DWELL-INTERVAL RETRANSMISSIONS FOR FREQUENCY-HOP COMMUNICATION SYSTEMS

Justin P. McNew
Motorola, Cellular Infrastructure Group
1701 Golf Rd., Rolling Meadows, IL 60008

Carl W. Baum
Clemson University, ECE Dept.
Riggs Hall, Clemson, SC 29634-0915

ABSTRACT

Retransmission protocols are commonly employed for data communications when low bit error rates are required. In this paper, retransmission protocols for frequency-hop communication systems are investigated in which the unit of retransmission is the data contained within a single dwell interval. A scheme is presented for determining the reliability of dwell intervals by counting corrected errors and decoder failures. The performance of this scheme is determined for a channel containing partial-band interference and noise. Comparisons of the dwell retransmission schemes to type-I hybrid ARQ show that dwell retransmission methods give significantly improved performance.

I. INTRODUCTION

Interference can significantly degrade the performance of frequency-hop (FH) spread-spectrum communication systems. Error control coding and interleaving are used to partially mitigate the detrimental effects, but retransmission protocols are often necessary to ensure acceptable performance of the system. Retransmissions are especially important for the transmission of data (as opposed to voice or video) with stringent requirements on bit error probability.

Retransmission protocols are implementations of error control for channels with feedback. The most basic type of retransmission protocol, pure automatic-repeat-request (ARQ), uses error detection codes and generates retransmission requests based on detected errors at the receiver [1]. ARQ used in conjunction with both error detection and error correction is known as type-I hybrid ARQ (HARQ-I). HARQ-I generally provides significantly better performance than ARQ. Both ARQ and HARQ-I protocols require retransmission of one or more codewords each time a retransmission request is generated, and the receiver discards all but the most recent transmission of a given block of data [2], [3], [4]. A number of more advanced retransmission schemes make use of older transmissions to improve performance [5], [6], [7], [8]. Some of these schemes employ some form of packet combining [5], [6], [7].

Another retransmission protocol is known as type-II hybrid ARQ (HARQ-II). This adaptive protocol uses incremental redundancy to more efficiently utilize channels with large variations in reliability over time [9]. The receiver generates requests for additional redundancy symbols to provide increased error correcting capability as needed. HARQ-II and the packet combining schemes require greater data buffering capabilities at the receiver than the other schemes, but can provide greatly improved performance [8], [10], [11].

In FH communication systems, packets are divided into sub-packets and the sub-packets are transmitted in pseudo-randomly chosen frequency slots. The time period in which a particular sub-packet is transmitted is called a dwell interval. In this paper, we refer to the contents of a sub-packet as a dwell. This paper focuses on retransmitting FH sub-packets instead of complete packets.

A ***dwell retransmission protocol*** is a selective retransmission scheme in which the receiver requests the retransmission of only those dwells suspected to be the most severely corrupted by noise or interference. The dwells are chosen for retransmission according to some form of receiver-generated side information. Simply examining the results of decoding is one approach to this retransmission selection process.

As with HARQ-II and packet combining schemes, dwell retransmission protocols also require the use of buffering in the receiver. However, dwells which require retransmission may be discarded, and thus the maximum storage required is no more than a single packet.

II. SYSTEM MODELS

The communication system considered in this paper employs FH spread-spectrum modulation with M-ary orthogonal signaling and a singly extended (n,k) Reed-Solomon (RS) code for error control. The blocklength and alphabet size of the code is n, where n is a power of two, and the size of the modulation alphabet is equal to the size of the code symbol alphabet. Each packet contains v codewords, and v code symbols are transmitted in each dwell interval. The code symbols are interleaved using an $n \times v$ block interleaver to mitigate the effects of interference.

A dwell retransmission protocol can be described generally as follows. A packet is transmitted over n successive dwell intervals. An acknowledgement is then transmitted by the intended receiver of the packet. The acknowledgement includes a list of dwell intervals to be retransmitted and, for the purpose of analysis, is assumed to require a transmission time equal to one dwell interval. In practice, more than one dwell interval may be required, particularly if v is small. The use of more than one dwell interval may also be desirable to increase the reliability of the acknowledgement.

The performance of the system is determined for a channel characterized by a combination of partial-band interference and wideband noise. The effects of the noise and interference are assumed to be independent from dwell to dwell.

A fraction ρ of the communication bandwidth contains both Gaussian noise and Gaussian interference. The noise

This work was supported by the Army Research Office under grant number DAAH04-95-1-0247.

0-7803-4249-6/97/$10.00 © 1997 IEEE

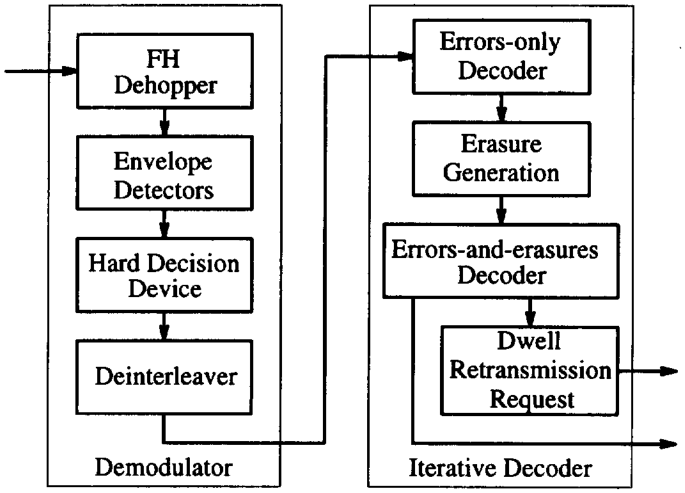

Fig. 1. Frequency-hop communications receiver.

power spectral density is defined to be $\frac{1}{2}\rho^{-1}N_I + \frac{1}{2}N_0$ in this fraction of the band. In the remainder of the band, only Gaussian noise is present, and the noise power spectral density is $\frac{1}{2}N_0$. With these definitions, the average noise over the entire band is $\frac{1}{2}N_I + \frac{1}{2}N_0$ a quantity that is independent of ρ.

A block diagram of the receiver is shown in Figure 1. The demodulator contains a bank of M parallel envelope detectors. The outputs of the envelope detectors are inputs to a decision device that makes hard decisions on the received symbols. The decision device is followed by a deinterleaver and an iterative bounded-distance decoder. A detailed discussion of the iterative decoder is given in Section 4.

III. ANALYTICAL MODELS

In this section we investigate the theoretical performance gains that may be possible by using dwell retransmissions as compared to HARQ-I (abbreviated as HARQ).

A. Perfect Dwell Error Detection

This analytical model is based on the assumption that if any errors occur in a dwell, the errors are detected and the entire dwell is erased. We call this the perfect dwell error detection model. Received words can be decoded if the number of erasures is less than $n - k$. If a decoding failure occurs, the erased dwells are simply retransmitted until decoding occurs. Note that the assumption that all errors are detected (and all corresponding dwells erased) implies that erasures-only decoding can be used. The assumption is equivalent to assuming that an ideal error-detecting "inner" code (with perfect error-detecting capability and negligible redundancy) is employed.

We assume retransmissions occur until the packet decodes, so for the perfect dwell error detection model, the probability of not decoding is zero for both HARQ and the dwell retransmission scheme. Furthermore, the fact that only erasures (i.e., no errors) are present implies that the probability of not decoding *correctly* is zero. Throughput is used as a measure of performance. In calculating throughput we assume that the acknowledgement message has a duration equal to that of one dwell interval.

We now define several variables associated with an analysis of dwell retransmissions and HARQ under the perfect dwell error detection model. Let N_T denote the number of transmissions and let D denote the average cumulative delay in dwell intervals. Denote the probability of one or more errors in a dwell (and thus, the probability of dwell erasure) by P_E. Define the normalized throughput to be the ratio of the number of data symbols to the average number of symbols required to send the packet (including retransmissions and acknowledgements), and denote this quantity by η.

We first analyze perfect dwell error detection with HARQ. Here, the delay increases by $n + 1$ on each successive transmission, and thus the average cumulative delay can be expressed as $D = (n + 1)E[N_T]$. The event that $N_T = i$ is the event that the first $i - 1$ transmissions fail (i.e., the number of erasures exceeds $n - k$), and the ith transmission succeeds (i.e., the number of erasures does not exceed $n - k$). It is straightforward to show that $P(N_T = i) = (1-p)^{i-1}p$, where p is the probability that all the received words in a packet decode, i.e., $p = \sum_{i=0}^{n-k} \binom{n}{i} P_E^i (1 - P_E)^{n-i}$. It follows that $E[N_T] = \sum_{i=1}^{\infty} iP(N_T = i) = \sum_{i=1}^{\infty} i(1-p)^{n-i}p = \frac{1}{p}$, and the average cumulative delay is therefore $D = (n+1)/p$. The relationship between throughput and delay can then be expressed as $\eta = k/D$.

We now analyze the performance of perfect dwell error detection with dwell retransmissions. Let E_m denote the number of dwells that are erased on the mth transmission. The following illustrates the behavior of the average cumulative delay for successive transmissions. If $E_1 \leq n - k$, then $N_T = 1$ and $D = n + 1$. If $E_1 = i > n - k$ and $E_2 \leq n - k$, then $N_T = 2$ and $D = (n + 1) + (i + 1)$. If $E_1 = i > n - k$, $E_2 = j > n - k$, and $E_3 \leq n - k$, then $N_T = 3$ and $D = (n+1) + (i+1) + (j+1)$, and the argument generalizes in the obvious way. Note that $j \leq i$ because the number of erasures on successive transmissions cannot increase.

Now, let $p_{i,j}$ denote the probability of j erasures on transmission $m + 1$ given that there are i erasures on transmission m, and note that this probability is not a function of m. Then $p_{i,j} = \binom{i}{j} P_E^j (1 - P_E)^{i-j}$ for $j \leq i$, and $p_{i,j} = 0$ for $j > i$. The average cumulative delay to go from a state in which i erasures are made to a state in which the number of erasures is less than or equal to $n - k$ can be expressed as

$$D_i = (i + 1) + \sum_{j=n-k+1}^{i} p_{i,j} D_j, \qquad (1)$$

and this expression is valid for $n - k + 1 \leq i \leq n$. When the number of erasures is less than or equal to $n - k$, decoding is possible and no additional retransmissions are needed. Eq. (1) can be written recursively by solving for D_i. The average cumulative delay is found by solving (1) for $D = D_n$. The throughput is $\eta = k/D$.

B. Perfect Side Information

The perfect side information model is similar to the perfect dwell error detection model in the sense that an entire dwell is erased whenever one or more symbol errors are detected in a dwell. However, in the perfect side information model, it is assumed that only errors caused by interference can be detected; errors due to background noise are undetectable. This model is an approximation for practical schemes for the generation of side information, including those which employ test symbols and parity-check symbols. However, note that the term "perfect side information" does not imply any sort of optimality or upper limit on performance; counterexamples prove that other practical schemes can outperform perfect side information [12].

In the perfect side information model, a certain percentage of the frequency slots are corrupted by interference. Dwells sent at these frequencies are referred to as "hit" dwells. These dwells are detected and erased. A received packet is decoded using a bounded-distance errors-and-erasures decoder with a decoding diameter of at most $n-k$[1]. A value less than $n-k$ reduces the probability of decoding into the wrong codeword. However, this improvement in error rate is achieved at the expense of throughput.

For the purpose of analysis, we assume that a decoding diameter of $n-k-2$ is used. With this assumption, we can reasonably assume that the probability of decoding into the wrong codeword is small compared to the probability of a decoding failure. It follows that retransmissions occur until the sum of the number of erasures and twice the maximum number of symbol errors in a codeword does not exceed $n-k-2$.

If a received packet fails to decode, the hit (erased) dwells are retransmitted until decoding is possible. In the event that there are no erasures and a decoding failure still occurs, the entire packet is retransmitted. This event occurs very infrequently if the symbol error probability of non-hit dwells is small.

Let ρ denote the probability a dwell is hit, and let p_t denote the probability of symbol error in dwells that are not hit. As with perfect dwell error detection, let N_T denote the number of transmissions, let D denote the average cumulative delay, and let η denote the throughput.

For HARQ, the delay can be expressed as $D=(n+1)/p$, where p is the probability that all of the received words in the packet decode. Received words can be decoded if the sum of the number of erasures and twice the number of errors is less than or equal to $n-k-2$. The probability that all received words in the packet decode is $p=\sum_{i=0}^{n-k-2}\binom{n}{i}\rho^i(1-\rho)^{n-i}P_i$, where $P_i=P(2\max_\ell X_\ell+i\le n-k-2 \mid i \text{ erasures})$ and X_ℓ is the number of errors in the ℓth codeword.

Symbol errors in non-hit dwells are independent and identically distributed. Therefore, $P_i=P(2X_1\le n-k-2-i \mid i \text{ erasures})^v$ which can in turn be expressed as

$$P_i=\left(\sum_{j=0}^{\lfloor\frac{n-k-i-2}{2}\rfloor}\binom{n-i}{j}p_t^j(1-p_t)^{n-i-j}\right)^v,$$

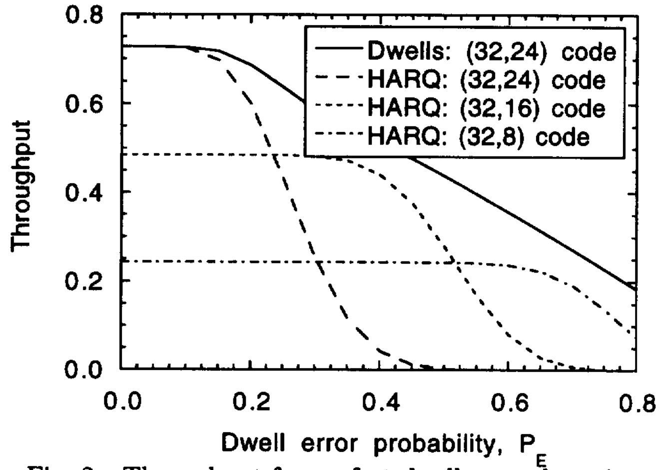

Fig. 2. Throughput for perfect dwell error detection.

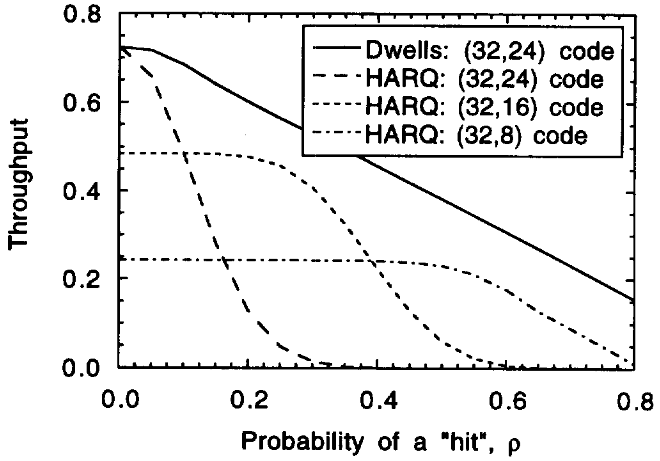

Fig. 3. Throughput for perfect side information, $p_t=10^{-2}$.

where v is the number of codewords in the packet. The average throughput can be expressed as $\eta=k/D=kp/(n+1)$.

Analysis of the dwell retransmission scheme with this model is possible by using an approach similar to the analysis of perfect dwell error detection. Unfortunately, the resulting expressions are numerically intractable except when used to analyze trivially short codes. For this reason, simulation is used to determine performance.

C. Performance Comparison

In this section, performance results are presented for a system using $(32,k)$ RS codes with 10 codewords in a packet. The dwell retransmission scheme is compared with HARQ for both the perfect dwell error detection and perfect side information models in Figures 2 through 4. In the comparisons, the dwell retransmission scheme with a given code rate is compared to HARQ with multiple code rates. HARQ performance is very sensitive to the code rate, and low-rate codes are required in severe channels. In contrast, the dwell retransmission scheme performs well with a single code over a wide range of channel conditions.

The performance of the dwell retransmission scheme and HARQ under the perfect dwell error detection model is shown in Figure 2. The analytical expressions derived in Sec-

---

[1] We define the decoding diameter to be the value d^* such that correct decoding is guaranteed if $2t+e\le d^*$, where t is the number of symbol errors and e is the number of erasures.

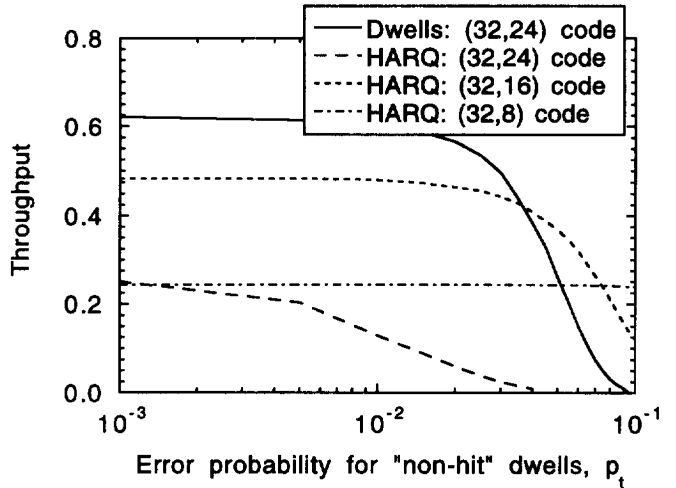

Fig. 4. Throughput for perfect side information, $\rho = 0.2$.

tion 3.2 are used to generate these results. For this model, the performance of the dwell retransmission scheme with a (32,24) code is significantly better than HARQ using *any* code rate for all values of P_E.

Simulation results for the perfect side information model are shown in Figures 3 and 4. The throughput of the dwell retransmission scheme is significantly better than the throughput of HARQ. Figure 3 shows performance as a function of the hit probability ρ for $p_t = 10^{-2}$. The results are quite similar to those for the perfect dwell error detection model. Figure 4 presents throughput as a function of the error probability for non-hit dwells. This figure shows that if the channel conditions are very poor, the throughput of the dwell retransmission scheme may be worse than that of HARQ. However, the background noise levels must be extremely large for this to occur.

IV. A SCHEME BASED ON ERROR COUNTING

The dwell retransmission scheme we propose is designed for a system in which there is no available knowledge about the reliability of symbols in a received packet (i.e., no side information). The key idea in our approach is that the results of decoding can be used to determine which dwells are the most likely to be severely corrupted. The scheme uses an iterative decoding algorithm to obtain and use information about the reliability of individual dwells in a received packet. One benefit of this approach is the fact that *no* additional redundancy (in the form of test symbols or an inner code) is required.

Figure 5 provides a flow-diagram representation of the dwell retransmission scheme integrated with the decoding algorithm. Prior to decoding, hard decisions are made on the received symbols. Following this, the first operation is errors-only decoding. If one or more decoder failures occur in a packet, a reliability factor, or "score," is calculated for each dwell. The score is calculated by adding the number of decoder failures to the number of symbol errors in each dwell. (The error locations can be found by simply re-encoding received words that did not fail to decode.) Dwells with high scores are considered unreliable. If the score for a dwell is greater than or equal to a threshold t^*, all symbols in received words that fail to decode in that dwell are erased. After erasures are inserted, the received words that originally resulted in decoder failures are decoded again, this time using errors-and-erasures decoding. If any decoder failures remain, the dwells with scores greater than or equal to t^* are retransmitted. After the retransmitted dwells are received, the decoder operation is performed only on the received words that failed to decode on the previous transmission. However, after every transmission, all of the new information that is available is used to determine dwell scores. If there are no erasures present at the redecoding stage and a decoding failure still occurs, the entire packet is retransmitted.

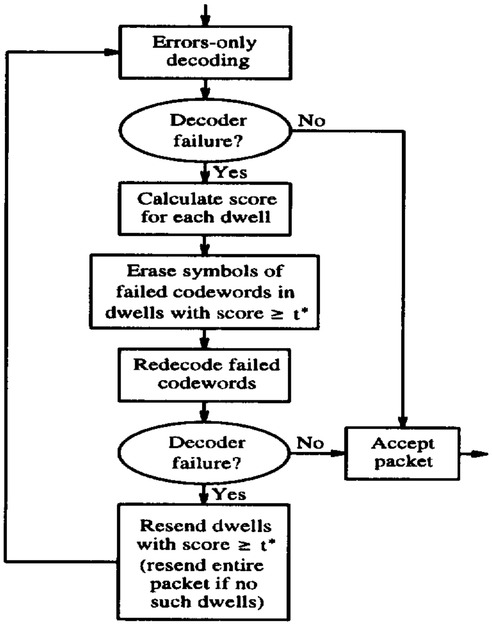

Fig. 5. Iterative decoding algorithm.

Modifications may be made to this dwell retransmission scheme to reduce complexity and processing delay. In particular, the system may be simplified by eliminating the redecoding stage completely. Without the redecoding stage, three blocks are eliminated from Figure 5. The system simply consists of an errors-only decoder and a device to compute the score for each dwell and generate retransmission requests accordingly. Of course, removing erasure generation and erasure correction from the system results in a degradation in performance, and thus there is a tradeoff between throughput and delay/complexity.

The iterative decoding operation may be used with HARQ systems as well. Here, however, each time a packet failure occurs, the entire packet is retransmitted. Thus, scores are used for erasure generation only.

V. NUMERICAL RESULTS

Simulation results are presented for a system that uses $(32, k)$ RS codes with ten codewords in a packet. The dwell retransmission scheme is again compared to HARQ systems that use the same and lower rate codes. Both the dwell retransmission scheme and HARQ are assumed to take advantage of error counting, erasure generation, and iterative decoding. For both schemes, the best performance is achieved

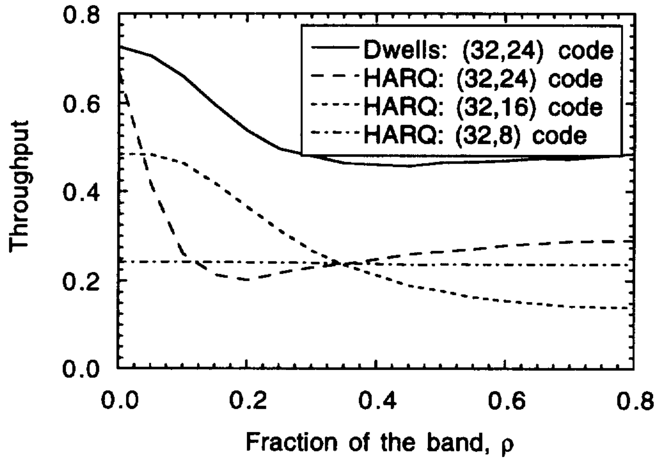

Fig. 6. System performance, $E_S/N_0 = 10\text{dB}$, $E_S/N_I = 12\text{dB}$.

when the erasure threshold is $t^* = 10$. Numerical results are presented as a function of E_S, the energy per *channel* symbol. In comparing systems with multiple code rates, we assume that the channel symbol rate and channel symbol energy are fixed constants, and the transmitter simply sends less data (and more redundancy) when the code rate is reduced.

Figure 6 presents the performance of the dwell retransmission scheme and HARQ in partial-band interference as a function of the fraction of the band with interference, ρ. The dwell retransmission scheme is seen to provide significant performance improvement. As with the models discussed in Section 3, the performance of the dwell retransmission scheme with the (32,24) code is better than HARQ with the same and lower code rates.

In this figure it can be seen that the worst-case interference (or jamming) for the dwell retransmission scheme occurs at approximately $\rho = 0.4$, and the worst-case interference for HARQ using the (32,24) code occurs at approximately $\rho = 0.2$. The resulting worst-case throughput of HARQ is less than half that of the dwell retransmission scheme. For HARQ with the other code rates, the performance is monotonic, and the worst-case interference occurs for $\rho = 1$ (broadband jamming). HARQ performance with the (32,8) code is especially insensitive to ρ. It is interesting to note that the worst-case HARQ performance of the (32,8) code is slightly better than that of the other codes (but still much worse than the performance of the dwell retransmission scheme).

Figure 7 presents the performance as a function of E_S/N_I with ρ fixed at 0.2. It is seen that the dwell retransmission scheme provides roughly a 10dB gain over HARQ performance at a throughput of 0.6. The performance of the dwell retransmission scheme is significantly better than that of HARQ unless interference levels are extremely high (E_S/N_I small). The dwell retransmission scheme can still outperform HARQ if a lower code rate is used with the dwell retransmission scheme.

Figure 7 shows that HARQ performance can be non-monotonic as a function of E_S/N_I. This may appear to be surprising, but non-monotonic performance of FH systems

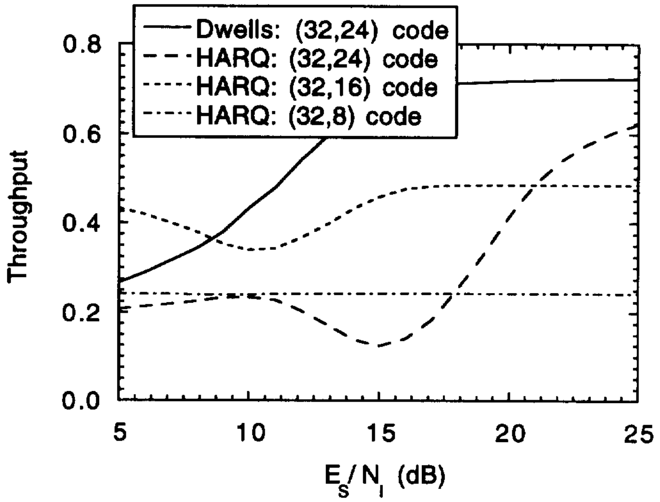

Fig. 7. System performance, $E_S/N_0 = 10\text{dB}$, $\rho = 0.2$.

subject to partial-band interference has been observed elsewhere [12]. This is due in part to the fact that strong interference is easier to detect than moderate interference. The iterative decoding algorithm attempts to detect and erase dwells corrupted by interference, and increases in interference levels may increase the capabilities of the decoding algorithm in this task.

In conclusion, we have shown that the performance of FH communication systems can be improved significantly by using dwell retransmission protocols. Furthermore, the use of error counting is an effective way to determine which dwells should be retransmitted. Because receiver processing requirements are relatively simple, the use of such protocols should be considered for future applications.

REFERENCES

[1] S. B. Wicker, *Error Control Systems for Digital Communication and Storage*. Prentice Hall, 1995.

[2] C. W. Baum and A. A. Daraiseh, "Performance analysis of Reed-Solomon encoded hybrid ARQ frequency-hop systems subject to frequency selective interference and fading," *IEEE Intern. Symp. Inform. Thy. Applic.*, pp. 669–672, 1996.

[3] R. A. Comroe and D. J. Costello, "ARQ schemes for data transmission in mobile radio systems," *IEEE J. Sel. Areas Commun.*, pp. 472–481, July 1984.

[4] S. Lin, D. J. Costello, and M. J. Miller, "Automatic-repeat-request error-control schemes," *IEEE Commun. Mag.*, pp. 5–17, Dec. 1984.

[5] R. J. Benice and A. H. Frey, "An analysis of retransmission schemes," *IEEE Trans. Commun. Tech.*, pp. 135–145, Dec. 1964.

[6] J. Metzner, "Improvements in block retransmission schemes," *IEEE Trans. Commun.*, pp. 524–532, Feb. 1979.

[7] J. Metzner and D. Chang, "Efficient selective-repeat ARQ strategies for very noisy and fluctuating channels," *IEEE Trans. Commun.*, pp. 409–416, May 1985.

[8] P. Sindhu, "Retransmission error control with memory," *IEEE Trans. Commun.*, pp. 473–479, May 1977.

[9] D. M. Mandelbaum, "An adaptive feedback coding scheme using incremental redundancy," *IEEE Trans. Inform. Thy.*, pp. 388–389, May 1974.

[10] H. Krishna and S. Morgera, "A new error control scheme for hybrid-ARQ systems," *IEEE Trans. Commun.*, pp. 981–990, 1987.

[11] S. Lin and P. S. Yu, "A hybrid-ARQ scheme with parity retransmission for error control of satellite channels," *IEEE Trans. Commun.*, pp. 1701–1719, July 1982.

[12] C. W. Baum and M. B. Purlsey, "Bayesian methods for erasure insertion in frequency-hop communication systems with partial-band interference," *IEEE Trans. Commun.*, pp. 1231–1238, July 1992.