Hybrid Spread Spectrum Signal Acquisition in the Presence of Worst Case Multitone Jamming

Siew-Kee Loh
Email: lsiewkee@dso.org.sg

Defence Science Organisation National Laboratories

Abstract

The acquisition performance of a hybrid slow frequency hopped direct sequence spread spectrum (SFH/DS SS) communication link in the presence of multitone jamming is analysed. Equations are derived to enable analysis of the optimisation of the jamming strategy. It is shown that the jamming strategy may be optimised by varying the number of tones. It is also shown that a higher DS spreading gain is preferable in improving the anti-jam capability of the sychronisation system against the multitone jammer.

1. Introduction

Recently, there is an increasing interest in hybrid slow frequency hopping/ direct sequence (SFH/DS) spread spectrum systems. [1] has studied the optimisation of DS spreading gain in the presence of worst case multitone jamming in order to minimise bit error rate. [1] has not analysed the effects of jamming against the synchronisation system.

It is well known that the synchronisation process is a critical part of any spread spectrum system [2]. Synchronisation in a hybrid system can be divided into two components: synchronisation to the hop sequence and to the DS code sequence. With the availability of very accurate clocks and the ability to access global time through GPS (global positioning system), we assume that synchronisation to the hop sequence is achieved by such accurate timing alone. This analysis thus focuses on the DS code sequence synchronisation required when a new packet of data is received at the start of every hop.

As in [3, pp. 292], the FH carrier frequency spacing (defined as a hop slot) is assumed to be equal to the DS spread bandwidth. Using a worst case assumption that the multitone jammer knows all of the hop frequencies, it will place, at most, a single tone centered in each jammed hop slot. The jammer will attempt to optimise its effectiveness by varying the number of tones N_J. With a small N_J, the SFH/DS system will not be able to synchronise to packets in a small number of hop slots; with increasingly larger N_J, more hop slots are subjected to jammer tones of ever decreasing powers. An optimised N_J will produce the largest degradation in synchronisation performance. This analysis enables such a trade-off to be studied.

The paper is organised as follows. In Section 2, the DS signal acquisition system is described. Derivation of the probability of synchronisation is presented in Section 3. Section 4 gives numerical results and concluding remarks are presented in Section 5.

2. System Model

Due to the short time available for data transmission in each hop dwell, reliable synchronisation should be achieved for each packet in the shortest possible time in order to increase data throughput. A good synchronisation scheme for such a system is based on matched filtering [4]-[6]. The parallel acquisition scheme analysed in [5] consists of a bank of N parallel I-Q matched filters (MF's) as shown in Fig. 1. The acquisition scheme has two modes of operation: a search mode and a coincidence detection (CD) mode. N different PN sequences, each with period T and chip duration T_c, are loaded into the bank of N MF's. Each PN code consists of $M = T/T_c$ chips. The number of taps in a MF is M/Δ where Δ is typically 1/2. It is assumed that the code uncertainty region is the total PN code length of $L = MN$ chips. In T seconds, L/Δ samples are collected from the N parallel MF's to be stored at the decision device. If the largest of the L/Δ samples exceeds a threshold γ_1, the corresponding PN code is assumed, tentatively, to be sent and the acquisition moves to the CD mode.

The first PN code in the preamble is for the search mode. This is followed by A PN codes used for CD to reduce the probability of false acquisition. Out of the A CD tests, if at least B tests cross the CD threshold, acquisition is complete and data demodulation begins. Otherwise, the synchronisation system repeats the above search and CD modes.

3. Performance Analysis

In sections 3a and 3b, the equations governing performance in a jammed or an unjammed hop slot are derived. These results are used in sections 3c and 3d to obtain the overall synchronisation performance.

A. In a jammed hop slot

Suppose the signal is transmitted over an AWGN channel with noise power spectral density $N_o/2$. The received signal is

$$r(t) = \sqrt{2S}c(t)\cos(\omega_o t + \theta_s) + \sqrt{2J}\cos(\omega_o t + \theta_J) + n(t)$$
$$= s(t) + j(t) + n(t)\,, \tag{1}$$

where S is the desired signal power, J is the on-tune jammer power, ω_o is the carrier frequency, θ_S and θ_J are the phases of the desired signal and jammer signal respectively. If the jammer divides its power equally into N_J tones, J is replaced by J/N_J.

The output y of the MF (fig. 1b) is given by

$$y = y_I^2 + y_Q^2\,, \tag{2}$$

where y_I and y_Q are the output of the in-phase and quadrature-phase MF correlators respectively. It can be easily shown that

$$y_I = \frac{1}{\sqrt{T}}\int_0^T r(t)\cdot\sqrt{2}c(t)\cos\omega_o t\, dt$$
$$= \sqrt{ST}\cos\theta_S - \frac{1}{M}\sqrt{JT}\cos\theta_J + n_I \tag{3}$$

and

$$y_Q = \frac{1}{\sqrt{T}}\int_0^T r(t)\cdot\sqrt{2}c(t)\sin\omega_o t\, dt$$
$$= -\sqrt{ST}\sin\theta_S - \frac{1}{M}\sqrt{JT}\sin\theta_J + n_Q\,, \tag{4}$$

where n_I and n_Q are independent, zero mean Gaussian random variables with variance $\sigma_n^2 = N_o/2$. In the above derivations, the PN sequences are assumed to be m-sequences and hence have the property that there is one more -1 than +1 chips in one period.

In order to make the analysis tractable, it is assumed that there is only one H_1 cell corresponding to code synchronisation, that the L/Δ samples are independent, and the small non-zero lag m-sequence autocorrelation values are ignored here. The probability density function (pdf) of the H_1 sample in a jammed hop slot is noncentral χ^2 with two degrees of freedom:

$$P(y\,|\,H_1, J) = \frac{1}{2\sigma_n^2}\exp\left(-\frac{m_1^2 + y}{2\sigma_n^2}\right)I_o\left(\frac{m_1\sqrt{y}}{\sigma_n^2}\right) \tag{5}$$

where

$$m_1^2 = ST + \frac{JT}{M^2} - \frac{2T}{M}\sqrt{JS}\cos(\theta_S - \theta_J). \tag{6}$$

$I_o(\,)$ is the modified Bessel function of the first kind and zero order. The pdf of the H_0 samples is also noncentral χ^2 with two degrees of freedom:

$$P(y\,|\,H_0, J) = \frac{1}{2\sigma_n^2}\exp\left(-\frac{m_2^2 + y}{2\sigma_n^2}\right)I_o\left(\frac{m_2\sqrt{y}}{\sigma_n^2}\right) \tag{7}$$

where

$$m_2^2 = \frac{JT}{M^2}. \tag{8}$$

The probability of detecting the first PN sequence in a jammed hop slot $P_{D,J}$ with detection threshold γ_1 for a given phase difference ϕ is, after some algebraic manipulations, given by

$$P_{D,J}(\phi) = \int_{\gamma_1}^{\infty} P(y|H_1, J)\left[\int_0^y P(x|H_0, J)\,dx\right]^{\frac{L}{\Delta}-1} dy$$
$$= \int_{\sqrt{\gamma'_1}}^{\infty} y\exp\left(-\frac{m_{11}^2 + y^2}{2}\right)I_0(m_{11}y)\cdot \left[1 - Q(m_{21}, y)\right]^{\frac{L}{\Delta}-1} dy \tag{9}$$

where

$$\gamma'_1 = \frac{\gamma_1}{\sigma_n^2}, \tag{10}$$

$$m_{11} = \frac{m_1}{\sigma_n} = \sqrt{2v\left(M + \frac{J}{MS} - 2\sqrt{\frac{J}{S}}\cos\phi\right)}\,, \tag{11}$$

$$m_{21} = \frac{m_2}{\sigma_n} = \sqrt{\frac{2J}{MS}v}\,, \tag{12}$$

$$\phi = \theta_S - \theta_J\,, \tag{13}$$

$$v = \frac{ST_c}{N_o}, \tag{14}$$

and Q(a,b) is the generalised Q-function. Assuming that ϕ is uniformly distributed over $[0,2\pi)$, the average probability of detection is given by

$$P_{D,J} = \frac{1}{2\pi}\int_0^{2\pi} P_{D,J}(\phi)\,d\phi\,. \tag{15}$$

The probability that the combination of noise and jamming signal in a H_0 cell crosses the detection threshold γ_1 is given by

$$P_{FA,J} = \int_{\gamma_1}^{\infty} P(y|H_0, J)\,dy = Q\left(m_{21}, \sqrt{\gamma'_1}\right). \tag{16}$$

The probability of false alarm during a search mode in a jammed hop slot is then approximated by [6]

$$P_{F,J} = 1 - (1 - P_{FA,J})^{\frac{L}{\Delta}} \approx \frac{P_{FA,J}L}{\Delta}. \tag{17}$$

During coincidence detection, the probability of a successful CD at each test is given by

$$P_{C,J} = \int_{\gamma_2}^{\infty} P(y|H_1, J)\,dy = Q\left(m_{11}, \sqrt{\gamma'_2}\right). \tag{18}$$

where γ'_2 is the CD threshold normalised by σ_n^2. The probability of successful CD is then given by

$$P_{CD,J} = \sum_{i=B}^{A}\binom{A}{i} P_{C,J}^i\,(1 - P_{C,J})^{A-i} \tag{19}$$

When a false acquisition occurs, the probability of a false CD at each test is given by

$$P_{FC,J} = Q\left(m_{21}, \sqrt{\gamma'_2}\right). \tag{20}$$

Then the probability of a false CD is

$$P_{FCD,J} = \sum_{i=B}^{A} \binom{A}{i} P_{FC,J}^{i} (1-P_{FC,J})^{A-i} . \quad (21)$$

B. In an unjammed hop slot

By setting J=0 in equations (5) - (8), pdf of the H_1 and H_0 samples in an unjammed hop slot are found. The detection probability during search mode is

$$P_{D,NJ} = \int_{\gamma_1}^{\infty} P(y|H_1, NJ) \left[\int_0^y P(x|H_0, NJ)\, dx \right]^{\frac{L}{\Delta}-1} dy$$
$$= \frac{1}{2}\exp(-Mv) \int_{\gamma_1}^{\infty} \exp\left(-\frac{y}{2}\right) I_0\left(\sqrt{2Mvy}\right) \cdot \left[1-\exp\left(-\frac{y}{2}\right)\right]^{\frac{L}{\Delta}-1} dy \quad (22)$$

The probability of false alarm in an H_0 cell is given by

$$P_{FA,NJ} = \int_{\gamma_1}^{\infty} P(y|H_0, NJ)\, dy = \exp\left(-\frac{\gamma'_1}{2}\right). \quad (23)$$

The results in (22) and (23) agree with equations (8) and (11) in [5].

Corresponding to (17) - (21), in an unjammed hop slot, the following equations apply:

$$P_{F,NJ} = 1-(1-P_{FA,NJ})^{\frac{L}{\Delta}} \approx \frac{P_{FA,NJ} L}{\Delta}. \quad (24)$$

$$P_{C,NJ} = \int_{\gamma_2}^{\infty} P(y|H_1, NJ)\, dy = Q\left(\sqrt{2Mv}, \sqrt{\gamma'_2}\right). \quad (25)$$

$$P_{CD,NJ} = \sum_{i=B}^{A} \binom{A}{i} P_{C,NJ}^{i} (1-P_{C,NJ})^{A-i} . \quad (26)$$

$$P_{FC,NJ} = \exp\left(-\frac{\gamma'_2}{2}\right). \quad (27)$$

$$P_{FCD,NJ} = \sum_{i=B}^{A} \binom{A}{i} P_{FC,NJ}^{i} (1-P_{FC,NJ})^{A-i} . \quad (28)$$

C. Probability of successful synchronisation

In this section, the subscript X attached to various variables are to be replaced by J or NJ (and the corresponding results in sections A and B are used) depending on whether a hop slot is jammed or unjammed.

The average blocked time due to a false acquisition decision is given by [6]

$$T_{B,X} = (A+1+L_d P_{FCD,X})T \quad (29)$$

where L_d is the number of data bits in a packet. The average number of acquisition decisions between receptions of two consecutive packets is given by

$$K_X = \frac{T_s}{T + P_{F,X} T_{B,X}} \quad (30)$$

where T_s is the duration between the arrival of two consecutive packets (equal to the inverse of the hop rate if there is only one preamble per hop dwell). The receiver is blocked for time $K_X P_{F,X} T_{B,X}$ on average during T_s. The probability of receiver blockage is

$$P_{B,X} = \frac{K_X P_{F,X} T_{B,X}}{T_s} = \frac{(A+1+L_d P_{FCD,X}) P_{FA,X} L/\Delta}{1+(A+1+L_d P_{FCD,X}) P_{FA,X} L/\Delta}. \quad (31)$$

Therefore, the probability of successful packet synchronisation is

$$P_{S,X} = (1-P_{B,X}) P_{D,X} P_{CD,X} \cdot \quad (32)$$

D. Jammer optimisation

Assuming the spread bandwidth is twice the chip rate (BPSK spreading), given a total transmission bandwidth of W_{ss}, there are a total of

$$N_{HS} = \frac{W_{SS}}{2/T_c} = \frac{W_{SS}}{2R_b M} \quad (33)$$

non-overlapping hop slots. R_b is the binary data rate. When N_J out of N_{HS} hop slots are jammed, the probability that a packet is able to synchronise successfully is given by

$$P_{sync} = \frac{N_J}{N_{HS}} P_{S,J} + \frac{N_{HS}-N_J}{N_{HS}} P_{S,NJ} \cdot \quad (34)$$

4. Numerical results

The following values are used in these numerical results: N = 1, Δ = 1/2, A = 8, B = 4, L_d = 1000. Fig. 2 plots probability of synchronisation in an unjammed hop slot against bit energy to noise power spectral density (psd) ratio, which is defined as Eb/No = Mv. As synchronisation performance improves with larger v (chip energy to noise psd ratio), for a given Eb/No, it is necessary to have low DS spreading gain to achieve better synchronisation performance.

Fig. 3 shows the synchronisation performance in a jammed hop slot with J/S = 25 dB (N_J = 1). More spreading gain reduces the effect of the jammer, resulting in better synchronisation performance. Thus, we see that the jammed and unjammed hop slots exert conflicting demands on the DS spreading gain.

An m-sequence generated by a k-stage shift register has period $M = 2^k-1$. For $W_{ss}/R_b = 2^{10}$, equation (33) can be rewritten into an approximation: $N_{HS} \approx 2^{9-k}$, and we have the following possible combinations of N_{HS} and M:

N_{HS}	1	2	4	8	16	32	64
M	511	255	127	63	31	15	7

Table 1

Note that N_J is bounded by the range $[1,N_{HS}]$. Fig. 4 shows the acquisition performance when J/S = 25 dB and M = 31 (N_{HS} = 16). We see that increasing N_J from 1 to 2 reduces P_{sync} for all plotted Eb/No values; but when N_J is further increased, P_{sync} is reduced at the lower Eb/No values but increases at the higher Eb/No. As an Eb/No of greater than 15 dB is necessary for reliable synchronisation in an unjammed AWGN channel (as shown by the dotted graph), the jammer will select N_J = 2 to cause the greatest damage over most Eb/No values in this region of interest.

With higher DS spreading gains (as shown in fig. 5 for M = 127, J/S = 25 dB), the system becomes significantly less vulnerable to tone jamming. Varying N_J also has little impact on P_{sync} -- further computations have shown that for a higher J/S, a larger M is required for P_{sync} to achieve such immunity to jammer optimization. For the values assumed in fig. 5 and 6, we found that a relatively low DS spreading gain of M ≥ 127 is sufficient to impart considerable immunity against a strong jammer of J/S = 25 dB. This clearly illustrates the powerful anti-jamming (AJ) capability of the proposed MF-based synchronisation system in a hybrid SFH/DS communication link.

5. Conclusions

The equations necessary for analysing the acquisition performance of a hybrid FH/DS communication link in the presence of multitone jamming have been developed. They enable the evaluation of synchronisation performance when the multitone jammer optimises the number of jammer tones to inflict the greatest damage. The conflicting demands on DS spreading gain was illustrated. The numerical results also show that for a given W_{ss}/R_b ratio, increasing the DS spreading gain will tend to improve synchronisation performance. In the case where J/S is very high, thus requiring very large DS spreading gain for jammer immunity, or in the case where a large DS spreading gain is not achievable (for example, in high speed data links), one can still resort to increasing W_{ss} to improve P_{sync} -- herein lies the well-known flexibility in using a hybrid FH/DS communication system to achieve high AJ capability.

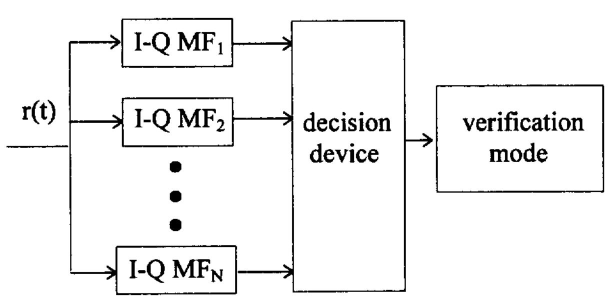

Figure 1a. Parallel matched-filter acquisition scheme.

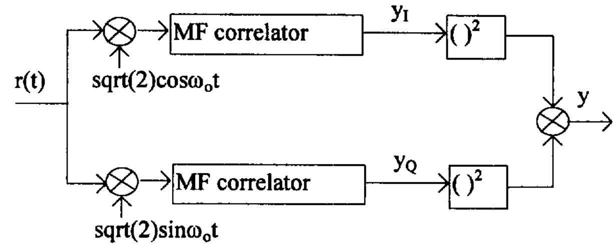

Fig. 1b. I-Q noncoherent matched filter (MF)

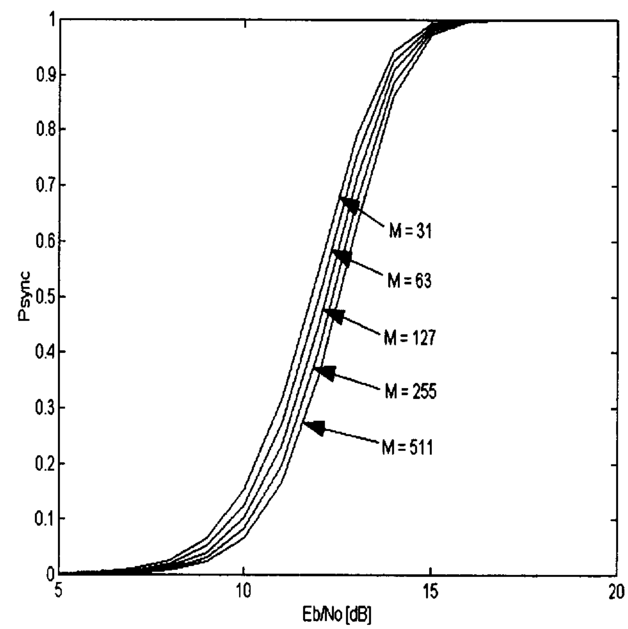

Fig. 2. Probability of synchronisation in an unjammed hop slot ($P_B = 10^{-5}$).

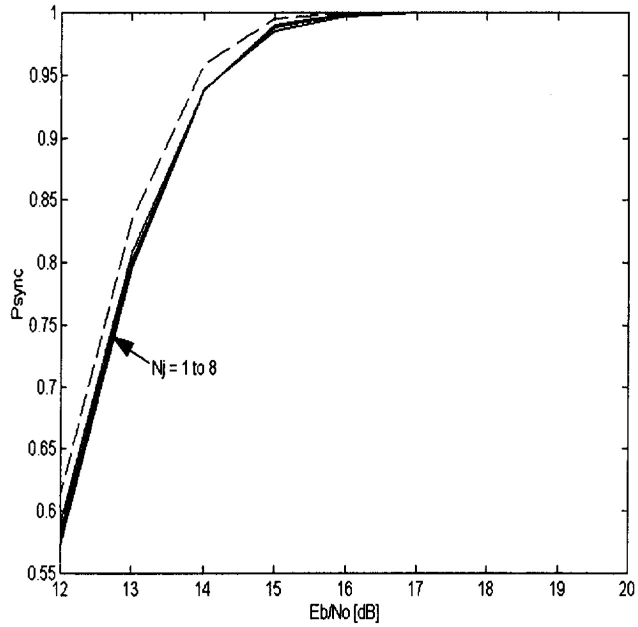

Fig. 3. Probability of synchronisation in a jammed hop slot ($P_B = 10^{-5}$, J/S = 25 dB, N_J = 1).

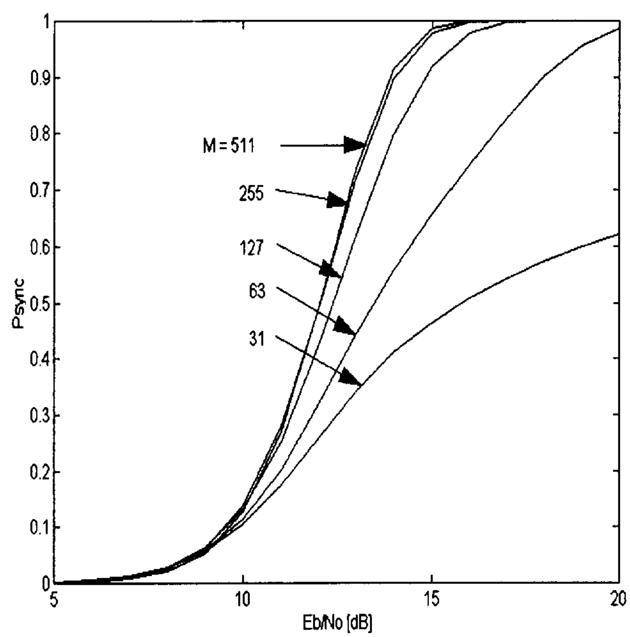

Fig. 4. Dotted line: P_{sync} with no jamming. Solid lines: P_{sync} in multitone jamming with different number of tones N_J (J/S = 25 dB, M = 31).

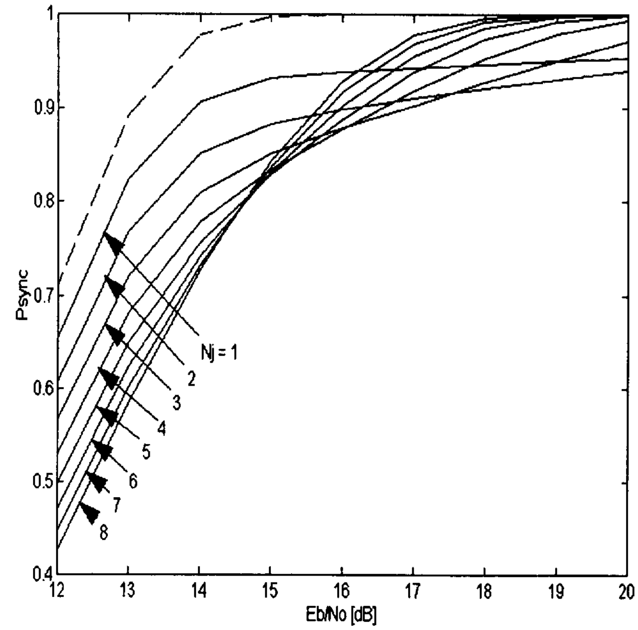

Fig. 5. Dotted line: P_{sync} with no jamming. Solid lines: P_{sync} in multitone jamming with different number of tones N_J (J/S = 25 dB, M = 127).

6. References

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