SOFT DECODING OF BCH CODES APPLIED TO MULTILEVEL MODULATION CODES FOR RAYLEIGH FADING CHANNELS

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Abstract

New 8PSK MultiLevel Codes (MLC) designed for fading channels with MultiStage Decoding (MSD) are reported in this paper. Binary block BCH codes of medium length have been chosen to construct powerful MLC 8PSK schemes and an efficient soft decision decoding algorithm [5] has been used in the MSD decoder. Error rate performances have been evaluated by simulations and compared favourably to the Cutoff Rate limit over Rayleigh channel.

The new MSD decoding scheme presents several advantages compared to the usual method based on the Viterbi algorithm. Among them the most interesting one is that the decoding complexity is not dependent of the trellis structure of the block codes, it can thus cope with the codes of medium length that are used to construct very powerful MLC 8PSK codes. Statistically, it is highly efficient in computation. In fact, very few computation is necessary when the channel is under good conditions. When the channel is worse, the computation effort can be pre-limited, without an important loss in performance. It has however a more complex structure and a variable processing delay.

Intensive computer simulations have been performed to evaluate the performance of the constructed MLC schemes. For the interesting bandwidth efficiencies ranged from 1.5 to 2.6 Bits/s/Hz, very good error rate performances have been obtained. Comparison with the known codes and the Cutoff Rate limit revealed that these new MLC 8PSK codes are very powerful for information transmission over Rayleigh fading channel. The adopted soft decision decoding algorithm can be adapted to different levels of performance/complexity degrees, so good performance/complexity trade-offs can be achieved for these MLC/MSD schemes for practical applications.

1. Introduction

The MultiLevel coding technique has been introduced in 1977 [1]. Progresses have been achieved latter with set partitioning approach using expanded signal constellations over AWGN channel. Much research work has been done on trellis coded modulations [11], but the performance was quite limited by both either very large trellis complexity or the subset size. MultiLevel coded modulation and MultiStage decoding have been applied recently to information transmission over fading channels [2]-[4].

The built-in time diversity (time diversity in short), the Squared Euclidean Product Distance (SEPD) are the two main design criteria for good coding schemes for Rayleigh fading channel [11]. It has been shown that the time diversity which dominates the asymptotic performance is closely related to the Hamming distance of the building codes in MLC schemes for fading channel. Block codes of medium length are used to construct powerful MLC coding schemes for fading channel. In particular, the recently reported "I-Q" approach [12] has been generalized to construct the 8PSK MLC codes of large built-in time diversity, showing also interesting properties in MultiStage decoder. The set partitioning is done so that the 8PSK constellation is encoded in two levels, the first one is encoded by a powerful code, and the next two levels are encoded by 2 codes using the "I-Q" approach so as to alleviate the error propagation between stages.

MultiStage decoding of MLC schemes is usually performed using Viterbi algorithm for either convolutional or block building codes in each stage. For decoding block building codes, however, the trellis complexity is a limiting factor for using the Viterbi algorithm, so that the MLC constructions could be only theoretically powerful, if constructed with block codes of medium length for a large time diversity. For decoding efficiently block codes of medium length, another soft decision decoding algorithm [5][6] is used in this study.

In Section 2, the basic soft decision decoding algorithm for block codes is described and explained. A brief recall of basic principles about the system model including transmitter, interleaved memoryless Rayleigh fading channel and receiver will be given in Section 3. Simulation results are given then in Section 4. The paper will be ended with some concluding remarks.

2. Soft Decoding of BCH codes

The algorithm used in this article was developed in 1986 and the interested readers find the details in [5][6]. In this section we briefly recall the principles of this decoding method. The systematic BCH encoder of rate K/N produces a codeword $C=(c_1,c_2,\dots,c_N)$ that is transmitted on the channel. At the receiver, the sequence $R=(r_1,r_2,\dots,r_N)$ is observed.

2.1 Likelihood coefficients

The likelihood coefficients are defined as follows :

$$v(i,j) = -\log \frac{p(r_j|i)}{p(r_j|\mathrm{hd}(r_j))} \qquad (1)$$

where $i \in \{0,1\}$ and $\mathrm{hd}(r_j)$ is the hard decision on r_j. The decoding metric used in the decoding algorithm is defined as :

$$\begin{aligned} z(R,\overline{C}) &= \sum_{l=1}^{N} z(r_l,\overline{c}_l) \\ &= -\sum_{l=1}^{N} \log\left(\frac{p(r_l / \overline{c}_l)}{p(r_l / \mathrm{hd}(r_l))}\right) \qquad (2) \\ &= \sum_{l=1}^{N} v(\overline{c}_l, l) \end{aligned}$$

where $\overline{C} = (\overline{c}_1, \overline{c}_2, \ldots, \overline{c}_N)$ is a codeword and :

$$-\log\left(\frac{p(r_l / \overline{c}_l)}{p(r_l / \mathrm{hd}(r_l))}\right) = \begin{cases} 0, \text{ if } \mathrm{hd}(r_l) = \overline{c}_l \\ > 0, \text{ if } \mathrm{hd}(r_l) \neq \overline{c}_l \end{cases}.$$

There must be a null entry in each column of the $\{v(i,j)\}$ matrix that we delete to obtain a vector $\{v(j)\}$ of size N.

2.2 Preprocessing

A sort is performed according to the decreasing order of the $\{v(j)\}$ vector. The associated permutation list T is to applied to the parity matrix H of the code and the received vector R. We apply the Gauss reduction to the rightmost subblock of the permuted matrix to obtain a systematic form. When this subblock is singular, it is necessary to modify as little as possible the permutation T. The final step is to create the non-decreasing ordered list U formed by the first K likelihood coefficients of the vector $\{v(j)\}$. At this point, an ordered list U, a vector R^T obtained by permutation of R and a systematic matrix H^T are available.

2.3 Systematic search for the optimum codeword

The systematic search operates on the equivalent code defined by the systematic parity matrix H^T. The decoding metric is then divided into the information part and the parity-check one :

$$\begin{aligned} z(R^T,\overline{C}^T) &= \sum_{l=1}^{N} z(r_l^T,\overline{c}_l^T) \\ &= z'(R^T,\overline{C}^T) + z''(R^T,\overline{C}^T) \end{aligned} \qquad (3)$$

where $z'(R^T,\overline{C}^T) = \sum_{l=1}^{K} z(r_l^T,\overline{c}_l^T)$, $z''(R^T,\overline{C}^T) = \sum_{l=K+1}^{N} z(r_l^T,\overline{c}_l^T)$.

The following properties are satisfied :

- $z''(R^T,\overline{C}^T) \geq 0$,
- it is possible to generate the $z'(R,\overline{C}^{T(i)})$ in increasing order by the "subset reordering with exclusion" procedure [5] which uses the list U, in such a way that $z'(R,\overline{C}^{T(i)}) \leq z'(R,\overline{C}^{T(i+1)})$.

The "subset reordering with exclusion" procedure is an algorithm which generates an ordered list of subsets with a linear complexity.

Optimum maximum likelihood decoding consists in finding a certain codeword $\overline{C}^{T(\bullet)}$ such that $z(R,\overline{C}^{T(\bullet)}) \leq z(R,\overline{C}^T)$ for all codewords $\overline{C}^T$. We perform an encoding operation on the first K symbols of $\mathrm{hd}(R^T)$ to obtain the first codeword $\overline{C}^{T(1)}$, which is the starting point of the systematic search. We set a threshold value σ as $z(R^T,\overline{C}^{T(1)})$. For this first candidate, we have $z'(R^T,\overline{C}^{T(1)}) = 0$ and so $\sigma = z''(R^T,\overline{C}^{T(1)})$. The systematic search is summarized as follows :

1. Set $i = 2$, $i^* = 1$, $j^* = 2$, $\sigma = z''(R^T,\overline{C}^{T(1)})$.
2. Call the "subset reordering with exclusion" subroutine to get $z'(R^T,\overline{C}^{T(i)})$.
3. $z'(R^T,\overline{C}^{T(i)})$ is compared to σ,
   - if $\geq$, $j^* = i$, $\overline{C}^{T(\bullet)} = \overline{C}^{T(i)}$, decoding is finished, we just have to inverse the permutation T in order to obtain the decoded codeword $\overline{C}^{(\bullet)}$,
   - if $<$, $z(R^T,\overline{C}^{T(i)}) = z'(R^T,\overline{C}^{T(i)}) + z''(R^T,\overline{C}^{T(i)})$ is computed and compared with σ,
     - ♦ if $<$, $\sigma = z(R^T,\overline{C}^{T(i)})$ and $i^* = i$,
     - ♦ if $\geq$, do nothing.
4. $i = i+1$, return to 2.

At the final step, j^* gives the number of codewords examined during the search procedure, while i^* gives the best codeword index. In this article, a suboptimal version of this algorithm is used by limiting i to a upper bound J. At the step 4., if $i > J$, the algorithm stops and gives the best codeword.

3. System Description

3.1 Rayleigh Fading Channel Model

The system under consideration is depicted in Fig. 1. The MultiLevel coder and the MultiStage Decoder are discussed in detail in next subsections. The channel model considered here is a frequency non-selective slowly varying Rayleigh Fading channel. This channel is considered memoryless (independent fading in each channel symbol) because of the use of ideal interleaving and deinterleaving. At the receiver, coherent demodulation is assumed, the effect on the phase is fully compensated. Only the degradation on the amplitude is considered in the channel model.

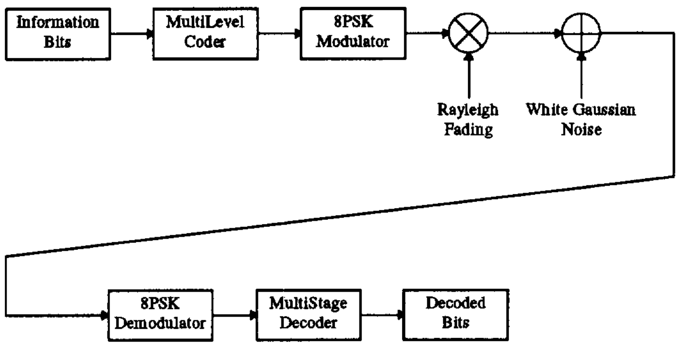

Fig. 1. Block diagram of the MultiLevel Block coded system.

Let x_i be the complex signal input to the channel at time i. The received signal is given by $y_i = \alpha_i x_i + n_i$ where n_i is complex white noise with zero mean and variance N_0. The random variable α_i is the magnitude of the fade at time i, it is Rayleigh distributed and normalized so that $E\{\alpha_i^2\} = 1$ to ensure that the average received signal energy is equal to the transmitted signal energy E_s. The MLC encoder produces a sequence of signals $X_N = (x_1, x_2, \ldots, x_N)$ that is transmitted over the Rayleigh fading channel. The receiver observes the sequence $Y_N = (y_1, y_2, \ldots, y_N)$. We assume that the amplitudes $A_N = (\alpha_1, \alpha_2, \ldots, \alpha_N)$ can be perfectly recovered at the receiver (a variety of methods are described in [7]-[10]). In the case of a maximum likelihood decoder, an upper bound is available on the pairwise error probability $P(X_N, \hat{X}_N)$, probability that the decoder chooses the sequence $\hat{X}_N = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_N)$ when the transmitted sequence was $X_N = (x_1, x_2, \ldots, x_N)$. This occurs when :

$$\sum_{j=1}^{N} \left\| y_j - \alpha_j \hat{x}_j \right\|^2 < \sum_{j=1}^{N} \left\| y_j - \alpha_j x_j \right\|^2 . \qquad (4)$$

Using (4) and applying a Chernoff bound results in :

$$P(X_N, \hat{X}_N) \le \prod_{j \in \eta} \frac{1}{1 + \frac{1}{4N_0} \left\| x_j - \hat{x}_j \right\|^2} \qquad (5)$$

where η is the set of all j for which $x_j \neq \hat{x}_j$. The symbolwise Hamming distance between X_N and $\hat{X}_N$ is equal to $\mathrm{Card}(\eta)$. The minimum of this distance in the set of all coded sequences L is called *minimum time diversity* or *effective code length* and noted L. From (5), it's clear that a coding scheme adapted to the Rayleigh fading channel should have a large *effective code length,* its minimum product distance among code sequences being a secondary criterion to optimize.

3.2 MultiLevel 8PSK Block Codes Description

3.2.1 "I-Q" Approach

[12] shows that encoding the in-phase and quadrature channels independently provides significant gain over the more traditional approach. We have generalized this approach to

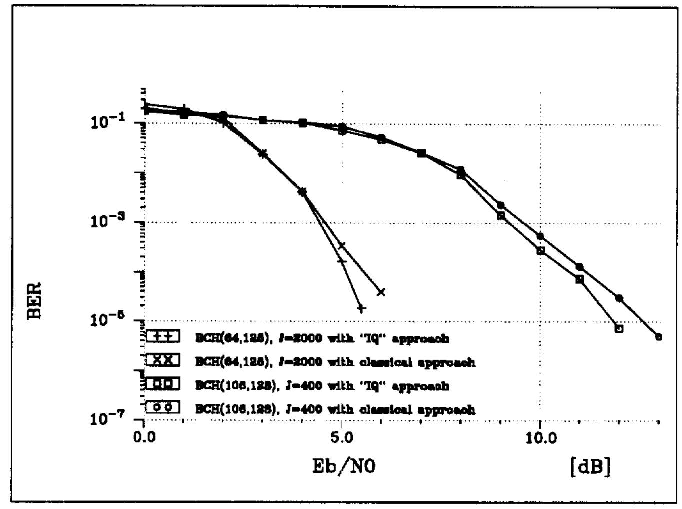

Fig. 2. Performance of BCH codes with "I-Q" approach and QPSK modulation.

block coding case so that a larger range of code rate is obtained. For example comparing the extended BCH (106,127) code Gray-mapped to a QPSK constellation with an "I-Q" code in which two BCH encoders are used to encode the in-phase/quadrature components, the latter approach provides 0.7 dB of coding gain at a BER of 10^{-5}. In fact, the diversity of the last system is equal to the Hamming distance of the building codes. The BER curves obtained by simulation for the two extended BCH codes (64,128) and (106,128) are presented in Fig. 2. This approach will be used to encode the second level of the following MLC schemes.

3.2.2 MultiLevel Encoder

The MultiLevel encoder is described in Fig. 3. The coding scheme consists of two component codes. The first code named BCH_1 selects a QPSK signal set. The second code, which uses the "I-Q" approach, is constituted by two encoders, named I-Encoder BCH_2 and Q-Encoder BCH_2 in the figure, that encodes the in-phase en quadrature components of the QPSK constellation preselected by the first code. The labelling of the 8PSK signal constellation is shown in Fig. 4. Each digit b_i is then independently addressed by the output of one of the binary BCH encoders. Let the minimum Hamming distance for each of the component codes be d_{Hi}, $i = 1,2$. Then the minimum Hamming distance between any two transmitted 8PSK coded sequence, i.e. the time diversity of the MultiLevel code, is :

$$L = \mathrm{Min}\{d_{H1}, d_{H2}\} . \qquad (6)$$

Moreover, for a such system, the effective code rate is defined as :

$$R_{eff} = \left(\frac{K_1}{N_1} + 2\frac{K_2}{N_2} \right) \qquad (7)$$

where K_i/N_i is the code rate of BCH_i. This quantity represents the number of information bits transmitted by 8PSK symbol.

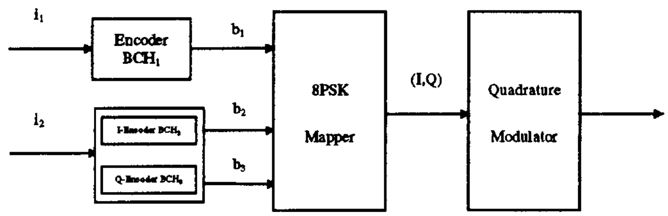

Fig. 3. Block diagram of the MultiLevel Block coded system.

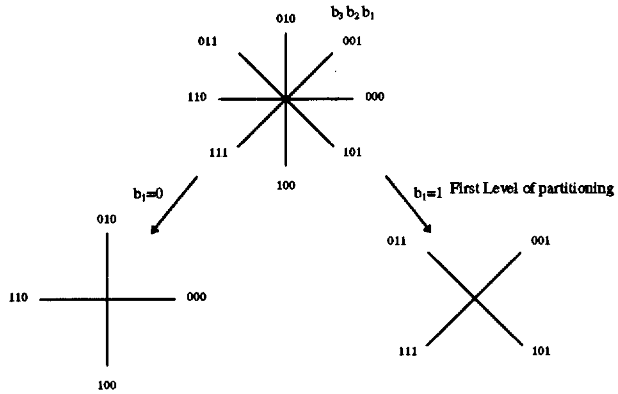

Fig. 4. The 8PSK constellation and its set partitions.

3.2.3 MultiStage Decoder

The optimal decoding of a MultiLevel code can be performed by a maximum likelihood decoder (MLD) that decodes jointly the three component codes. Suboptimal techniques such MultiStage decoding are preferred because the complexity of the MLD is too high. The MultiStage decoder is shown in Fig. 5.

The delay buffer #1 stores the information sequence Y_N until decoding in the first stage is accomplished, the delay buffer #2 synchronizes the outputs of the different BCH decoders. For the first decoder, BCH$_1$ Decoder, the metric is based on the distance to each QPSK subset. Let the eight points of the 8PSK be labelled as S_k, the index k is given by $4b_3 + 2b_2 + b_1$. The decoder computes at time j the following likelihood coefficients :

$$v(i,j) \approx -\log\left(\frac{\exp\left(-\operatorname*{Min}_{k/b_1=i}\left(\|y_j - \alpha_j S_k\|^2\right)\right)}{\exp\left(-\operatorname*{Min}_{k/b_1=\mathrm{lsb}(\mathrm{hd}(y_j))}\left(\|y_j - \alpha_j S_k\|^2\right)\right)}\right)$$
$$\approx \operatorname*{Min}_{k/b_1=i}\left(\|y_j - \alpha_j S_k\|^2\right) - \operatorname*{Min}_{k/b_1=\mathrm{lsb}(\mathrm{hd}(y_j))}\left(\|y_j - \alpha_j S_k\|^2\right)$$

where $\mathrm{hd}(y_j)$ is the index of the 8PSK point corresponding to a hard decision on y_j. After decoding at the first stage, the estimation of the output bits of the first coder is sent through the QPSK demodulator. This block gives to each BCH$_2$ Decoder the in-phase or the quadrature component of the QPSK selected by the information received from the first decoder. This method is equivalent to two decoders that would

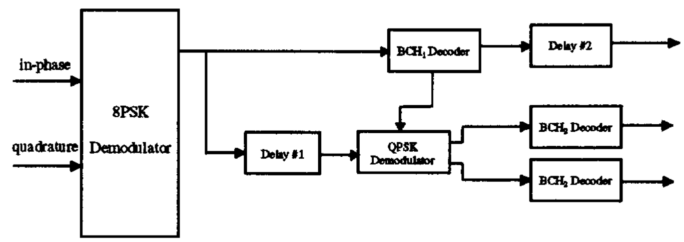

Fig. 5. Block diagram of the MultiStage Decoder.

compute the following likelihood coefficients :

$$v(i,j) = -\log\left(\frac{\exp\left(-\operatorname*{Min}_{k/b_3=i,b_2=i,b_1=\hat{b}_{1j}}\left(\|y_j - \alpha_j S_k\|^2\right)\right)}{\exp\left(-\operatorname*{Min}_{k/2b_3+b_2=\mathrm{hd}(y_j)>>1,b_1=\hat{b}_{1j}}\left(\|y_j - \alpha_j S_k\|^2\right)\right)}\right) \text{ for the}$$

decoder working on the in-phase component and :

$$v(i,j) = -\log\left(\frac{\exp\left(-\operatorname*{Min}_{k/b_3=i,b_2=1\oplus i,b_1=\hat{b}_{1j}}\left(\|y_j - \alpha_j S_k\|^2\right)\right)}{\exp\left(-\operatorname*{Min}_{k/2b_3+b_2=\mathrm{hd}(y_j)>>1,b_1=\hat{b}_{1j}}\left(\|y_j - \alpha_j S_k\|^2\right)\right)}\right) \text{ for the}$$

decoder working on the quadrature component, $\hat{b}_{1j}$ corresponding to the estimation of the jth output bit of the BCH$_1$ Encoder. In the last formula, the symbol $\oplus$ represents the addition modulo-2.

3.3 MultiLevel 8PSK Block Codes for the Rayleigh Fading Channel

We choose the effective code length of a MultiLevel block code as the design criterion. Since the first decoded bit level partitions the signal constellations for the next decoding steps, it is obvious that the binary block code for the first bit level should be the most powerful one. An improvement is achieved by proper selection of component codes to meet the error correcting requirements of different bit levels. We should also care about choosing error correcting capability of component codes in order to achieve a trade-off between error performance and effective code rate.

By applying this criteria, two MultiLevel block codes are proposed for the Rayleigh fading channel. The Table 1 shows these different MultiLevel codes, all the component codes are extended binary BCH codes.

Spectral efficiency (Bits/Symbol)	Code BCH$_1$	Code BCH$_2$	L
2.16	BCH(64,128)	BCH(106,128)	≥8
2.21	BCH(71,128)	BCH(106,128)	≥8

Table 1. 8PSK MultiLevel codes.

4. Simulation Results

The results for the two block coded modulations are presented in Fig. 6 and Fig. 7 for different values of the parameter J.

This last parameter allows us to limit the complexity of the decoding. So for the two coded modulations, the value 400 seems to be a trade-off between complexity and performance.

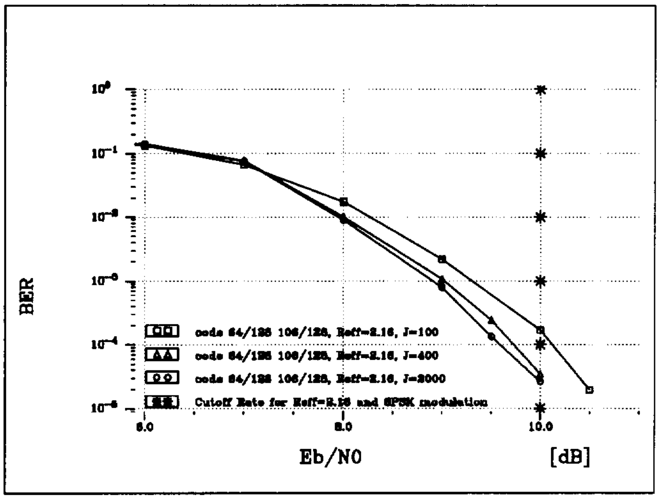

Fig. 6. Simulation results for (64,128) (106,128) 8PSK coded modulation.

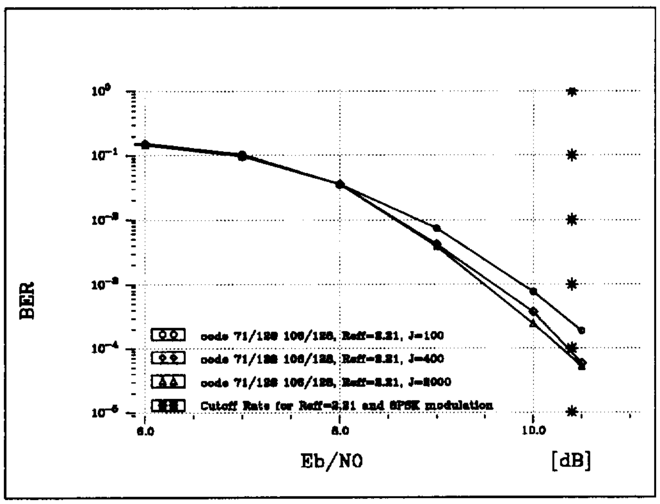

Fig. 7. Simulation results for (71,128) (106,128) 8PSK coded modulation.

Coding gains close to the Cutoff Rate limit are achieved. The BER curves of this two coding schemes show in fact that at a BER=10^{-4} the requested E_b/N_0 is less than the Cutoff Rate limit.

5. Concluding Remarks

New block coded 8PSK MLC schemes as well as their efficient MultiStage decoding are reported for Rayleigh fading channel. Simulated performances are shown to be excellent and compared favourably to the Cutoff Rate limit.

The new MultiLevel construction with an 8PSK modulation combines a first level block code with an "I-Q" trellis code, achieving very large time diversities in a wide range of code rate. These schemes are then decoded in MultiStage using an efficient soft decision block decoding algorithm that has been used in one of its sub-optimal versions with a pre-limited computation.

We can conclude that these new coding schemes achieve very good performance over fading channel, and the proposed decoding scheme is efficient for their decoding. Nevertheless, further studies would be done for real-time implementations, especially for achieving the best performance/complexity trade-offs for the practical applications. The basic decoding algorithm could be further simplified under specific implementation conditions. Finally, it is interesting to indicate that the constructed MLC codes perform very well over the AWGN and the Rice channels.

6. Acknowledgement

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7. References

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