AJ PERFORMANCE OF A FAMILY OF SELF-NORMALIZED DIVERSITY COMBINERS

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ABSTRACT

A family of self-normalized diversity combiners, which are generalizations of the self-normalized combiner previously analyzed in the literature, are introduced and their anti-jam (AJ) capabilities assessed in the presence of worst-case tone jamming.

It is shown that every combiner in the family performs at least as well as the standard self-normalized combiner, which is one particular member of the family; also, it is shown that as the parameter which defines the family is increased without bound, a combiner with jammer state information (JSI) is essentially realized.

INTRODUCTION

In the presence of optimized or worst-case jamming, it is known that the choice of the diversity combiner used can significantly affect the bit error rate (BER) performance of a fast frequency hopped (FFH) spread spectrum system [1, 2, 3].

In previous works [1, 2, 3], it was shown that, in the context of a FFH binary frequency-shift-keying (BFSK) system, a nonlinear diversity combiner, referred to as the self-normalized combiner, offered marked improvement in BER performance over that of a system using a linear combiner. More recently, it has been shown that the self-normalized combiner is also effective in a fading environment [4]. The anti-jam (AJ) capability of the self-normalized combiner is due to its response characteristics, which suppresses strong energy inputs (which usually correspond to the jammed frequencies). The result of this is that the combiner cannot be 'swamped' by a single jammer hit.

The AJ capabilities of the self-normalized combiner may be enhanced by a slight generalization of the combining algorithm. This is done by taking the m-th root of the normalized test statistic at each hop prior to summing (the algorithm will be precisely stated shortly). The special case of $m = 1$ yields the standard self-normalized combiner as analyzed in [3]. It will be shown that by choosing $m > 1$, the BER performance is improved; the performance enhancement, in fact, is seen to be monotonic with m, and as m increases without bound, the resulting combiner has the characteristics of a combiner with jammer state information (JSI), forcing the jammer to jam the entire hopping band in order to be effective.

SYSTEM MODEL

A block diagram of the system model used in this study is shown in Figure 1, the transmission signal set is standard MFSK, where one of M tones, at frequencies $\{f_i\}$, $i = 1, 2, ..., M$, is selected to represent $\log_2 M$ information bits. The frequencies in the signal set are uniformly spaced at R Hertz and are positioned around the center (or carrier) frequency f_c as shown in Figure 2 (for the case of $M = 8$), and are assumed to be equally probable.

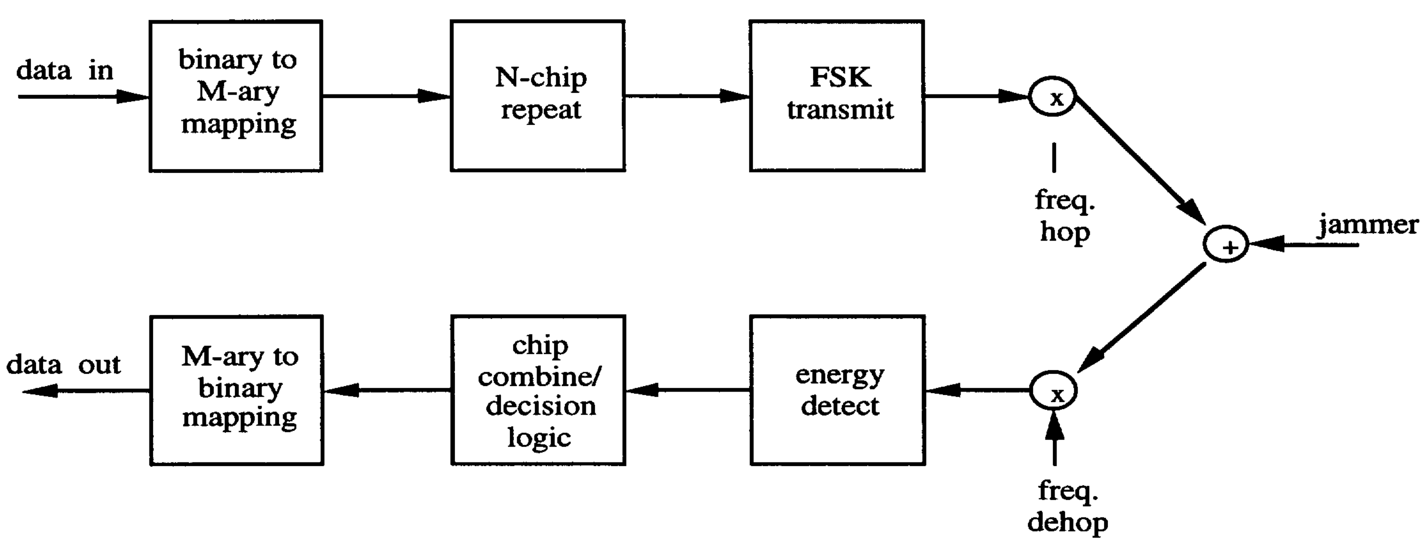

Figure 1: Fast Frequency Hopped System Model .

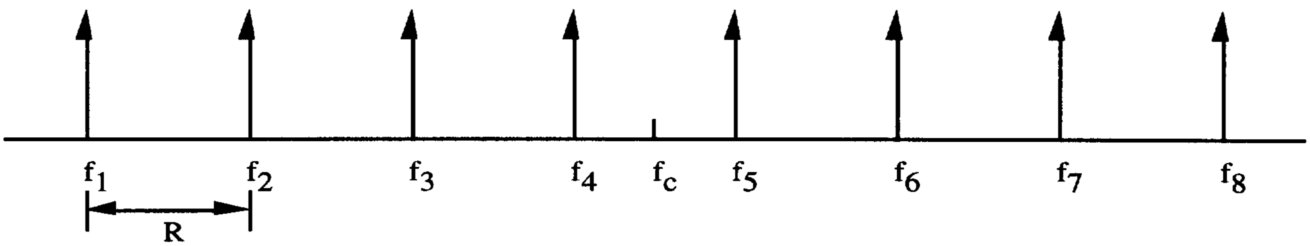

Figure 2: 8FSK Signal Set.

Prior to transmission, the center frequency is pseudo-randomly hopped among a set of possible hop frequencies $\{f_{c,i}\}$ where $| f_{c,i} - f_{c,j} | \geq M R$ for $i \neq j$. (This implies that the hopping signal sets do not overlap.) Also, the ordering of the signal set $\{f_i\}$ around f_c is not changed from hop to hop (i.e., no scrambling or shuffling within the signal set). At the receiver, the signal is dehopped and energy detected at the M frequency bins, the outputs of which are then processed and combined in some manner to arrive at a final test statistic for each bin. The frequency corresponding to the bin with the greatest test statistic is selected as the combiner output. Detail descriptions of the combiner processing are contained in later sections.

JAMMER MODEL

The jammer assumed here is a 'smart' multi-tone jammer which has complete knowledge of the signal set and hopping structure, except for the hopping sequence. The jammer has finite total power J, and over a hop period T_h has total energy JT_h, which, if spread evenly among the total number of possible hop frequencies, N_{ss}, yields the 'spectral density' of the jammer, denoted by J_0.

Rather than jamming the entire hop bandwidth (i.e., emitting energy in all N_{ss} hop frquencies), the jammer instead jams only a fraction ρ, $0 < \rho \leq 1$, of the band, with effective spectral density J_0/ρ. The 'optimized' or 'worst-case' jammer is that jammer which chooses a ρ such that the resulting bit error rate (BER) of the communicator is greatest.

Given the hopping and signal structure as described above, the jammer can concentrate its attack on one particular symbol (say f_2) by choosing the jammed frequencies such that they have constant frequency separation (2.5 R in the example shown in Fig. 2) from the closest hop frequency.

PERFORMANCE ANALYSIS

For simplicity, it will be assumed that the only noise source is the jammer. Also, to facilitate the discussion, often-used variables and parameters are defined below.

N	---	order of diversity.
N_J	---	number of jammed hops; $N_J \leq N$.
ρ	---	jammed band fraction; also equal to the probability of a hop being jammed.
J_0	---	jammer 'spectral density'.
E_c	---	signal energy detected in one hop (or chip).
γ	---	E_c/J_0
$\{z_{i,j}\}$	---	energy detector output for hop i (i = 1, 2, ..., N), frequency bin j (j = 1, ..., M).

Using these notations, the **m-th root self-normalized combiner** may be described mathematically as,

$$u_k = \sum_{i=1}^{N} \left(\frac{z_{i,k}}{\sum_{j=1}^{8} z_{i,j}} \right)^{\frac{1}{m}}, \quad k = 1,2,..,M; \qquad (1)$$

where $\{u_k\}$ are the final test statistics upon which the combiner bases its decision, specifically, the output of the combiner is f_k (see Figure 2) whenever $u_k = \max\{u_j\}$.

Letting f_1 be the information-bearing symbol, and f_2 the jammed symbol, the test statistics after combining may be written as,

$$u_1 = N_J \left(\frac{E_c}{E_c + \frac{J_0}{\rho}} \right)^{\frac{1}{m}} + (N - N_J), \qquad (2)$$

$$u_2 = N_J \left(\frac{\frac{J_0}{\rho}}{E_C + \frac{J_0}{\rho}} \right)^{\frac{1}{m}}.$$

$$u_i = 0,\ i = 3, ..., M.$$

An error occurs whenever $u_2 \geq u_1$ (for simplicity, it will be assumed that the case where $u_2 = u_1$ will cause an error; this assumption have negligible effect on the final results), therefore, the probability of symbol error is,

$$P_s = pr[u_2 \geq u_1]. \quad (3)$$

After some algebra, it is easily verified that $u_2 \geq u_1$ is equivalent to,

$$N_J \geq \frac{N(1+\rho\gamma)^{1/m}}{1-(\rho\gamma)^{1/m}+(1+\rho\gamma)^{1/m}} = N\,T(\rho\gamma, m), \quad (4)$$

where $T(\rho\gamma, m)$ is the normalized error threshold and may be used to conveniently compare AJ performance. Figure 3 depicts $T(\rho\gamma, m)$ as a function of $\alpha \equiv \rho\gamma$, for m = 1, 2, ..., 6. It is seen that for a given α, the threshold (and therefore AJ performance) increases monotonically with m; and in the limit as m increases without bound, it may be verified that $T(\alpha, m)$ assumes the characteristics of a (scaled) step function, with $T(\alpha, m) = 0.5$ at $\alpha = 0$, $T(\alpha, m) = 1$ at $0 < \alpha \leq 1$, and undefined elsewhere. Having $T(\alpha, m) = 1$ implies that the jammer must jam all N of the combined hops in order to cause an error; this is the best that any combiner is able to do against an intelligent jammer, and is achievable by the so-called 'combiner with jammer state information (JSI)', in which the combiner knows (somehow) whether each hop is jammed or not, and disregards those hops which are jammed. The traditional implementation of an ideal combiner with JSI involves binary-decision-making; the algorithm to accomplish this with minimal probability of error is not immediately apparent and may be quite complex. The m-th root self-normalized combiner, on the other hand, is fairly straight-forward to implement, and, with an appropriate choice of m, can achieve a level of AJ performance comparable to that of a combiner with JSI.

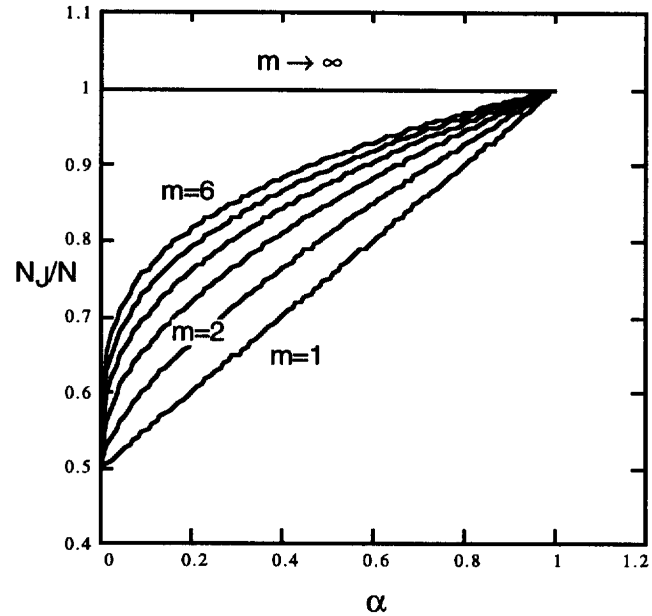

Figure 3: Error thresholds.

To obtain the BER performance, equations (3) and (4) are evaluated to yield,

$$P_s(\rho,\gamma) = \frac{(M-1)}{M} \sum_{k=k_m}^{N} p(N_J = k), \quad k_m = \lceil N\,T(\rho\gamma, m) \rceil \quad (5)$$

where

$$p(N_J = k) = \binom{N}{k} \rho^k (1-\rho)^{N-k},$$

and the factor of (M-1)/M is due to the fact that conditioned on a hop being jammed, there is a 1 out of M chance that the targeted symbol is the information-bearing symbol, in which case the jammer cannot cause an error. Figures 4 and 5 show the symbol error probabilities of FFH 8FSK using the m-th root self-normalized combiner, for m = 1 and 4, respectively, and N = 8. On each plot, the individual (unlabelled, stair-case-shaped) curves are parametrized by ρ; the quantity of interest is the worst-case performance curve, which is the upper envelope of these curves and is indicated by the straight line drawn at the upper right corner of the plot. The slope of this line is a measure of the AJ effectiveness of that particular combiner configuration, a steeper line implies more AJ capability. Plots similar to Figures 4 and 5 have been produced for other values of m, including the limiting case of $m \rightarrow \infty$, and their asymptotic slopes were noted. Table I below summarizes the pertinent numerical results.

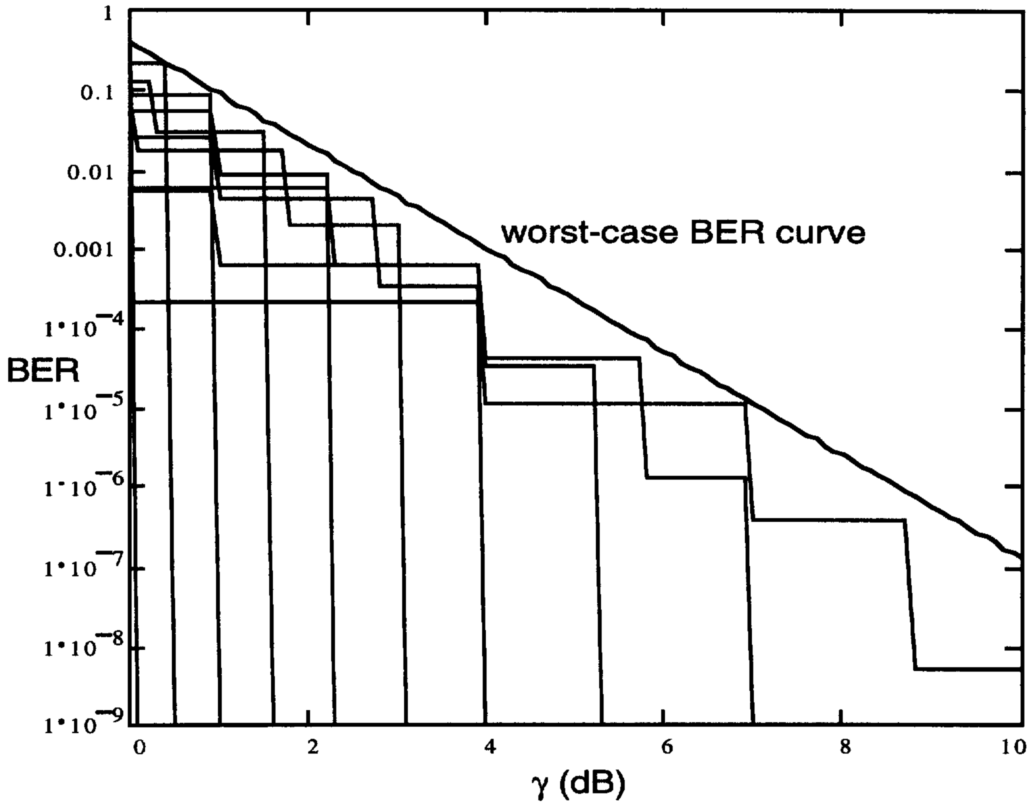

Figure 4: Worst-case BER, N = 8, m = 1.

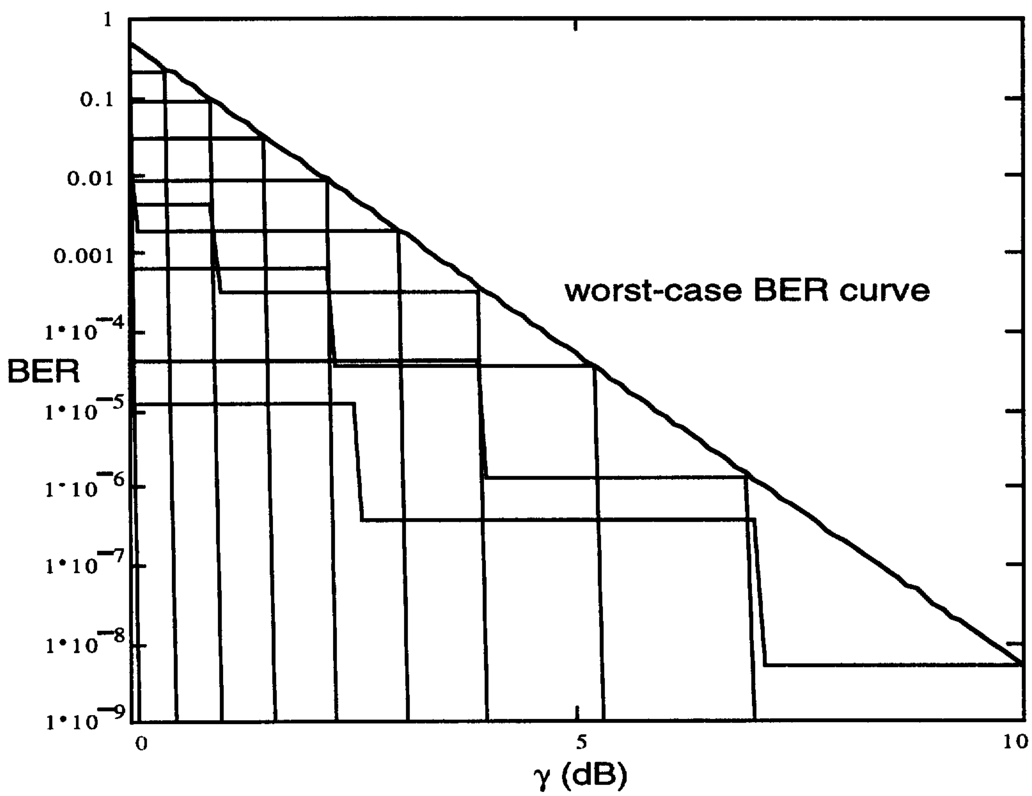

Figure 5: Worst-case BER, N = 8, m = 4.

N	m = 1	m = 2	m = 3	m = 4	m → ∞
2	20	20	20	20	20
4	35	37	40	40	40
8	65	71	77	80	80
16	120	140	150	150	160

Table I : Asymptotic fall-off of worst-case BER (dB/decade).

From Table I, it is evident that even for m as low as 4, the combiner performance already approaches that of the combiner with JSI.

CONCLUSION/SUMMARY

A family of self-normalized diversity combiners, referred to as the m-th root self-normalized combiners, has been described and analyzed . The results indicate that as the parameter m is increased from its low-end value of 1, the corresponding AJ performance is improved, and for m-values as low as 3 or 4, the resulting performance approaches the performance achievable by a combiner with JSI.

REFERENCES

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