulation method

In order to reduce ICI between users with various doppler shifts, we choose the Hanning window. Thus, for fixed transmitter power, the 1.6 dB loss implies a reduction of the symbol rate over the channel. Moreover, the  $2/T_s$  spacing between users drastically reduces the spectral efficiency. Therefore, it can be interesting, in non stressed conditions and for high data rates, to use a coherent phase digital modulation that preserves the constant envelope property desired for the use of non linear amplifiers. With QPSK, spectral efficiency is doubled and substantial gain in  $E_b/N_0$  may be expected with coherent demodulation (compared to differential demodulation of SDBPSK). In our case, the difficulty arises from the frequency hopping (FH) and the small number of symbols per hop (36) which does not enable classical coherent demodulation techniques. However, the few *pilot symbols* already present for differential demodulation, and which value is known to the receiver, can provide an estimate of the absolute phase of the signal to carry out the coherent demodulation of the QPSK symbols. We present four techniques using this *pilot symbols* approach and analyse their performances against doppler shift. When comparing the results to theoretical QPSK, one has to keep in mind that the loss due to the insertion of pilot symbols must be added (i.e. 0.51 dB for 4 pilots among 36 symbols).

### 3.1 Order 0 compensation

All four pilot symbols are located at the beginning of the hop and the receiver computes their complex mean. The conjugate of this value, multiplied by the expected value, gives the correction to apply to useful symbols until the end of the hop. Figure 6 gives the performance of this technique as a function of  $D = f_d * T_H$ , where  $f_d$  is the doppler shift and  $T_H$  is the duration of the hop. In our case  $T_H = 36T_s$ . Without doppler shift ( $D = 0$ ) we can estimate the incompressible loss which is around  $0.5 + 0.51 \approx 1$  dB, an acceptable value. For a small doppler value,  $D = 0.04$ , we further loose 1dB, which means that we are 2 dB above theoretical QPSK performances. When doppler increases to  $D = 0.2$ , we get

catastrophic results and the BER is stable around 0.25 whatever the value of  $E_b/N_0$ . This is due to constant phase compensation (order 0) over the whole hop while it should vary linearly.

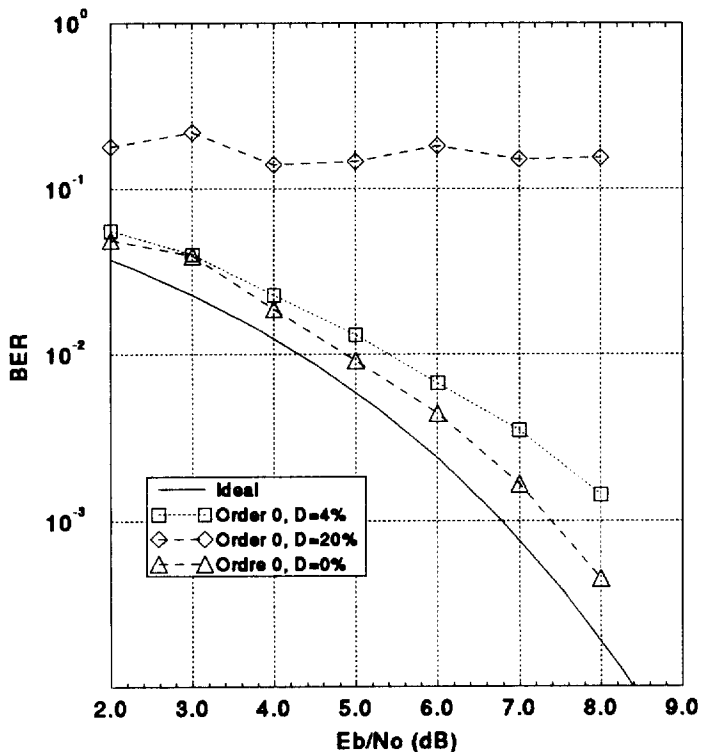


Figure 6: Absolute phase estimation and doppler compensation with order 0 technique. All 4 pilot symbols are at the beginning of the hop.

In order to resist higher doppler values, with the same simple method, it is possible to spread the pilots over the hop with, for example, 1 pilot every 8 symbols. With this modification, both doppler test values are compensated. However, the respective losses of  $2.8 + 0.51$  dB and  $3.2 + 0.51$  dB are unacceptable. Moreover, without doppler shift, the backward from theoretical performance is around  $2.7 + 0.51 \approx 3.2$  dB. The correction, estimated over a single value is far less accurate than with the first technique, where averaging on four pilots reduces the noise by 6 dB on the channel estimate.

### 3.2 Order 1 compensation

The above methods consider that the doppler shift is small enough to have a constant phase compensation over the whole hop. A more efficient technique should compensate a linear phase change (order 1). The following technique has been simulated and requires two steps. In a first step, the initial phase at the beginning of the hop is estimated, over four pilot symbols, with

the order 0 method explained above. Then, we take the fourth power of all symbols in the hop in order to suppress the QPSK phase modulation. Without noise, the symbols describe an arc of a circle with points located at  $\theta_i = 8\pi f_d i T_s$ . Knowing the absolute phase at the beginning of the hop, and the doppler shift by linear regression, it is then easy to remove the channel phase change. The technique compensates for both  $D = 0.04$  and  $D = 0.2$  doppler values. However, the incompressible loss is quite large, about  $1.0 + 0.51 \approx 1.5$  dB. Furthermore, the number of operations is important and the demodulation is deferred to the end of the hop, while this was not the case with the methods described before.

### 3.3 Feedback compensation

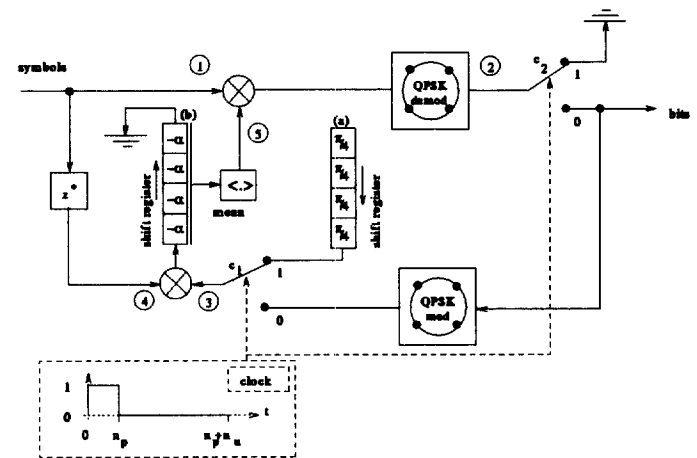


Figure 7: QPSK doppler compensation with feedback technique.

A third technique, involving a small number of operations, is shown on figure 7. The explanation, in a first step, supposes that the doppler is small enough to consider that the channel phase is constant over a hop. The shift register (a) is initialized with the expected pilot symbol values at the beginning of each hop (i.e.  $\pi/4$ ). The clock<sup>2</sup> at the bottom of the figure drives commutators  $c_1$  and  $c_2$ . The first received symbol is a pilot of known phase plus the channel phase  $\alpha$  we wish to suppress. The expected value (point 3) is multiplied by the conjugate of the received value (point 4) and the output  $-\alpha$  is pushed into shift register (b). The bits corresponding to pilot symbols are discarded by commutator  $c_2$ . When  $n_p$  pilots are received, register (b) contains the values  $(-\alpha, -\alpha, -\alpha, -\alpha)$  and  $c_1$  and  $c_2$  commute. The first useful symbol, with phase  $\phi + \alpha$  (point 1) is multiplied by the shift register's mean  $-\alpha$  (point 5) so that the QPSK demodulator is given the right value  $\phi$ . The output bits are fed to the loopback QPSK modulator which output is multiplied (point 4) with the conjugate of the current input value. This gives

<sup>2</sup>We recall that  $n_p$  and  $n_p + n_u$  respectively refers to the number of pilot symbols and the total number of symbols in a hop.

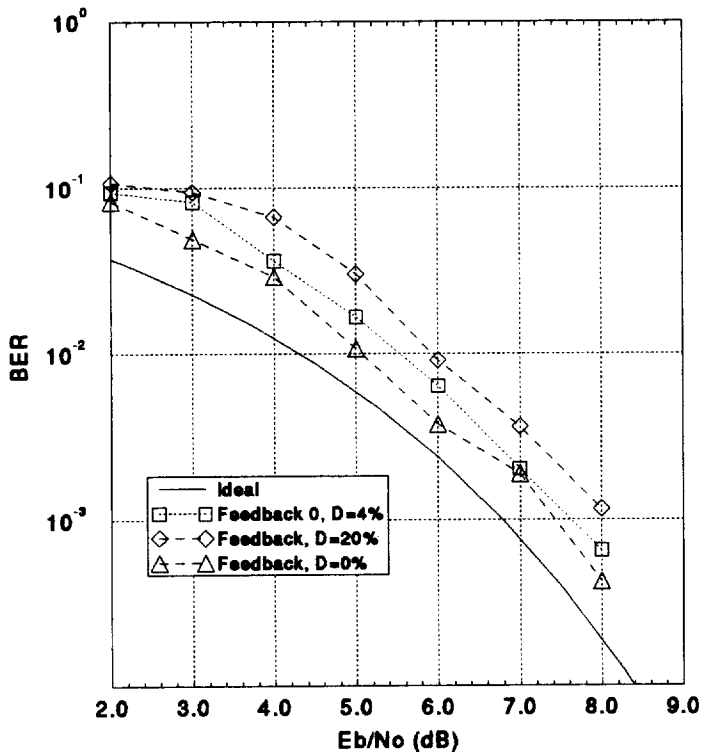


Figure 8: QPSK feedback technique performances under AWGN for doppler values  $D = 0.04$ ,  $D = 0.2$  and  $D = 0.0$ .

the right value  $-\alpha$  which is again pushed into shift register (b). The process is repeated until the end of the hop.

Now we suppose that the channel phase changes by  $d\alpha$  between two consecutive symbols (constant doppler shift). If the absolute phase estimate was correct at the preceding symbol and if  $d\alpha$  is small enough, the QPSK demodulator input symbol  $\phi + d\alpha$  remains in the right decision region, so that the value  $\phi$  is obtained at point 3. Therefore, the value  $-\alpha - d\alpha$  is pushed into shift register (b). Hence, for the next symbol, the correction value at point 1 is closer to the true correction value. An integrator is required in the loopback; it is the shift register (b) with computation of the mean. The longer the shift register, the smaller the effect of noise on the estimate. However, the estimated correction is delayed with regard of the true correction.

Simulations of the algorithm have been performed and a register length of 4 or 5 seems an appropriate compromise between doppler correction and performances under AWGN. Figure 8 shows the performances of this technique as a function of  $D = f_d * T_H$ . Without doppler the incompressible loss is less than  $0.7 + 0.51 \approx 1.2$  dB for a BER of  $10^{-3}$ . For a small doppler value,  $D = 0.04$ , the loss is about  $0.9 + 0.51 \approx 1.4$  dB above theoretical QPSK performances. When doppler increases to

$D = 0.2$ , the performances are still acceptable, with a loss about  $1.2 + 0.51 \approx 1.7$  dB. In these simulations, the number of pilot symbols and the register length were both equal to 4.

It is clear that the feedback technique has the best performances. The incompressible loss is close to that of order 0 technique and reduces when  $E_b/N_0$  increases. For example, assuming a hopping frequency  $f_H = 1/T_H = 9.10^3$  Hz and 36 symbols per hop, the method is able to compensate a doppler shift  $f_d = 1800$  Hz with a loss smaller than 1.7 dB if  $E_b/N_0$  is better than 8.0 dB. Moreover, the technique only requires two shift registers, a QPSK mapper and its inverse mapper, two multiplications, a conjugation and a four elements summation. Therefore, it does not require much computations and DSP implementation is possible.

## 4 Conclusion

We presented a possible implementation of a Multi Carrier Demodulator of the future military frequency hopped MDR. In its current implementation, derived from OFDM, doppler shift and synchronization errors introduce a very awkward ICI. We have seen that the performances can be improved with the use of weighting functions. The Hanning window seems to be a good compromise to resist both effects with a 1.6 dB loss. Such a loss was non negligible and we began to think of a coherent QPSK modulation mode. As the number of symbols per hop is small (36 symbols including pilot symbols in the highest mode), efficient demodulation is not an easy task, especially with doppler shift. We studied a demodulator, based on a feedback technique, which shows little incompressible loss and resists high enough doppler values for geostationary satellite applications.

### ACKNOWLEDGMENT

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## References

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