PERFORMANCE ANALYSES OF THE POWER-DOMAIN KERNEL ADAPTIVE LOCALLY OPTIMUM PROCESSOR (ALOP) AGAINST INTERFERENCE WAVEFORMS WITH CONTINUOUS POWER DENSITY

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Abstract: *The Power Kernel Adaptive Locally Optimal Processor (ALOP) is an implementation of non-linear locally optimum detection in the presence of non-Gaussian noise using a kernel estimation technique to model the noise power density function. Methods of analyzing the expected performance of this algorithm are developed for a class of noise processes with continuous power density functions. These methods are used to illustrate the performance and the parameter tradeoffs that can be made with the Power Kernel ALOP.*

1. Introduction

In previous papers, a family of interference suppression signal processing algorithms, the Kernel Adaptive Locally Optimum Processors (ALOP), has been defined [1,2,3,4,5]. These algorithms implement an adaptive realization of the locally optimum detection (LOD) process for detection of small communications signals in the presence of large, non-Gaussian interference signals [8]. The unknown density function of the non-Gaussian interference signal is adaptively modeled using a sum of Gaussian kernel functions centered on the sample values of the interference signal in real time. This formulation will be defined in section 3.

An important aspect of the practical application of Kernel ALOP algorithms is the selection of the parameters of the algorithm: the number of Gaussian kernel functions in the approximation of the density function, and the standard deviation of the Gaussian kernel function. In previous work [1], the tradeoffs between performance and these parameters were developed for interference signals whose density functions had the structure of finite number of discrete states. In this paper, the tradeoffs between the algorithm parameters and the expected performance of the algorithm against interference signals with a continuous density function are presented.

2. General Power Domain Non-Linear Processing

Given complex received samples, $r_k = s_k + n_k$, of a small signal of interest (SOI), s_k, corrupted by arbitrary additive interference/noise process, n_k, we seek a multiplicative gain factor, $g_k(|r_k|^2)$, as a function of the magnitude squared of the sample and such that the resulting complex SNR ratio is maximized. Following Kullstam [6], the mean and variance of symbols formed by correlation of this product against a known SOI sequence (such as a DSSS spreading sequence) can be computed as

$$z = \sum_k g(|r_k|^2) r_k s_k^*$$

$$SNR_g = |E[z]|^2 / Var[z]$$

$$E[z] = E_s \int_0^\infty - v\, g(v)\, p'(v)\, dv$$

$$Var[z] = E_s \int_0^\infty v\, g(v)^2\, p(v)\, dv$$

follows:

Here p(v) is the density function of the magnitude squared (power) of the noise process, s^*_k is the complex conjugate of the known signal sequence, and E_s is the total energy of the signal. Thus, the SNR can be maximized by selection of a gain function that is related to the density function of the power of the interference signal as follows:

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$$g_{opt}(v) = -p'(v)/p(v)$$

$$SNR_{opt} = E_s \int_0^{\infty} v\,(g_{opt}(v))^2\, p(v)\, dv$$

$$Improvement_{g,1} = 10 \log(SNR_g / SNR_1)$$

$$SNR_1 = E_s / \int_0^{\infty} v\, p(v)\, dv$$

The more useful, normalized measure of the performance of any gain function is the SNR improvement factor (in dB) of the SNR using the gain function relative to the SNR using a unity gain function.

Thus if we can accurately estimate the power density function from the received samples in real time, the optimal gain factor can be calculated and applied to the received sequence. Development of interference suppression algorithms using this principle have been reported [5,7]. The Power Kernel ALOP defined in Section 4 is a particularly elegant member of this class.

3. A Test Class of Interference/Noise Signals with Simple Continuous Power Density Functions

Define the following class of non-Gaussian interference/noise signals, $N = \{N(P1,P2,d)\}$, where a sample noise process, $\{n_k\}$, consists of independent, identically distributed complex values whose phase is uniformly distributed and whose magnitude squared is distributed as follows:

$$p(v) = \begin{cases} M\{1-\cos(\pi(v-P1+d)/d)\}/2 & \text{for } P1-d < v < P1 \\ M & \text{for } P1 < v < P2 \\ M\{1-\cos(\pi(v-P2-d)/d)\}/2 & \text{for } P2 < v < P2+d \\ 0 & \text{otherwise.} \end{cases}$$

$$\text{where } M = 1/((P2-P1)+d)$$

The power density functions of this class have the property of being continuous, compactly supported, and differentiable. The two ends of the density feature a raised cosine shape. For P1 = P2, noise processes of this class are unimodal with the spread of the power density controlled by the d parameter. For P1 <> P2, the noise process has a uniform distribution of the power between P1 and P2. A small d parameter (relative to the P1 and P2 parameters) indicates increased steepness of the density function on both ends of the distribution.

Using the formulation of the optimal gain factor in section 2, we can form the optimal non-linear processor for densities of this class and calculate the expected improvement in SNR over a detection process that uses unity gain:

$$\text{Improvement (dB)} = 10 \cdot \log\left(\frac{\pi^2 (P1+P2)^2}{4d(P2-P1+d)}\right)$$

It is clear from this formulation that we expect maximum improvement when P1 = P2 and when d is small. This agrees with the intuition that these methods should provide significant improvement when the interference/noise signal is of constant (or near constant) instantaneous power.

Table 1 defines a subset of noise processes from class *N* and their corresponding improvement calculations along with the their unity gain SNR relative to the first noise waveform. Two families of waveforms will be considered: a set of noise waveforms with unimodal density functions and increasing spread (e.g. A,B,C,D) and a set of noise waveforms with increasing flat region (e.g., A,E,F,G).

4. Definition of the Power Kernel ALOP

To define the Power Kernel ALOP, we assume that we have a finite sequence of noise samples, n_k for k = -K..0..K, and we let $p_k = |n_k|^2$ be the magnitude squared of each sample. The estimate of the noise power density is formed by taking a Gaussian density centered at each p_k with variance s^2 and averaging all of these kernel functions:

$$p(v) = \frac{1}{2K+1} \sum_{-K}^{K} \frac{e^{-\frac{(v-p_k)^2}{2s^2}}}{s\sqrt{2\pi}}$$

From this estimate of the power density function, the optimal gain function can be computed as in section 1. The final formulation of the gain function assumes that the SOI component of the received samples is small and that the magnitude squared of the received sample is a good approximation of the magnitude squared of the noise sample. The final formulation is:

$$g_{PK}(v) = \frac{\sum_{-K}^{K} -\frac{1}{s^2}(v - p_k)\, e^{-\frac{(v-p_k)^2}{2s^2}}}{\sum_{-K}^{K} e^{-\frac{(v-p_k)^2}{2s^2}}}$$

It is clear that the overall quality of the power density estimate can be significantly influenced by the parameters K and s^2. Since the parameter K is closely tied to the cost (in hardware components or in computer execution time) of implementing the algorithm, it is important to ascertain the effect of K on the performance of the algorithm and to understand the extent to which optimization of the s^2 parameter effects the K requirement.

5. Performance Indicators for the Power Kernel ALOP on the Test Class of Interference/Noise Signals

Since the Power Kernel ALOP is a naturally adaptive algorithm with explicit use of a sliding window of sample points, the computation of its resulting SNR or improvement factor by means of the integrals of section 1 would (at a minimum) involve a 2K+1 dimensional integration to average over all of the possible sequences of noise samples. A more practical approach is to compute a canonical improvement measure that is likely to be an upper bound on the performance and to compute a measure of the expected shortfall of the improvement due to stochastic estimation losses from the canonical situation. This last measure may be referred to as the expected improvement after consideration of implementation losses or stochastic losses of the algorithm.

For the canonical improvement measure, the calculations of section 1 are performed numerically using the 2K+1 quantiles of the noise distribution as the sample points. This selection of sample points generates a canonical kernel approximation of the noise density and represents a uniform sampling of the inverse noise power distribution function. In contrast, if we divide the interval [0,1] into 2K+1 equal bins and we select 2K+1 of these at random with replacement, it is much more probable that we will get clustering of our bin choices (with some chosen multiple times and some not chosen at all). Since this clustering phenomenon happens much more often, we can get a measure of its degrading effect on the expected SNR improvement by averaging the mean and variance integrals over a Monte Carlo sample of sample point selections.

The following tables contain the results of such a calculation. Table 2 illustrates the results of computing the canonical improvement measure using the indicated values of K and s^2 for the same subset of *N* noise processes computed whose optimal performance was presented in Table 1. Table 3 illustrates the results of using the Monte Carlo averaging of 100 sets of random sample point selections as part of the improvement calculation (including stochastic losses) for the same subset of *N* noise processes.

From Table 2 and 3 the expected general trends are that the major factor in performance for each family of scenarios is the proper optimization of the s parameter to the scenario waveform. Secondarily, an increase of the K parameter from 12 to 24 leads to a consistent increase in expected performance (but only marginally so compared to the effects of the s parameter optimization). In addition, for noise processes with wide power densities, the stochastic losses in performance are expected to be larger and to have limited dependence on K.

6. Performance of the Power Kernel ALOP on Monte Carlo Simulations of the Test Class

To ascertain the reliability of the analysis approach presented above to predict the achievable improvement for various values of the K and s^2 parameters, the noise processes of class *N* were implemented in a Monte Carlo baseband simulation of a DSSS communication system with the power kernel ALOP signal processing and demodulation of the DSSS symbols. Simulation runs were made for the sets of parameters and noise processes in the previous Tables. The resulting measured SNR improvement is presented in Table 4.

7. Assessment

From an overall perspective, the simulation performance of the power kernel ALOP (when parameters are optimized) can meet or exceed the theoretical performance of Table 1 for the scenarios in family 1 and can come within 1-2 dB of the theoretical performance for the scenarios of family 2. There are tradeoffs, however; since the small s values that maximize performance in scenario A also cause significant degradation in scenario D.

With respect to optimization of the s parameter, the best strategy is dependent on the requirements of a specific application, but a robust approach would be to minimize the degradation in performance in scenarios D and G by using the value s=0.2. Can this anticipated by the canonical or stochastic analysis procedures we have introduced? Both the canonical and the stochastic analyses indicate this parameter tradeoff for family 1 scenarios. For family 2 scenarios, the canonical analysis is a counter-indicator and the stochastic analysis is a consistent, but understated indicator of the parameter tradeoff. Therefore, we recommend using the stochastic analysis procedure over the canonical one to evaluate trends in s parameter tradeoffs.

With respect to evaluation of the marginal value of increasing the K parameter, the simulation performance indicates that there is a 1-2 dB increase in performance when K is increased from 12 to 24. The strongest gain is in scenarios D and G with small s values. The canonical analysis procedure provides a consistent but weak indicator of additional performance. The stochastic analysis procedure provides a consistent and generally stronger indication of increased performance.

8. Summary

Using a class of noise processes with continuous power density functions, the expected performance of the Power Kernel ALOP interference suppression algorithm for a range of parameter values can be assessed. The performance assessment includes evaluation of a canonical calculation of performance representing an upper bound on improvement and the calculation of performance that includes the stochastic estimation losses inherent in the algorithm. For use in predicting the parameter tradeoffs impacting simulated power kernel ALOP performance, the stochastic analysis procedure is shown to be a more consistent and accurate indicator.

Practical implications of the expected performance and parameter tradeoffs of the Power Kernel ALOP against noise processes with continuous power density can be derived from the calculations.

9. References

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Table 1: Performance Improvement for Optimal Gain Factor			
Noise Process ID	Noise Parameters	Improvement (dB)	Unit Gain SNR rel A.
A	P1=1, P2=1, d=0.1	29.9	---
B	P1=1, P2=1, d=0.25	21.9	0 dB
C	P1=1, P2=1, d=0.5	15.9	0
D	P1=1.0, P2=1.0, d=1.0	9.9	0
E	P1=0.75, P2=1, d=0.1	23.3	-0.58
F	P1=0.5, P2=1, d=0.1	19.6	-1.25
G	P1=0.25,P2=1, d=0.1	16.6	-2.04

Table 2: Canonical Improvement Measure for Power Kernel ALOP (dB)						
Noise Process ID	K=12			K=24		
	s = 0.2	s = 0.1	s = 0.05	s = 0.2	s = 0.1	s = 0.05
A	28.8	28.8	28.9	28.8	28.8	28.9
B	20.9	21.0	21.3	20.9	21.0	21.3
C	14.9	15.3	15.7	14.9	15.2	15.7
D	9.3	9.6	7.8	9.2	9.7	8.7
E	18.6	19.3	21.3	18.6	19.3	21.3
F	13.1	15.4	17.7	13.1	15.4	17.6
G	9.6	12.3	14.7	9.6	12.3	14.5

Table 3: Improvement Measure for Power Kernel ALOP (dB) including stochastic losses						
Noise Process ID	K=12			K=24		
	s = 0.2	s = 0.1	s = 0.05	s = 0.2	s = 0.1	s = 0.05
A	28.6	28.7	28.6	28.7	28.8	28.7
B	20.7	20.7	20.2	20.8	20.8	20.6
C	14.5	14.2	12.2	14.7	14.7	12.9
D	8.2	6.1	4.0	8.6	7.3	3.9
E	18.4	18.8	19.9	18.5	19.1	20.3
F	12.7	14.0	14.3	12.9	14.8	15.3
G	8.9	10.1	9.5	9.1	11.1	10.7

Table 4: Improvement Measure for Power Kernel ALOP (dB) from Simulation Runs						
Noise Process ID	K=12			K=24		
	s = 0.2	s = 0.1	s = 0.05	s = 0.2	s = 0.1	s = 0.05
A	30.1	31.2	32.2	31.1	32.5	34
B	25.3	25.7	24.7	25.9	26.5	26.2
C	19.5	18.5	14.1	20.0	19.7	16.9
D	12.1	8.0	0.1	13.1	10.6	3.9
E	23.2	23.7	23.1	23.7	24.5	25
F	17.4	18.3	15.9	18.0	19.5	18.8
G	12.9	13.4	9.6	13.7	15.2	13.1