PERFORMANCE ANALYSIS OF MULTI-TIER ASYNCHRONOUS M-PSK DS/CDMA SYSTEMS WITH A NONLINEAR AMPLIFIER *

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Abstract

The performance of asynchronous M-PSK CDMA systems with a nonlinear amplifier is considered in this paper. The in-band degradation due to nonlinearities is of the major concern. Gaussian approximation with random spreading sequence is adopted to evaluate the system. The symbol error rates of systems with different number of users and different order of data modulation are presented with respect to the input backoff of the nonlinear amplifier.

1 Introduction

Nonlinear distortions, existing mostly in the power amplifiers, generally reduce system performances by as large as 10 dB being one of the major sources of degradation[2]. While most of the analyses of CDMA systems are based on the assumptions that signal waveforms are ideally/linearly retransmitted over the power amplifiers, it has been known that the existence of nonlinearities impacts a real system design.

The effects of nonlinear distortions on CDMA systems can be categorized into two classes – *out-band degradation* and *in-band degradation*. Due to high demand on frequency bandwidth, stringently regulatory emission requirements have always been enforced to prevent interference with other communication systems. To accommodate more users simultaneously in the designated frequency bandwidth, signals transmitted over wireless channels are always shaped so as to have a compact spectrum within this frequency bandwidth. It also means the out-band emission has to be below the regulated level. However, nonlinear distortions reshape the signals so that they lose their compactness in spectrum which leads to out-band spectral regrowth[1][4]. A band-pass filter then has to be utilized before the signal transmission in order to reject this undesired out-band power. The inefficiency of power utilization consequently results and the filtered signal experiences higher intersymbol interference. The BER then increases due to this extra undesired interference. We call this *out-band degradation* since the degradation is caused by the rejection of out-band power.

The second effect of nonlinear distortions is the *in-band degradation* [2][3]. Suppose the power gained from the nonlinear amplifier is totally consumed while signals travel through the channel. This nonlinear amplifier can then be regarded as a nonlinear transformation between the transmitters and the receivers. Considering a fully orthogonal system, i.e. all users are fully synchronized with a set of orthogonal spreading codes, orthogonality are preserved only if the channel is linear. In other words, in a non-fading linear channel, the only interference to each user is the thermal noise; no multiple-access interference (MAI) exists. Therefore, with or without the out-band filter, the nonlinear transformation has messed part of inter-user correlations, which introduces MAI to an originally orthogonal system or more MAI to an originally non-orthogonal system such as the asynchronous CDMA systems utilizing PN spreading codes. The impacts on systems are higher BER which further leads to lower system capacity.

This paper focuses on the *in-band degradation*. We analyzed the performance of a multi-tier M-PSK CDMA systems in the presence of nonlinear distortions. This is an extension of our previous work reported in [2], which considers the effects of nonlinear distortions on synchronous CDMA systems in a single cell or a single beam.

2 System Model

As depicted in Fig.1, the input signal $S(t)$ consists of a sum of CDMA signals from different beams in satellite systems or different cells in cellular systems. There are K_0 users in the cell/beam with the desired user, K_1 users in the first tier and K_2 users in the second tier. The nonlinear amplifier can be seen as a bent pipe, which is especially well

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known in the satellite communications. We further assume the downlink/forward channel is an Additive White Gaussian Noise (AWGN) channel with attenuation equal to the power gained through the nonlinear amplifier. This can be easily achieved since, in the case of unequal attenuation, its effect can be easily incorporated into the parameters of the model of the nonlinear amplifier.

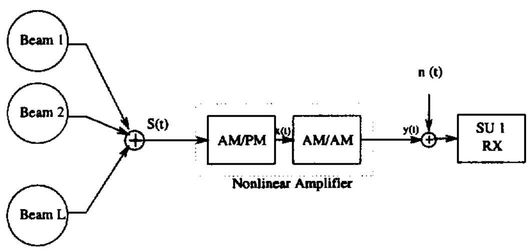

Figure 1. System Model

This system model can be viewed as a satellite system with high uplink power which leads to the omission of uplink noise. It can otherwise be seen as a forward link of cellular systems. In this case, the phases of all the local oscillators might be the same which is a special case of this model.

2.1 Transmitter

Suppose each user sends a M-PSK signal with inphase (I) and quadrature (Q) components spread by the respective user code sequence (see Fig.2). For the kth user, the M-PSK data signal of I and Q components can be represented respectively as

$$b_I^{(k)}(t) = \sum_{i=-\infty}^{\infty} b_I^{(k)}[i]\mathcal{I}_{T_s}(t - iT_s)$$

$$b_Q^{(k)}(t) = \sum_{i=-\infty}^{\infty} b_Q^{(k)}[i]\mathcal{I}_{T_s}(t - iT_s)$$

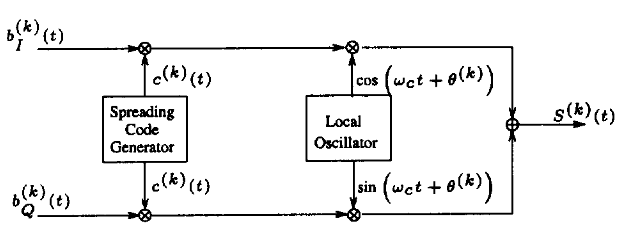

Figure 2. Transmitter of user k

Let p_k be the kth user's transmitted power. $\left(b_I^{(k)}[i], b_Q^{(k)}[i]\right) \in \left\{\left(\sqrt{2\mu_k p_k}\cos\frac{(2m-1)\pi}{M}, \sqrt{2\mu_k p_k}\sin\frac{(2m-1)\pi}{M}\right), m = 1, .., M\right\}$ with equal probability $\frac{1}{M}$ and μ_k is defined as

$$\mu_k = \begin{cases} \lambda_0 & k \le K_0 \\ \lambda_1 & K_0 < k \le K_0 + K_1 \\ \lambda_2 & K_0 + K_1 < k \le K_0 + K_1 + K_2 \end{cases}$$

λ_i is the *attenuation coefficient* of ith tier, which is the percentage of interference contributed from the ith tier. Usually λ_0 is assumed to be 1. This attenuation is the result of the geological separation of different beams[5]. T_s is the symbol period. $\mathcal{I}_T$ is defined as

$$\mathcal{I}_T(t) = \begin{cases} 1 & \text{if } 0 \le t \le T \\ 0 & \text{otherwise.} \end{cases}$$

The spreading code signals of the kth user can be represented as

$$c^{(k)}(t) = \sum_{i=-\infty}^{\infty} c^{(k)}[i]\mathcal{I}_{T_c}(t - iT_c)$$

where T_c is the chip duration and $c^{(k)}[i] \in \{-1, +1\}$. Taking into account the phase of each user's local oscillator, the output signal of user k can be represented as

$$\begin{aligned} S^{(k)}(t) = {} & b_I^{(k)}(t)c_I^{(k)}(t)\cos\left(\omega_c t + \theta^{(k)}\right) \\ & + b_Q^{(k)}(t)c_Q^{(k)}(t)\sin\left(\omega_c t + \theta^{(k)}\right) \end{aligned}$$

By assuming perfect synchronization, i.e. $\theta^{(1)} = 0, \tau^{(1)} = 0$, we can have

$$\begin{aligned} S(t) &= \sum_{k=1}^{K} S^{(k)}(t - \tau^{(k)}) \\ &= \sum_{k=1}^{K} A^{(k)}(t)\cos(\omega_c t) + \sum_{k=1}^{K} B^{(k)}(t)\sin(\omega_c t) \\ &= V(t)\cos(\omega_c t - \Phi(t)) \end{aligned}$$

where

$$\begin{aligned} A^{(k)}(t) &= b_I^{(k)}\left(t - \tau^{(k)}\right) c_I^{(k)}\left(t - \tau^{(k)}\right)\cos\varphi^{(k)} \\ &\quad + b_Q^{(k)}\left(t - \tau^{(k)}\right) c_Q^{(k)}\left(t - \tau^{(k)}\right)\sin\varphi^{(k)} \\ B^{(k)}(t) &= -b_I^{(k)}\left(t - \tau^{(k)}\right) c_I^{(k)}\left(t - \tau^{(k)}\right)\sin\varphi^{(k)} \\ &\quad + b_Q^{(k)}\left(t - \tau^{(k)}\right) c_Q^{(k)}\left(t - \tau^{(k)}\right)\cos\varphi^{(k)} \\ \varphi^{(k)} &= \theta^{(k)} - \omega_c \tau^{(k)} \end{aligned}$$

$$V(t) = \sqrt{\left[\sum_{k=1}^{K} A^{(k)}(t)\right]^2 + \left[\sum_{k=1}^{K} B^{(k)}(t)\right]^2}$$

$$\Phi(t) = \tan^{-1}\left[\frac{\sum_{k=1}^{K} B^{(k)}(t)}{\sum_{k=1}^{K} A^{(k)}(t)}\right]$$

2.2 Nonlinear Amplifiers

The model of the nonlinear amplifier consists of two factors: AM/AM and AM/PM[2]. AM/AM is represented by a 3rd-order polynomial which is a memoryless nonlinearity affecting only the amplitude of the input signal. The coefficients of the polynomial are determined by placing the local maximum or minimum at the saturation point of the nonlinear amplifier. In the mean time, AM/PM introduces a phase distortion which is proportional to the square of the envelope of the input signal.

The drive power at which the output power saturates is called the *input saturation power*. In Fig.3, the corresponding baseband input amplitude is 2.3. The ratio of input saturation power to desired drive power is called the *input backoff* (IBO). Similarly, the *output saturation power* is the maximum output power of an amplifier. Its corresponding baseband output amplitude in Fig.3 is 1. *Output backoff* (OBO) is therefore the ratio of the output saturation power to the actual output power. Increasing IBO or OBO leads to less output power but reduces the nonlinearities introduced during signal amplifiyication. The tradeoff between lower power and more nonlinearities results in the highest effective signal-to-noise ratio or, equivalently, the highest effective Es/No and the lowest BER.

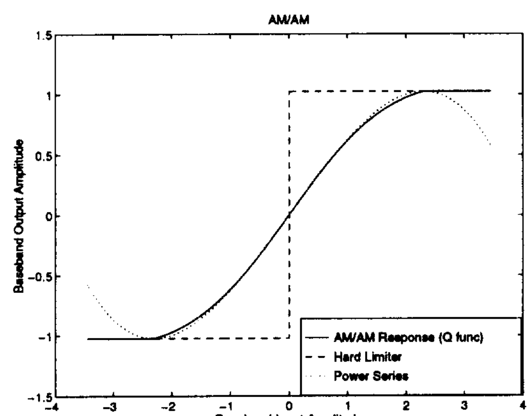

Figure 3. AM/AM curve

Therefore, AM/PM : $\Theta[A(t)] = \eta A^2(t) \ll 1$

$$\begin{aligned} x(t) &= V(t)\cos\{\omega_c t - \Phi(t) + \Theta[A(t)]\} \\ &\approx S(t) + D(t) \end{aligned}$$

where

$$D(t) = -\eta V^3(t)\sin[\omega_c t - \Phi(t)] \qquad (1)$$

AM/AM : $y(t) = \alpha_1 x(t) + \alpha_3 x^3(t)$,

$$y(t) = \alpha_1\left[S(t) + D(t)\right] + \alpha_3\left[S(t) + D(t)\right]^3 \qquad (2)$$

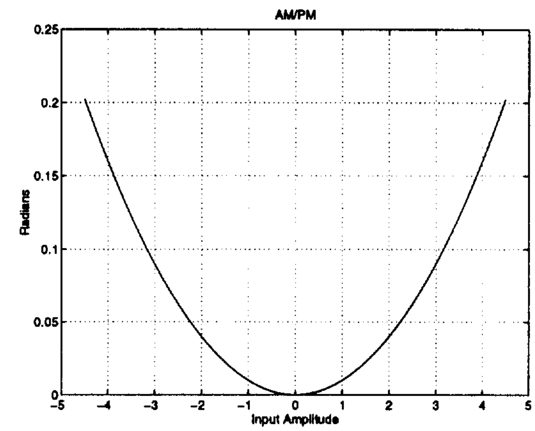

Figure 4. AM/PM curve

$$\begin{aligned} y(t) &\approx \left[\alpha_1 + \alpha_3 V^2(t)\right] S(t) + \left[\alpha_1 + \alpha_3 V^2(t)\right] D(t) \\ &\quad + n_{I,d}(t)\cos(\omega_c t) + n_{Q,d}(t)\sin(\omega_c t) \\ &= \left[S_I(t) + D_I(t) + n_{I,d}(t)\right]\cos(\omega_c t) \\ &\quad + \left[S_Q(t) - D_Q(t) + n_{Q,d}(t)\right]\sin(\omega_c t) \end{aligned}$$

where

$$S_I(t) = \left[\alpha_1 + \alpha_3 V^2(t)\right]\sum_{k=1}^{K} A^{(k)}(t)$$

$$D_I(t) = \eta\left[\alpha_1 V^2(t) + \alpha_3 V^4(t)\right]\sum_{k=1}^{K} B^{(k)}(t)$$

$$S_Q(t) = \left[\alpha_1 + \alpha_3 V^2(t)\right]\sum_{k=1}^{K} B^{(k)}(t)$$

$$D_Q(t) = \eta\left[\alpha_1 V^2(t) + \alpha_3 V^4(t)\right]\sum_{k=1}^{K} A^{(k)}(t)$$

In-phase RX of user 1:

$$\mathbf{Z_I} = \frac{1}{T_s}\int_0^{T_s} y(t) c_I^{(1)}(t) 2\cos(\omega_c t) dt \qquad (3)$$

The matched filter will then ignore the high frequency portion of the signal, so

$$\mathbf{Z_I} = \mathbf{S_I} + \mathbf{D_I} + \mathbf{N_I} \qquad (4)$$

where

$$\mathbf{S_I} = \frac{1}{T_s}\int_0^{T_s} S_I(t) c_I^{(1)}(t) dt$$

,

$$\mathbf{D_I} = \frac{1}{T_s}\int_0^{T_s} D_I(t) c_I^{(1)}(t) dt$$

,

$$\mathbf{N_I} = \frac{1}{T_s}\int_0^{T_s} N_I(t) c_I^{(1)}(t) dt$$

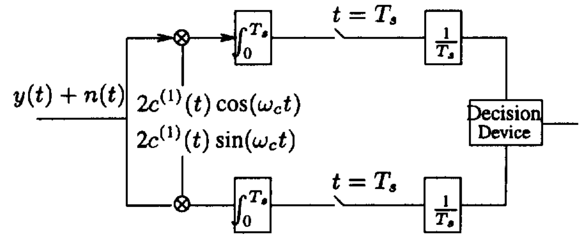

Figure 5. Receiver Model of user 1

. The mean of $\mathbf{N_I}$ is 0 and its variance is $\sigma_N^2 = \frac{N_0}{T_s}$. $\frac{N_0}{2}$ is the two-sided power spectral density.

Therefore,

$$\mathbf{S_I} = \frac{1}{T_s}\int_0^{T_s} S_I(t)c_I^{(1)}(t)dt$$
$$= \alpha_1 \frac{1}{T_s}\int_0^{T_s} \sum_{k=1}^{K} A^{(k)}(t)c_I^{(1)}(t)dt$$
$$+\alpha_3 \frac{1}{T_s}\int_0^{T_s} \sum_{k_1,k_2,k_3=1}^{K} A^{(k_1)}(t)A^{(k_2)}(t)A^{(k_3)}(t)c_I^{(1)}(t)dt$$
$$+\alpha_3 \frac{1}{T_s}\int_0^{T_s} \sum_{k_1,k_2,k_3=1}^{K} A^{(k_1)}(t)B^{(k_2)}(t)B^{(k_3)}(t)c_I^{(1)}(t)dt$$

Similarly,

$$\mathbf{D_I} = \frac{1}{T_s}\int_0^{T_s} D_I(t)c_I^{(1)}(t)$$
$$= \eta\alpha_1 \frac{1}{T_s}\int_0^{T_s} \sum_{k_1,k_2,k_3=1}^{K} B^{(k_1)}(t)A^{(k_2)}(t)A^{(k_3)}(t)c_I^{(1)}(t)dt$$
$$+\eta\alpha_1 \frac{1}{T_s}\int_0^{T_s} \sum_{k_1,k_2,k_3=1}^{K} B^{(k_1)}(t)B^{(k_2)}(t)B^{(k_3)}(t)c_I^{(1)}(t)dt$$
$$+\eta\alpha_3 \frac{1}{T_s}\int_0^{T_s} \sum_{\substack{k_1,k_2\\k_3,k_4,k_5=1}}^{K} \Big\{B^{(k_1)}(t)A^{(k_2)}(t)A^{(k_3)}(t)A^{(k_4)}(t)A^{(k_5)}(t)$$
$$+2B^{(k_1)}(t)B^{(k_2)}(t)B^{(k_3)}(t)A^{(k_4)}(t)A^{(k_5)}(t)$$
$$+B^{(k_1)}(t)B^{(k_2)}(t)B^{(k_3)}(t)B^{(k_4)}(t)B^{(k_5)}(t)\Big\}\, c_I^{(1)}(t)dt$$

3 Performance Analysis

Suppose the data is equiprobable and the phase of each user's local oscillator is uniformly distributed. The Gaussian Approximation method with random spreading sequence model is adopted to evaluate this system for the reason of its success on evaluation of synchronous CDMA systems[2][3]. Only results of QPSK systems are presented here. The same principle can be applied to general M-PSK systems.

Given the transmitted data to be $\{d_I, d_Q\}$ for the first user, the first moment of received signal can be written as

$$W_I = a_{00}K_0^2 + a_{11}K_1^2 + a_{22}K_2^2 + a_{01}K_0K_1 + a_{02}K_0K_2$$
$$+a_{12}K_1K_2 + a_0K_0 + a_1K_1 + a_2K_2 + a_{rem}$$

where

$$a_{00} = 24\,\eta\,\alpha_3\,\lambda_0{}^2\,d_Q\,p^2$$
$$a_{11} = 24\,\eta\,\alpha_3\,\lambda_1{}^2\,d_Q\,p^2$$
$$a_{22} = 24\,\eta\,\alpha_3\,\lambda_2{}^2\,d_Q\,p^2$$
$$a_{01} = 48\,\eta\,\alpha_3\,\lambda_0\,\lambda_1\,d_Q\,p^2$$
$$a_{02} = 48\,\eta\,\alpha_3\,\lambda_0\,\lambda_2\,d_Q\,p^2$$
$$a_{12} = 48\,\eta\,\alpha_3\,\lambda_1\,\lambda_2\,d_Q\,p^2$$

$$a_0 = -60\,\eta\,\alpha_3\,\lambda_0{}^2\,d_Q\,p^2 + (12\,\eta\,\alpha_3\,\lambda_0\,d_Q{}^3 + 12\,\eta\,\alpha_3\,\lambda_0\,d_I{}^2\,d_Q$$
$$+ 4\,\alpha_3\,d_I\,\lambda_0 + 4\,\eta\,\alpha_1\,\lambda_0\,d_Q)p$$

$$a_1 = \left(-12\,\eta\,\alpha_3\,\lambda_1{}^2\,d_Q - 48\,\eta\,\alpha_3\,\lambda_0\,\lambda_1\,d_Q\right)p^2 + (4\,\alpha_3\,d_I\,\lambda_1$$
$$+ 4\,\eta\,\alpha_1\,\lambda_1\,d_Q + 12\,\eta\,\alpha_3\,\lambda_1\,d_Q{}^3 + 12\,\eta\,\alpha_3\,\lambda_1\,d_I{}^2\,d_Q)p$$

$$a_2 = \left(-12\,\eta\,\alpha_3\,\lambda_2{}^2\,d_Q - 48\,\eta\,\alpha_3\,\lambda_0\,\lambda_2\,d_Q\right)p^2 + (4\,\eta\,\alpha_1\,\lambda_2\,d_Q$$
$$+ 12\,\eta\,\alpha_3\,\lambda_2\,d_Q{}^3 + 12\,\eta\,\alpha_3\,\lambda_2\,d_I{}^2\,d_Q + 4\,\alpha_3\,d_I\,\lambda_2)p$$

$$a_{rem} = 36\,\eta\,\alpha_3\,\lambda_0{}^2\,d_Q\,p^2 + (-4\,\eta\,\alpha_1\,\lambda_0\,d_Q$$
$$- 12\,\eta\,\alpha_3\,\lambda_0\,d_Q{}^3 - 12\,\eta\,\alpha_3\,\lambda_0\,d_I{}^2\,d_Q - 4\,\alpha_3\,d_I\,\lambda_0)p$$

And the second moment is

$$\sigma_I^2 = b_{000}K_0^3 + b_{111}K_1^3 + b_{222}K_2^3 + b_{012}K_0K_1K_2$$
$$+b_{011}K_0K_1K_1 + b_{112}K_1K_1K_2 + b_{022}K_0K_2K_2$$
$$+b_{122}K_1K_2K_2 + b_{00}K_0^2 + b_{11}K_1^2 + b_2K_2^2 + b_{01}K_0K_1$$
$$+b_{02}K_0K_2 + b_{12}K_1K_2 + b_0K_0$$
$$+b_1K_1 + b_2K_2 + b_{rem}$$

where

$$b_{000} = 24\,\frac{\alpha_3{}^2\,\lambda_0{}^3\,p^3}{N}$$
$$b_{111} = 24\,\frac{\alpha_3{}^2\,\lambda_1{}^3\,p^3}{N}$$
$$b_{222} = 24\,\frac{\alpha_3{}^2\,\lambda_2{}^3\,p^3}{N}$$

$$b_{012} = \alpha_3{}^2\left(5\,\lambda_1{}^2\,\lambda_0 + 4\,\lambda_2\,\lambda_1{}^2 + 4\,\lambda_1\,\lambda_2{}^2 + 85\,\lambda_0\,\lambda_1\,\lambda_2\right.$$
$$\left.+ 4\,\lambda_0{}^2\,\lambda_1 + 4\,\lambda_0{}^2\,\lambda_2 + 4\,\lambda_2{}^2\,\lambda_0\right)p^3/N$$

$$b_{001} = 4\frac{\alpha_3^2 \lambda_0 \left(12\lambda_0\lambda_1 + \lambda_0^2 + 2\lambda_1^2\right) p^3}{N}$$

$$b_{002} = 4\frac{\alpha_3^2 \lambda_0 \left(2\lambda_2^2 + \lambda_0^2 + 12\lambda_0\lambda_2\right) p^3}{N}$$

$$b_{011} = \frac{\alpha_3^2 \lambda_1 \left(47\lambda_0\lambda_1 + 4\lambda_1^2 + 9\lambda_0^2\right) p^3}{N}$$

$$b_{112} = 4\frac{\alpha_3^2 \lambda_1 \left(2\lambda_2^2 + 12\lambda_1\lambda_2 + \lambda_1^2\right) p^3}{N}$$

$$b_{022} = 4\frac{\alpha_3^2 \lambda_2 \left(2\lambda_0^2 + 12\lambda_0\lambda_2 + \lambda_2^2\right) p^3}{N}$$

$$b_{122} = 4\frac{\alpha_3^2 \lambda_2 \left(\lambda_2^2 + 12\lambda_1\lambda_2 + 2\lambda_1^2\right) p^3}{N}$$

$$b_{01} = \alpha_3 p^2 \Big(-4\alpha_3 p\lambda_1^3 - 12\alpha_3 p\lambda_0^3 - 117\alpha_3 p\lambda_0^2\lambda_1 - 35\alpha_3 p\lambda_1^2\lambda_0 + 88\lambda_0\lambda_1\alpha_3 d_I^2 + 24\lambda_0\lambda_1\alpha_3 d_Q^2 + 16\lambda_0\lambda_1\alpha_1\Big)/N$$

$$b_{02} = 4\alpha_3 p^2\Big(-\alpha_3 p\lambda_2^3 - 9\alpha_3 p\lambda_2^2\lambda_0 - 3\alpha_3 p\lambda_0^3 - 29\alpha_3 p\lambda_0^2\lambda_2 + 22\lambda_0\lambda_2\alpha_3 d_I^2 + 6\lambda_0\lambda_2\alpha_3 d_Q^2 + 4\lambda_0\lambda_2\alpha_1\Big)/N$$

$$b_{12} = \alpha_3 p^2 \Big(-4\alpha_3 p\lambda_2^2\lambda_0 - 24\alpha_3 p\lambda_2\lambda_1^2 - 5\alpha_3 p\lambda_1^2\lambda_0 - 4\alpha_3 p\lambda_0^2\lambda_1 - 24\alpha_3 p\lambda_1\lambda_2^2 - 4\alpha_3 p\lambda_1^3 - 4\alpha_3 p\lambda_2^3 - 85\alpha_3 p\lambda_0\lambda_1\lambda_2 - 4\alpha_3 p\lambda_0^2\lambda_2 + 88\lambda_1\lambda_2\alpha_3 d_I^2 + 24\lambda_1\lambda_2\alpha_3 d_Q^2 + 16\lambda_1\lambda_2\alpha_1\Big)/N$$

$$b_0 = \frac{1}{2}\lambda_0 p\Big(318\alpha_3^2\lambda_0^2 p^2 - 40\alpha_1\alpha_3\lambda_0 p + 9\alpha_3^2\lambda_0 p\, d_I^2 N + 4\alpha_3^2\lambda_0 p\, d_Q^2 N - 168\alpha_3^2\lambda_0 p\, d_I^2 - 64\alpha_3^2\lambda_0 p\, d_Q^2 + 4\alpha_1\alpha_3 d_Q^2 + 9\alpha_3^2 d_I^4 + 2\alpha_1^2 + 12\alpha_1\alpha_3 d_I^2 + 80\alpha_3^2 d_I^2 d_Q^2 + \alpha_3^2 d_Q^4\Big)/N$$

$$b_1 = \frac{1}{2}p\Big(138\alpha_3^2 p^2\lambda_0^2\lambda_1 + 16\alpha_3^2 p^2\lambda_0^3 + 38\alpha_3^2 p^2\lambda_1^3 + 54\alpha_3^2 p^2\lambda_1^2\lambda_0 - 48\alpha_3^2\lambda_1 p\, d_Q^2\lambda_0 + 9\alpha_3^2\lambda_1^2 p\, d_I^2 N + 4\alpha_3^2\lambda_1^2 p\, d_Q^2 N - 16\alpha_3^2\lambda_1^2 p\, d_Q^2 - 8\alpha_3\lambda_1^2 p\alpha_1 + 8\alpha_3^2\lambda_1^2 p\, d_I^2 - 176\alpha_3^2\lambda_1 p\, d_I^2\lambda_0 - 32\alpha_3\lambda_1 p\alpha_1\lambda_0 + 4\alpha_1\alpha_3 d_Q^2\lambda_1 + 9\alpha_3^2\lambda_1 d_I^4 + 2\alpha_1^2\lambda_1 + 12\alpha_1\alpha_3 d_I^2\lambda_1 + 80\alpha_3^2 d_I^2 d_Q^2\lambda_1 + \alpha_3^2\lambda_1 d_Q^4\Big)/N$$

$$b_2 = \frac{1}{2}p\Big(38\alpha_3^2 p^2\lambda_2^3 + 56\alpha_3^2 p^2\lambda_2^2\lambda_0 + 16\alpha_3^2 p^2\lambda_0^3 + 136\alpha_3^2 p^2\lambda_0^2\lambda_2 + 9\alpha_3^2\lambda_2^2 p\, d_I^2 N - 32\alpha_3\lambda_2 p\alpha_1\lambda_0 + 4\alpha_3^2\lambda_2^2 p\, d_Q^2 N - 48\alpha_3^2\lambda_2 p\, d_Q^2\lambda_0 + 8\alpha_3^2\lambda_2^2 p\, d_I^2 - 16\alpha_3^2\lambda_2^2 p\, d_Q^2 - 176\alpha_3^2\lambda_2 p\, d_I^2\lambda_0 - 8\alpha_3\lambda_2^2 p\alpha_1 + 4\alpha_1\alpha_3 d_Q^2\lambda_2 + 9\alpha_3^2\lambda_2 d_I^4 + 2\alpha_1^2\lambda_2 + 12\alpha_1\alpha_3 d_I^2\lambda_2 + 80\alpha_3^2 d_I^2 d_Q^2\lambda_2 + \alpha_3^2\lambda_2 d_Q^4\Big)/N$$

$$b_{00} = 4\frac{\alpha_3\lambda_0^2 p^2(-27\alpha_3\lambda_0 p + 2\alpha_1 + 3\alpha_3 d_Q^2 + 11\alpha_3 d_I^2)}{N}$$

$$b_{11} = \alpha_3\lambda_1 p^2\Big(-47\alpha_3 p\lambda_0\lambda_1 - 40\alpha_3 p\lambda_1^2 - 9\alpha_3 p\lambda_0^2 + 44\lambda_1\alpha_3 d_I^2 + 12\alpha_3 d_Q^2\lambda_1 + 8\alpha_1\lambda_1\Big)/N$$

$$b_{22} = 4\alpha_3\lambda_2 p^2\Big(-2\alpha_3 p\lambda_0^2 - 12\alpha_3 p\lambda_0\lambda_2 - 10\alpha_3 p\lambda_2^2 + 2\alpha_1\lambda_2 + 3\alpha_3 d_Q^2\lambda_2 + 11\lambda_2\alpha_3 d_I^2\Big)/N$$

$$b_{rem} = -\frac{1}{2}\lambda_0 p\Big(150\alpha_3^2\lambda_0^2 p^2 - 80\alpha_3^2\lambda_0 p\, d_I^2 - 24\alpha_1\alpha_3\lambda_0 p + 4\alpha_3^2\lambda_0 p\, d_Q^2 N - 40\alpha_3^2\lambda_0 p\, d_Q^2 + 9\alpha_3^2\lambda_0 p\, d_I^2 N + 4\alpha_1\alpha_3 d_Q^2 + 9\alpha_3^2 d_I^4 + 2\alpha_1^2 + 12\alpha_1\alpha_3 d_I^2 + 80\alpha_3^2 d_I^2 d_Q^2 + \alpha_3^2 d_Q^4\Big)/N$$

4 Numerical Results & Simulations

In this system considered, $K_1 = 6K_0$ and $K_2 = 12K_0$, i.e. L=19 in Fig.1. K_0 is the number of users in one beam. $\lambda_1 = 1.16 \times 10^{-1}$ and $\lambda_2 = 1.226 \times 10^{-3}$. System performances are considered with the nonlinear amplifier shown in Fig.3 and Fig.4 and fixed variance of additive white gaussian noise. In other words, instead of fixing the Es/No of each case, the total power transmitted and noise added are fixed, which is the real case in communication systems.

Fig.6 shows the IBO versus Es/No and effective Es/No in single beam for QPSK modulation and processing gain 127. Fig.7 is for multiple tiers. For both of these cases, less users always get higher Es/No as well as higher effective Es/No because the amount of power gained through the amplifier by all users is fixed. Also note that the power gained through the amplifier is not linearly proportional to the reciprocal of number of users because of the different dynamic range. Fig.8 shows the difference of effective Es/No between single beam and multiple tiers. It shows that the maximum is about 1.6 dB at IBO 2 dB. Fig.9 gives the IBO versus symbol error rate (SER) in single beam and Fig.10 is in multiple tiers.

5 Conclusions

Mathematical analysis of the performance of multi-tier M-PSK CDMA systems with a nonlinear amplifier is presented. The optimal operating point of the nonlinear amplifier for QPSK DS/CDMA systems with processing gain 127 is around 4 dB IBO. At this operating point, the difference of effective Es/No between single beam and multiple tiers is about 1 dB.

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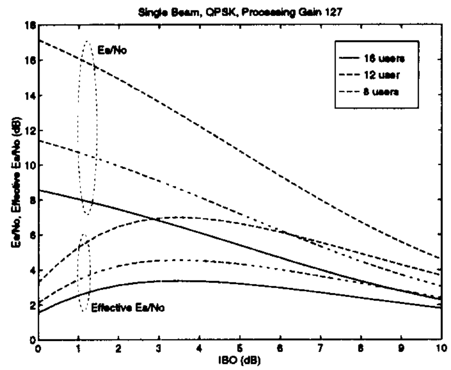

Figure 6. IBO vs. Es/No and Effective Es/No in Single Beam

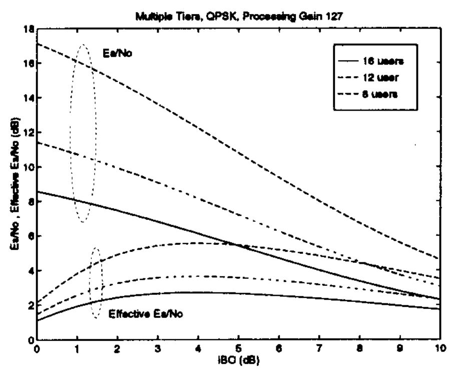

Figure 7. IBO vs. Es/No and Effective Es/No in Multiple Tiers

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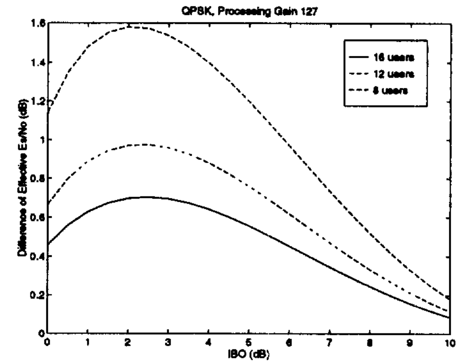

Figure 8. IBO vs. Difference of Effective Es/No between Single Beam and Multiple Tiers

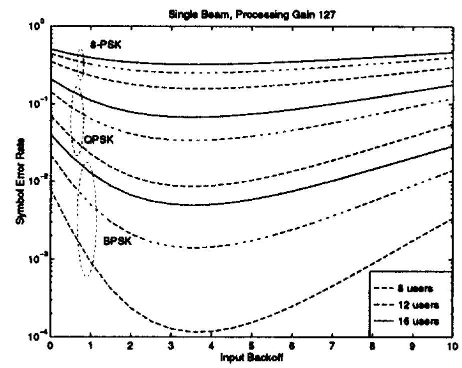

Figure 9. IBO vs. SER in Single Beam, Processing Gain 127

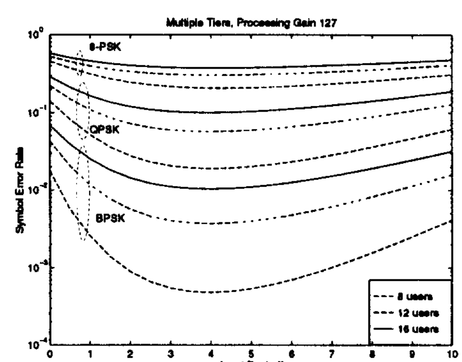

Figure 10. IBO vs. SER in Multiple Tiers, Processing Gain 127