USE OF MATHEMATICAL PROGRAMMING TO OPTIMIZE EFFECTIVELY-MEMORYLESS BANDPASS NONLINEARITIES

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Abstract *This paper addresses problems of synthesizing effectively memoryless bandpass nonlinearities to reduce jamming (interference) and intermodulation effects that occur in satellite and terrestrial communications. Instead of using classical calculus to synthesize (i.e., to optimize) effectively-memoryless bandpass nonlinearities, a different approach is described here to formulate synthesized problems as modern optimization problems that can be solved with the use of a digital computer and existing mathematical programming algorithms. As an illustrative example, the paper shows how the problem of optimizing an FDMA/SCPC transponder is formulated as a set of four linear programming problems and solved using the revised Simplex method.*

1.0 Introduction

In satellite and terrestrial communications, carriers are passed through one or more bandpass nonlinearities (e.g., traveling wave tube amplifiers (TWTAs), solid state power amplifiers (SSPAs)) and experience nonlinear gains, nonlinear phase shifts, and spectrum spreading. Nonlinearities also cause carriers to interact with each other to create intermodulation products, crosstalk, and modulation transfer which degrade the quality of information sent along with the carriers. To reduce communication impairment, characteristics of nonlinearities are sometimes altered (e.g., "linearized") using techniques such as feedforward [1, 8], feedback [2, 8], predistortion [3-8], or postdistortion [8]. In the benign environment (i.e., environment with the absence of jammers (i.e., strong (high power) interfering signals)), it has been widely accepted that the altered characteristics are those of the ideal envelope limiter (IEL)[1]; there are not, however, any proofs of this conjecture. In a hostile environment (i.e., an environment with the presence of jammers), the folk-theorem that "linear is best" has been proven false [9]. A few memoryless bandpass nonlinearities [9-12] (e.g., "biased inverting limiter," "biased power law rectifier," "dead zone limiter,") have also been presented and shown, through analyses or measurements, to perform better than the IEL, in terms of carrier-to-interference ratio (C/I)*, carrier-to-interference-plus-intermod ratio C/(I+IM)*, or carrier-to-noise-plus-interference-and-intermod ratio C/(N+I+IM)*. With these memoryless bandpass nonlinearities (instead of the IEL or common nonlinearities such as TWTAs or SSPAs), strong signals (i.e., jammers), instead of weak signals (i.e., desired carriers) are suppressed.

Through classical calculus, Heytrakul and Taylor [4] have found the optimal bandpass nonlinearity for a case where the input consists of a coherent phase shift keying (CPSK) carrier corrupted with white Gaussian noise. The optimum criterion was to maximize the carrier output power for a given output C/N. They also found that IEL produces suboptimum results.

Minkoff has shown in [13] that the presence of phase nonlinearities (i.e., AM/PM functions, defined Section 2) can only degrade, never improve, the intermodulation performance of memoryless nonlinear systems.

Recently, Blachman [9] has solved the general memoryless bandpass nonlinearity synthesis problem that maximizes C/(N+I+IM)*. Optimum memoryless bandpass nonlinearity, in the form of the AM/AM function (see Section 2), was provided as a ratio of two intergrals. In [9], simpler expressions have also been obtained for special cases (e.g., no noise, no interferor). An interesting result is that, in the absence of noise, the optimum C/(I+IM)* is 0 dB when the carrier is weaker than the interferor (at the input).

In this paper, a different approach to synthesize effectively-memoryless[2] bandpass nonlinearities is described. Instead of using classical calculus, optimum effectively-memoryless bandpass nonlinearity problems are formulated as modern optimization problems and solved with the use of a digital computer and a mathematical programming algorithm. Section 2, for clarity, provides definitions of memoryless bandpass nonlinearity and other nonlinearities. Section 3 shows how an optimum bandpass nonlinearity problem can, in general, be formulated as a mathematical programming problem. Section 4 provides, as an example, specific formulation of a practical synthesis problem of a satellite communication FDMA/SCPC system where the transponder (i.e., nonlinearity) is accessed by M equal-power, equal-frequency-spacing carriers. The optimum criterion is to maximize C/IM for a given carrier output power C. Section 4 also provides optimum results which are obtained by using the revised Simplex method along with results generated from a typical TWTA.

[1] IEL is also called the piecewise linear limiter, a hypothetical model exhibiting no phase characteristic and a linear relationship between the output envelope and the input envelope up to saturation and constant output envelope thereafter.

* Note that C, N, I, and IM are respectively output powers of the carrier (wanted signal), noise, composite interferor, and total intermodulation products. While under the worst conditions, N and I may fall entirely in the carrier bandwidth and are all accounted for BER-degradation, a portion of IM products always fall outside the carrier bandwidth and are filtered out.

[2] For definitions and distinction of memoryless and effectively memoryless, see Section 2 below.

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2.0 Nonlinearity Classification

Nonlinearities can be classified as: memoryless, effectively-memoryless, and with memory [14, 15]. Nonlinearities with memory are nonlinearities that cannot be memoryless or effectively-memoryless.

Memoryless nonlinearities respond to input signals without delay. They can be characterized by the following equation:

$$v(t) = g[u(t)] \tag{1}$$

where, u(t) and v(t) are input and output voltages respectively. If u(t) consists of narrowband signals[3] which can be put into a sinusoidal form,

$$u(t) = A(t)\cos[\omega t + \phi(t)] \tag{2}$$

then through the use of the first order Chebyshev transformation [16] and the bandpass[4] assumption, v(t) can be put into the following alternate form,

$$v(t) = G(A)\cos[\omega t + \phi(t)] \tag{3}$$

where g(u) and G(A) are the first order Chebyshev transform pair. The bandpass assumption is often satisfied automatically by microwave devices (e.g., TWTAs, SSPAs), because they are often narrowband. G(A) is the so-called AM/AM function that characterizes memoryless bandpass nonlinearities (MBNs). The ideal hard limiter and piecewise-linear limiter (i.e., the IEL) are two most common representative of MBNs. MBNs do not exist in reality as they must respond instantaneously and their output signals should exhibit no phase shifts. However, MBNs have provided simple and classical means to analyze nonlinearities.

Effectively-memoryless bandpass nonlinearities (EMBNs) are nonlinearities characterized by not only the AM/AM function G(A), but also the so-called AM/PM function F(A). That is, if input u(t) to EMBNs is of the form described by Eq. (1), then output v(t) from EMBNs can be described by Eq. (4),

$$v(t) = G(A)\cos[\omega t + \phi(t) + F(A)] \tag{4}$$

Bandpass nonlinearities whose bandwidths are much wider than the signals' bandwidths can be treated as EMBNs. The AM/PM function was first introduced by Shimbo [17] to correct the deficiency of modeling nonlinearities (TWTAs) by just the AM/AM function.

To perform analyses of EMBNs and to facilitate baseband simulation, EMBNs are often put into the quadrature form [3]. With the quadrature form, EMBNs are characterized by two AM/AM functions, namely the inphase AM/AM function I(A) and the quadrature AM/AM function Q(A), instead of one AM/AM function G(A) and one AM/PM function F(A). In the quadrature form, v(t) is shown by Eq. (5),

$$v(t) = I(A)\cos[\omega t + \phi(t)] + Q(A)\sin[\omega t + \phi(t)] \tag{5}$$

where

$$I(A) = G(A)\cos[F(A)] \tag{6}$$

$$Q(A) = -G(A)\sin[F(A)] \tag{7}$$

3.0 Mathematical Programming Formulation

The general synthesis problem is to find the AM/AM and AM/PM functions, G(A) and F(A), so that a certain performance index $\mathcal{P}$ is maximized,

$$\underset{G(A),\,F(A)}{\text{Max.}}\ \mathcal{P} = f[G(A), F(A), \underline{\gamma}] \tag{8}$$

where the vector $\underline{\gamma}$ is relevant parameters associated with the input signals (e.g., power of carriers and noise). Depending on the criterion of optimality chosen, the performance index $\mathcal{P}$ can be C, C/N, C/I, C/IM, C/(N+I+IM), ... where C can be the output power of all carriers or just a particular carrier of interest and N, I, and IM are the output power of all noise, interference, and intermodulation products, respectively or just their components that fall into the bandwidth of a particular carrier of interest. There are also constraints associated with the maximization of $\mathcal{P}$ which will be addressed later in this section.

The problem of maximizing the functional (function of functions) $\mathcal{P}$ with respect to two functions G and F (and subject to appropriate constrained functions) is in general difficult to solve directly from the classical calculus (of variations). The problem can be simplified to a mathematical programming problem by parameterizing the unknown functions G and F, i.e., by replacing G and F by two known functions p and q with two unknown sets of parameters $\underline{\alpha}$ and $\underline{\beta}$,

$$G(A) = p(A, \underline{\alpha}) \tag{9}$$

$$F(A) = q(A, \underline{\beta}) \tag{10}$$

and the replacement must be valid for all possible values of the composite input envelope, i.e.,

$$\forall A \in (A_{min}, A_{max}) \tag{11}$$

For instance, if the input u(t) consists just M constant envelope carriers,

$$u(t) = \sum_{i=1}^{M} A_i\cos[\omega_i t + \phi_i(t)] \tag{12}$$

then

$$A_{min} = \text{Max.}\{0, [2A_m - \sum_{i=1}^{M} A_i]\} \tag{13}$$

$$A_{max} = \sum_{i=1}^{M} A_i \tag{14}$$

where

$$A_m = \text{Max.}\{A_1, A_2, \ldots, A_M\} \tag{15}$$

When the envelopes of the carriers are all equal (and equal to A_I), then obviously $A_{min} = 0$ and $A_{max} = MA_I$.

[3] Signals are considered narrowband if their bandwidths are small relative to their frequencies.

[4] The word bandpass associated with nonlinearity is used to mean that the nonlinearity is narrowband, i.e., it only allows components at or in the vicinity of the signal fundamental frequencies to pass through.

With the replacement, the optimization problem is reduced to the mathematical programming problem of finding $\underline{\alpha}$ and $\underline{\beta}$ that maximize $\mathcal{P}$,

$$\max_{\underline{\alpha}, \underline{\beta}} \mathcal{P} = f[p(A, \underline{\alpha}), q(A, \underline{\beta}), \underline{\gamma}] \quad (16)$$

The most popular functions that have been used to replace (i.e., to model) G and F for analysis purposes are probably due to Fuenzalida, Shimbo and Cook [18] which are described by Eq. (17),

$$G(A)e^{jF(A)} = \sum_{s=1}^{L} b_s J_1(\alpha s A) \quad (17)$$

where the vectors $\underline{\alpha}$ and $\underline{\beta}$ are represented by L complex coefficient parameters b_s's and a real scaling parameter α. With the model of G(A) and F(A) defined by Eq. (17), the output power of each carrier, IM product, and noise can be expressed in simple mathematical forms [18], [19]. Other popular models for G and F are their power series, i.e.,

$$G(A) = \sum_i \alpha_i A^i \quad (18)$$

$$F(A) = \sum_i \beta_i A^i \quad (19)$$

Models due Saleh [20] have only four parameters involved,

$$G(A) = \alpha_1 A/(1 + \alpha_2 A^2) \quad (20)$$

$$F(A) = \beta_1 A^2/(1 + \beta_2 A^2) \quad (21)$$

With respect to constraints associated with the mathematical programming problem, G(A) is typically limited to its allowable maximum G_m (e.g., G at saturation),

$$0 \le G(A) \le G_m \quad \forall A \,\varepsilon\, (A_{min}, A_{max}) \quad (22a)$$

Other constraints, if they exist, are due to the criterion of optimality chosen. For instance, in commercial communication systems where jamming is not of concern, the performance index may be the output power of a particular C and the associated constraint may be the ratio C/N, C/IM, or C/(N+IM), associated with the carrier, to be a constant In another criterion, the performance index may be C/IM and the constraint is C to be a constant.

The constraint described by Eq. (22a) is a continuous constraint which must be valid for all values of A continuously from A_{min} to A_{max}. The continuous constraint is, in general, not acceptable directly from mathematical programming algorithms. One approach, proposed in [22] and used as an example shown below, is to replace the continuous constraint defined by Eq. (22a) by K discrete constraints,

$$0 \le G(A_k) \le G_m \quad A_k \,\varepsilon\, (A_{min}, A_{max}) \text{ for } k = 1, 2, ..., K \quad (22b)$$

To properly represent Eq. (22a), the number of discrete constraints, K, must be large and the A_k 's must span the interval (A_{min}, A_{max}). The approach proposed is to first solve the problem with a reasonable small value for K. The validity of the results are then checked by scanning over the interval (A_{min}, A_{max}) with a set of large closely spaced points (e.g., $K \ge 100$) to determine where the constraint represented by Eq. (22a) is violated. Additional discrete constraints are then added at regions of A where the violations occur. The problem is then solved again. The process is repeated until a solution is obtained for which no constraint violations occur at the set of closely spaced test points. The approach was found to be effective and efficient in computer run time.

Note that when noise exists, the noise envelope may approach infinity (but not likely) and accordingly, $A_{max} = \infty$. Thus, the approximation of the continuous constraint defined by Eq. (22a) by the discrete constraints defined by Eq. (22b) may fail to work (because there are too many discrete constraints involved). One approach that can be used for cases involving noise is to truncate the maximum noise real envelope to a constant which practically accounts for, say 99.5 % or 99.9 %, of the time of real envelope values and use Eq. (14) to calculate A_{max}.

Note also that the optimization problem involved is in general nonlinear (in $\underline{\alpha}$ and $\underline{\beta}$). However, by proper selection of the functions p and q and of the criterion of optimality, the problem may become quadratic or even linear. In an example shown in Section 4 below, the optimization problem can be formulated as a set of four linear programming problems.

4.0 Example

As an illustrative example, a practical synthesis problem of a satellite transponder is considered [22]. The transponder is shared in a frequency division multiple access (FDMA) manner by M SCPC (single (baseband) channel per carrier) carriers. The carriers are equal in power and are equally spaced in frequency. The effects of the uplink thermal noise and the uplink interference are assumed here to be negligible, so that input to the transponder can be described by Eq. (23).

$$u(t) = \sum_{i=1}^{M} A_i \cos[\omega_i t + \phi_i(t)] \quad (23)$$

The transponder is modeled as an effectively-memoryless nonlinearity characterized by unknown AM/AM and AM/PM functions G(A) and F(A). Due to the transponder nonlinearity, intermodulation (IM) products are generated at the output of the transponder. Of these, the third-order IM products are the highest in power level. Two types of these products exist: the (2A-B) IM products whose frequency is the difference between twice the frequency of one carrier and the frequency of the other; and the (A+B-C) IM products whose frequency is the difference between the sum of two carrier frequencies and a third carrier frequency. Because the transponder is assumed to be effectively-memoryless and the carriers are assumed to have the same power at the input, the carriers also have the same power at the output, all of the (2A-B) IM products have the same power, and all of the (A+B-C) IM products also have the same power. For the FDMA/SCPC case considered where the carriers are equal in power and equally spaced in frequency, the effects of the (A+B-C) IM products are much more dominant than those of the (2A-B) IM products [23] because there are many more of the (A+B-C) products [M(M-1)(M-2)/2 versus M(M-1)] and the power of an (A+B-C) product is much higher than that of a (2A-B) product[5]. For the FDMA/SCPC case, the center channel (channel containing the center carrier) receives the most number of IM products. Accordingly, the power of total (A+B-C) products falling into the center channel relative to the center carrier output power is of prime concern and is chosen as the criterion of optimality for this example. The constraint associated with the criterion is that the total carrier output power be a constant.

Note that the total number of (A+B-C) IM products falling into the center channel can be calculated from the equations below [21],

$$N_c = (3M^2-10M+8)/8 \qquad \text{M even} \qquad (24a)$$

$$N_c = [(3M^2-10M+9) + 2(-1)^{(M+1)/2}]/8 \qquad \text{M odd} \qquad (24b)$$

From [13] and as discussed earlier in Section 1, the AM/PM function F(A) cannot improve intermodulation performance. Accordingly, the optimum F(A) is a constant for all A,

$$F(A) = \text{constant} \quad \forall A \in (0, MA_I) \qquad (25)$$

To find G(A), G(A) is parameterized as a power series in A. Since the nonlinearity is bandpass, even order components in the power series will be filtered out, G(A) can be expressed by just odd order components of A,

$$G(A) = \alpha_1 A + \alpha_3 A^3 + \alpha_5 A^5 + \alpha_7 A^7 + \ldots \qquad (26)$$

and from trigonometry, it can easily be shown that [24], [25] the carrier envelope (A_o) and the (A+B-C) IM product envelope (A_{im}) at the nonlinearity output can be calculated from the following equations,

$$A_o = |A'_o| \qquad (27)$$

$$A_{im} = |A'_{im}| \qquad (28)$$

$$A'_o = \alpha_1 A_I + (2M-1)\alpha_3 A_I^3 + [(1+(M-1)(6M-3)]\alpha_5 A_I^5 + \{1+(M-1)[34+24M(M-2)]\}\alpha_7 A_I^7 + \ldots \qquad (29)$$

$$A'_{im} = 2\alpha_3 A_I^3 + (12M-6)\alpha_5 A_I^5 + [144+32(M-3)(2M-1)]\alpha_7 A_I^7 + \ldots \qquad (30)$$

Thus, with respect to the chosen criterion of optimality, the optimization problem can be stated as

$$\min_{\alpha} \; \eta = A_{im} \qquad (31)$$

subject to

$$A_o = \lambda \text{ (known constant)} \qquad (32)$$

$$0 \le G(A) \le G_m \qquad \forall A \in (0, MA_I) \qquad (33)$$

The solution to this problem can be obtained by solving the following four linear programming (LP) problems,

$$\text{Min. } A'_{im} \text{ subject to } A'_{im} \ge 0,\ A'_o = \lambda \qquad (34a)$$

$$\text{Min. } A'_{im} \text{ subject to } A'_{im} \ge 0,\ A'_o = -\lambda \qquad (34b)$$

$$\text{Max. } A'_{im} \text{ subject to } A'_{im} \le 0,\ A'_o = \lambda \qquad (34c)$$

$$\text{Max. } A'_{im} \text{ subject to } A'_{im} \le 0,\ A'_o = -\lambda \qquad (34d)$$

where each of these four LP problems is also subject to the following set of inequality constraints,

$$0 \le G(A_k) \le G_m \quad A_k \in (0, MA_I) \text{ for } k = 1, 2, \ldots, K \qquad (35)$$

The solution of the problem is that of the LP problem that yields the lowest value for the performance index η defined by Eq. (31).

To generalize results, the problem was normalized so that the normalized maximum output power $(G_m)^2/2$ and input power $(MA_I)^2/2$ were both unity, i.e., $G_m = \sqrt{2}$ and $A_I = (\sqrt{2})/M$.

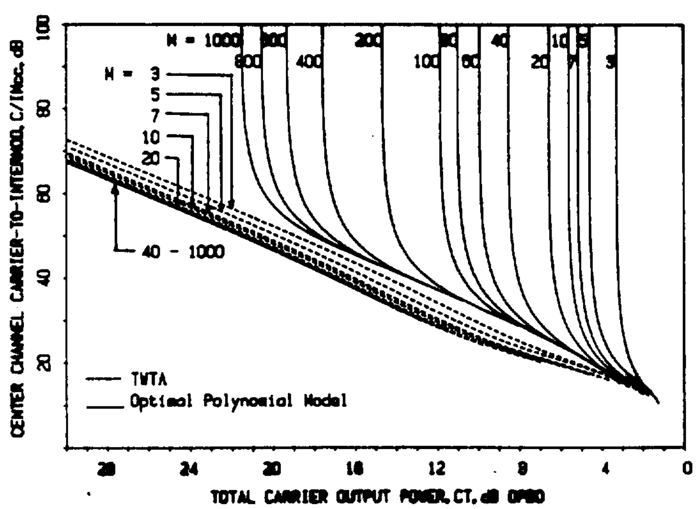
A FORTRAN computer program was developed using the revised Simplex method [24]. The problem solutions, obtained with G(A) being modeled as a seventh order polynomial in A, are shown in Figure 1. C_T and C/IM_{cc} are, respectively, the total carrier output power in dB OPBO (output backoff from saturation (maximum output power)) and the carrier-to-third-order intermodulation power ratio at the nonlinearity's center channel in dB,

$$C_T = -10 \log_{10}\{M\lambda^2/2\} \qquad (35)$$

$$C/IM_{cc} = 10 \log_{10}\{\lambda^2/(N_c A^2_{im})\} \qquad (36)$$

Figure 1 displays the plots of C/IM_{cc} versus C_T for various values of M ranging from 3 to 1000. For comparison, the corresponding plots of a typical TWTA are also shown in the figure, using both of its AM/AM and AM/PM functions and the Fuenzalida-Shimbo-Cook analysis technique [18].

Figure 1. Performance (C/IM_{cc} Vs. C_T) of The Optimal Memoryless Bandpass Nonlinearity G(A), Modeled as a Polynomial, and of a Typical TWTA.



From the figure, it is noted that there are not much improvement (in C/IM_{cc} over a TWTA) when the nonlinearity is operated close to saturation. However, the improvement increases significantly as the total carrier power backoff

[5] When G(A) can be represented by a third order polynomial in A, the power of an (A+B-C) product can easily be shown to be exactly 4 times (i.e., 6 dB) higher than that of a (2A-B) product.

increases. For instance, for M = 20, the improvement varies from 1.3 dB (15.3 dB vs. 14.5 dB) at 3 dB OPBO, to 5.6 dB (23.6 dB vs. 18.0 dB) at 5 dB OPBO, and to ∞ dB (∞ dB vs. 22.0 dB) at 8 dB OPBO. Corresponding to M = 20 and 3 dB OPBO, the optimizing parameter vector $\underline{\alpha} = [\alpha_1\ \alpha_3\ \alpha_5\ \alpha_7]$ = [4.319857 -7.534797 6.352725 -1.832645]. Values of $\underline{\alpha}$ for other cases can be found from [22].

Note that from Eqs. (29) and (30), it can be concluded that C/IM_{cc} goes to infinitive (i.e., $A_{im} = 0$) when $\lambda \leq (\sqrt{2})/M$ or equivalently when $C_T \geq 10 \log_{10}(M)$. The corresponding values for the α's are

$$\alpha_1 = (\sqrt{M})10^{-C_T/20} \quad (37a)$$

$$\alpha_{2i+1} = 0 \quad \text{for } i > 0 \quad (37b)$$

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