THE EFFECT OF MOBILE VELOCITY ON A COMMUNICATION SYSTEM OPERATING OVER MULTIPATH FADING CHANNELS

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Abstract: In this paper, we present an analytical model to determine the probability of error for a communication system as a function of mobile velocity. We investigate the performance of a system model with power control and error control coding. In a multipath fading environment, the rate of fading is highly dependent on the velocity of the mobile. To incorporate the effect of velocity, we apply a Markovian model to the channel that captures the correlation between successive bits. We then develop an error recursion that accounts for the power control and the channel coding. At low speeds, we find that effective power control is crucial to maintain low error rates. At higher speeds where power control is ineffective, error control coding is necessary to mitigate the channel impairments. For more intermediate speeds, the model gives the worst performance because neither power control nor error control coding can counteract the effects of the channel.

1 Introduction

Wireless communication systems suffer from a variety of channel impairments. Multipath fading has a particularly deleterious effect because it causes deep fades in the signal amplitude that lead to decoding errors at the receiver. Different techniques have been devised to counteract this type of signal degradation. However, because the rate of fading is strongly dependent on the speed of the mobile, the effectiveness of these different schemes vary considerably as a function of mobile velocity. In this paper, we study the effect of mobile velocity on system performance and then show how a combination of power control and error control coding can help mitigate the effects of the time varying channel.

There are two basic types of power control for the reverse link of a cellular system. However, only closed loop power control is effective against fading. In this feedback loop, a base station monitors the signaling power of a mobile and then transmits power control information to adjust the mobile's signal strength according to the state of the channel. Although closed loop power control can counteract the effects of fading, it is far from perfect. Because it is a feedback loop, the power control incurs a delay between when the channel is measured by the base station and the resulting update is sent back to the mobile. This delay results in imperfect power control because the level of fading changes during the execution of the control loop. Furthermore, if the mobile is moving at a high velocity, the rate of fading has a correspondingly high rate of change and thus the power control will be even less effective.

Channel coding also counteracts the effects of multipath fading. By adding redundancy to the transmitted information, errors that result from deep fades due to the channel can be corrected. However, like power control, this scheme is not entirely effective. If a mobile remains in a deep fade for too long, this can cause a burst of errors which may surpass the error correcting capability of the code. If the data can tolerate some delay at the receiver, interleaving can be used to break up the information bits and thus make the data more robust against bursts of errors. However, if the mobile is moving slowly and thus experiences a slowly changing channel, it may stay in a fade longer than the depth of interleaving which will then cause decoding errors.

Based on the observations above, by combining these two methods, we would expect to find that the overall system is more robust over a larger range of mobile velocities. In this paper, we present a model to evaluate the performance of a communication system as a function of velocity. We show how power control performs

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well at low velocities while channel coding works well at high velocities. Then, we demonstrate how the performance of the cellular system improves by combining these two techniques.

The organization of this paper is as follows. In Section 2, we introduce the system model and state various assumptions. In Section 3, we present an analytical framework to evaluate the performance of a cellular system. Numerical results are presented in Section 4. Finally, Section 5 summarizes our findings and comments on future work.

2 System Model

2.1 System Fundamentals

In this paper, we consider the link from the mobile user to the base station–usually the limiting link in cellular communications. Using a baseband model, the transmitted signal at the mobile is given by $s(t) = \sum_{n=-\infty}^{\infty} b_n l_n p(t - nT_b)$ where b_n is the data bit, $p(t)$ is some pulse shape resulting in no intersymbol interference at the receiver, and l_n is the correction factor, in volts, received from the power control loop, which will be defined later.

In our system, we model the fading as frequency nonselective; that is the frequency response of the channel does not vary over the signal bandwidth. We use the Gaussian Wide-Sense Stationary Uncorrelated Scattering (GWSSUS) fading model from [1] where the underlying components of the fading process, $X_r(t)$ and $X_i(t)$, are modeled as zero-mean Gaussian with autocorrelation function $R(\tau)$. Hence, $X(t)e^{j\Theta(t)} = X_r(t) + jX_i(t)$ is complex Gaussian, $E[X_r(t)] = E[X_i(t)] = 0$ and $E[X_r(t)X_r(t+\tau)] = E[X_i(t)X_i(t+\tau)] = R(\tau)$. Throughout this paper, we will employ the autocorrelation function $R(\tau) = J_0(2\pi f_m \tau)$ from [2] where $J_0(\cdot)$ is the zeroth order Bessel function and f_m is the Doppler frequency given as $f_m = \frac{v * f_c}{c}$ where v is the speed of the mobile, f_c is the carrier frequency, and c is the speed of light. Thus, there is a clear relationship between the fading process and the velocity of the mobile. Finally, we assume that the fading varies slowly enough so that it stays constant over a bit duration T_b.

2.2 Power Control

With the transmitter and the channel as defined, we now describe the model used for closed loop power control. Let R_p be the rate of power control updates, then $M = \frac{1}{T_b R_p}$ is the number of bits transmitted between power control updates. Let i denote the ith power control group and j denote the jth bit in the ith power control group where $1 \leq j \leq M$. Using results from [3], we develop a simplified power control loop that captures the essential points of the feedback loop. The model of the power control is shown in Figure 1. Note that to impose linearity onto the entire loop we have transformed the measurements from linear units into power. Thus, l_n is related to the transmitted power $L_{i,j}$ by $L_{i,j} = 20\log_{10} l_n$ where $n = iM + j$.

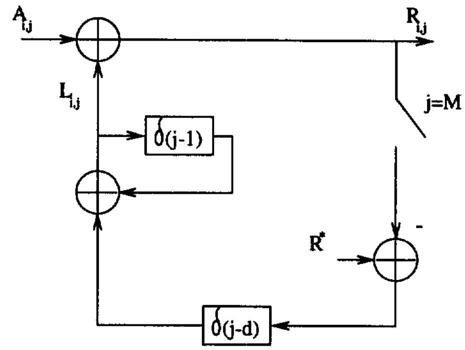

Figure 1: Basic Power Control Model

In the model, $L_{i,j}$ is the corrected signaling power at the mobile receiver. $A_{i,j}$ is the loss due to Rayleigh fading. $R_{i,j}$ is the received power at the base station where we assume that the base station can perfectly estimate the signal loss due to fading. R^* is some constant, desired received power. Furthermore, the processing and propagation delay is given as d.

The power control loop works as follows. When $j = M$, the switch closes and the received power $R_{i-1,M}$ is compared to R^*. A correction factor is computed and sent back to the mobile. After the correction is delayed by time d due to propagation and processing, it is combined with the previous transmitted power to form the new signaling power $L_{i,1}$. At all other times ($j \neq M$), the switch stays open and the transmitted power does not change. With this simple model for the closed loop power control, we can now develop the analytical framework to evaluate system performance.

3 System Performance

With the system model as defined, we develop an analytical model to characterize the performance of the system. To this end, we first develop a Markovian channel model that lends to a tractable analysis. Next, we consider the power control loop in our derivation and obtain an expression for the performance of the system which can be evaluated in a recursive fashion.

3.1 Channel Model

To characterize the Rayleigh fading as a Markov process, we compute a steady state distribution and a transition matrix. First partition the fading process $A_{i,j}$ (in dB) into L discrete levels $U_1, U_2, \ldots, U_L$. The steady state probability is denoted by $\underline{P}_A$ where the i-th element is given by:

$$P_A^{(i)} = \int_{U_i} p_A(a)da \tag{1}$$

In order to use the power control model described above, $p_A(a)$ is the Rayleigh probability density function (pdf) transformed into dB and is given as $p_A(a) = \frac{2}{20\log_{10} e} 10^{a/10} \exp(-10^{a/10})$ as in [4].

To compute the transition matrix of the Rayleigh fading process, we use the standard GWSSUS assumptions to obtain a closed form representation of the pdf for the current state of the faded channel conditioned on a past estimate. Given that the in-phase and quadrature processes are zero-mean Gaussian with variance σ^2, we can obtain the conditional pdf where s is the current level of fading given some previous level r [5]:

$$p(s|r) = \frac{s \exp\left(-\frac{\rho^2 r^2 + s^2}{2\sigma^2(1-\rho^2)}\right)}{\sigma^2(1-\rho^2)} I_0\left(rs\frac{\rho}{\sigma^2(1-\rho^2)}\right)$$

where $\rho = R(\tau)/\sigma^2$ and $I_0(\cdot)$ is the zeroth order modified Bessel function. We can then obtain the transition probability matrix $\mathbf{P}$ of the fading process. After employing a transformation to change $p(s|r)$ to dB units and then setting $\tau = T_b$ as the duration of one bit. A single transition in $\mathbf{P}$ is given by:

$$P_{a,b} = \int_{U_a}\int_{U_b} p_{db}(s|r) ds dr \tag{2}$$

$$= \int_{U_a}\int_{U_b} \frac{10^{s/10}}{10\log_{10}(e)(1-\rho^2)} \exp\left(\frac{10^{s/10} + \rho^2 10^{r/10}}{\rho^2 - 1}\right) I_0\left[10^{\frac{r+s}{20}} \frac{2\rho}{1-\rho^2}\right] ds dr \tag{3}$$

Clearly, the transition matrix varies as a function of vehicular speed through ρ. Note that even though the fading process is not Markovian, as we will see later, using this model yields accurate results. Thus, (1) and (3) give a steady state distribution and a transition matrix that characterize the quantized Rayleigh fading.

3.2 Power Control Model

To account for the power control in determining the performance of the communication system, we develop a model that computes the received power at the base station. From Figure 1, we can obtain the following expression for the received power of the jth bit in the ith power control group:

$$\begin{aligned} R_{i,j} &= A_{i,j} + L_{i,j} \\ &= A_{i,j} - A_{i-1,M} + R^* \\ &= A_{i,j} - B_{i-1} + R^* \end{aligned} \tag{4}$$

where we have used the fact that $L_{i,y} = L_{i,z} \;\; \forall y, z \neq M$ and we let $B_{i-1} = A_{i-1,M}$, the delayed version of the update. Thus, (4) gives an expression for the received power that is dependent only on the fading process and the power control.

3.3 Error Recursion

With the channel and power control as modeled, we can now obtain an error recursion for our system. Note that if we limit ourselves to $L = 2$ quantized levels, the channel model is similar to the Gilbert Elliot model and a relatively simple error recursion follows in [6]. We would like to develop a similar error recursion except to incorporate power control and channel coding. Let us define $P(m,n)$ as the probability of m errors in n bits. We will restrict the recursion to $n \leq M$; that is the length of the codeword is less than the size of one power control group. In the ith power control group consisting of M bits (for $n \leq M$, i is arbitrarily chosen), the value of the received power $R_{i,j}$ in (4) depends on the update from the previous power control group, B_{i-1}. Therefore the $P(m,n)$ recursion can be computed by conditioning over the range of values that B_{i-1} can take on:

$$P(m,n) = \sum_{l=1}^{L} P(m,n|B_{i-1} = U_l) * P(B_{i-1} = U_l)$$

A recursion for the probability of m errors in n steps conditioned on B_{i-1} is obtained by partitioning the recursion over the quantized levels of fading $(U_1, U_2, \ldots, U_L)$. This gives:

$$P(m,n|B_{i-1}) = \sum_{l=1}^{L} P(m,n,A_{i,n} = U_l|B_{i-1})$$

Letting $P_l(m,n|B_{i-1}) = P(m,n,A_{i,n} = U_l|B_{i-1})$ a recursion can now be obtained. The recursion for $P_l(m,n|B_{i-1})$ is dependent on the two terms $P_l(m,n-1|B_{i-1})$ and $P_l(m-1,n-1|B_{i-1})$ for all L states. Thus, the probability of m errors in n bits for state U_k is:

$$P_k(m,n|B_{i-1}) =$$

$$\sum_{l=1}^{L} P_l(m, n-1|B_{i-1})P(U_k|U_l)P(\bar{\xi}|B_{i-1}, A_{i,n} = U_k) +$$
$$\sum_{l=1}^{L} P_l(m-1, n-1|B_{i-1})P(U_k|U_l)P(\xi|B_{i-1}, A_{i,n} = U_k)$$

where ξ is the event of a bit error for the nth symbol. Each summation consists of 3 terms: $P_l(\cdot, n-1|B_{i-1})$, $P(U_k|U_l)$ and $P(\cdot|B_{i-1}, A_{i,n} = U_k)$. The second term, $P(U_k|U_l)$, is given by the transition matrix from (3). The third term is simply the bit error probability given the current level of fading $A_{i,j} = U_k$ and some B_{i-1}. For BPSK, with coherent demodulation and the received power from (4), the bit error probability becomes $P(\xi|B_{i-1}, A_{i,n} = U_k) = Q([2 * 10^{R_{i,n}/20}]^{1/2})$. Lastly, $P_l(\cdot, n-1|B_{i-1})$ depends on the recursion itself and the initial values:

$$\begin{aligned} P_k(0,1|B_{i-1}) &= P(\bar{\xi}|B_{i-1}, A_{i,1} = U_k)P(U_k|B_{i-1}) \\ P_k(1,1|B_{i-1}) &= P(\xi|B_{i-1}, A_{i,1} = U_k)P(U_k|B_{i-1}) \\ P_k(m,n|B_{i-1}) &= 0 \text{ for } m < 0 \text{ and } m > n \end{aligned}$$

In these initial terms, $P(\cdot|B_{i-1}, A_{i,1} = U_k)$ is defined as above and $P(U_k|B_{i-1})$ is identical to the transition probabilities given by $\mathbf{P}$ in (3) with the exception that $\tau = d$, the processing and propagation time.

Thus, we now have a recursion for $P(m, n)$, the probability of m errors in n steps, that incorporates the effect of power control and fading. If we assume a (n, k) binary linear code with t error correcting capability, the probability of packet error is $P_{\text{packet error}} = \sum_{m=t+1}^{n} P(m, n)$. From [7], a relatively loose upper bound for the probability of bit error can be computed as $P_{\text{bit error}} \leq \sum_{m=t+1}^{k} P(m,n)\frac{m}{k} + \sum_{m=k+1}^{n} P(m, n)$.

4 Numerical Results

Numerical results are presented for coherent reception of BPSK in a faded channel using (23, 12) Golay code where the fading is quantized over 32 levels. In this section, we will show how channel coding and power control mitigate the effects of multipath fading by studying the individual effects of coding and power control and then evaluating the combined effects of these two schemes.

Figure 2 compares the performance as a function of SNR of our uncoded communication system with no power control and the exact analysis of the same system from [8]. Thus, even though we restrict the Rayleigh fading process to be a Markov process quantized over 32 levels, this plot validates the accuracy of our model for reasonable levels of SNR (e.g. SNR $< 15dB$).

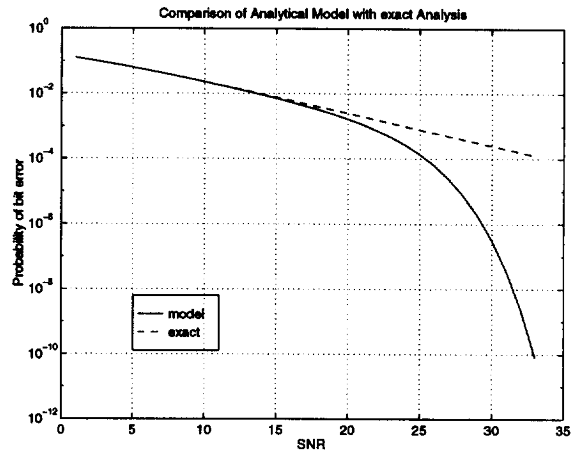

Figure 2: Performance vs SNR of analytical model

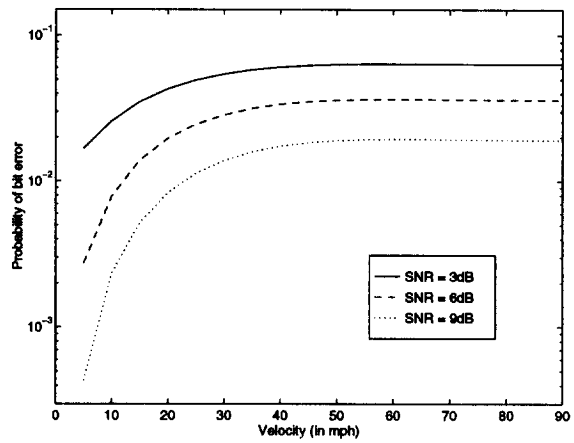

Figure 3: Performance of power controlled model

Figure 3 shows the performance versus mobile velocity for a power controlled system with no coding at 3 levels of SNR. This system with no coding performs well at low velocities but much worse at higher velocities. For slow mobile speeds, because the rate of fading is relatively slow, the fading stays relatively constant over many bit durations. This is reflected in the transition matrix given by (3); numerical analysis shows that given the channel is at some level of fading, the Markov process will most likely transition to the same state or to immediately adjacent states. Thus, power control will be very effective in counteracting the effects of the channel when mobile speeds are low. However, as speeds increase, the transition probabilities approach the steady state distribution of the Rayleigh process. Thus, the power control cannot track the change in the channel and system performance worsens.

The performance of a communication system with coding but no power control is shown in Figure 4. The performance of this system improves with increasing

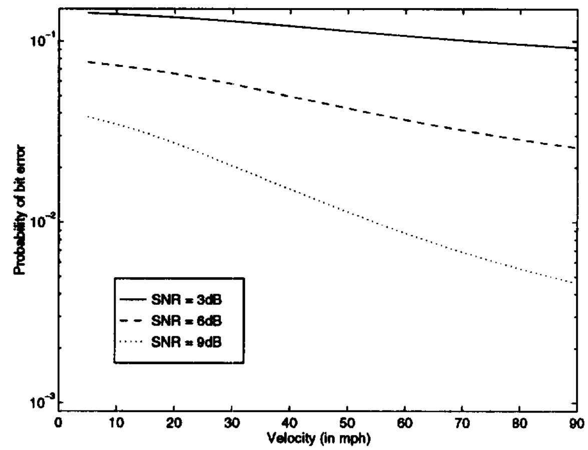

Figure 4: Performance of model with coding

mobile velocity. At low mobile speeds where the rate of fading is slow, deep channel fades cause bursts of errors in the transmitted data that exceed the error correcting capability of the code. However, as the mobile increases speed, the fading changes more quickly–thus, even if the mobile experiences a deep fade, it will quickly move out of it and only a few bit errors will occur. The coding works well in this situation and can correct the intermittent errors. This was observed in the numerical analysis of our system model. At low speeds, $P(m,n)$ contained most of the errors for high values of m suggesting long bursts of errors. Conversely, at high speeds, $P(m,n)$ was largest for low values of m suggesting that errors were spread out.

Finally, Figure 5 shows the performance of the system with both coding and power control as a function of mobile velocity. The plots confirm that the overall performance of the system will be more robust to fluctuations in mobile speed. One interesting observation

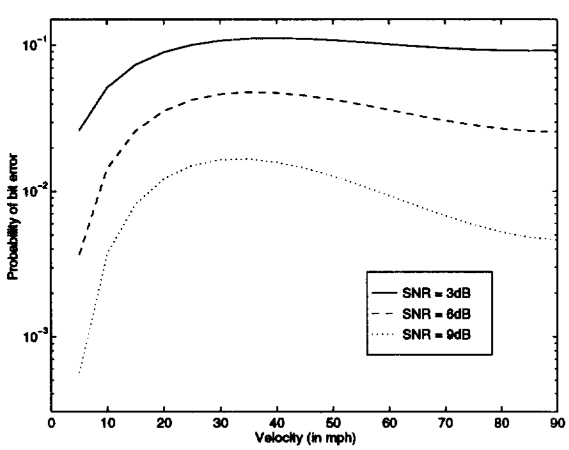

Figure 5: Performance of combined model

is that, with the coding scheme and power control parameters that we have used, the worst case performance occurs at a typical velocity for automobiles–at roughly 35 mph. Clearly, for vehicular applications, a proper mix of power control and coding parameters needs to be selected so that the worst case performance is outside the range of typical mobile speeds.

5 Conclusion

In this paper, we have presented a method to evaluate the performance of a communication system as a function of mobile velocity. From our results, we see that power control and coding are critical components to ensure the quality of service requirements in a wireless communication system. The combination of these two schemes enable the system to be far more robust in a Rayleigh fading environment.

A number of extensions are currently under way. We are studying how to extend the analytical model to incorporate interleaving, an effective way to spread out bursts of errors if some delay can be tolerated. We are also studying how the system performs when using more robust codes such as Reed Solomon codes.

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