

THE PERFORMANCE ENHANCEMENT OF SEVERELY IMPAIRED UNGUIDED LASER COMMUNICATION SYSTEMS USING TURBO CODES

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I. Abstract

Parallel concatenated convolutionally coded (PCCC) optical binary pulse-position modulation (PPM) communication systems are introduced and investigated. It is assumed that the laser channel is an intensity modulated (IM) channel and that the received optical signal is detected using a shot-noise limited direct-detection (DD) scheme. With the aid of the best available upper bounds, the performance of rate $\frac{1}{n}$ PCCC encoded optical PPM systems is assessed in terms of the upper bound on the system bit error rate (BER) for shot noise limited IM/DD channels. The numerical results demonstrate the enormous potential of this novel coding scheme in enhancing the performance of the aforementioned optical channel by a sizeable margin across the board.

II. Introduction

In 1993, a low complexity channel coding scheme, known as Turbo coding, was introduced by Berrou *et al.* [1]. This remarkable coding scheme, which has also been referred to as parallel concatenated convolutional coding, yields an astonishing performance at low signal-to-noise ratio (SNR) levels (bit error rates as low as 10^{-6} for a fraction of a dB have been reported). This coupled with the low complexity of this coding scheme makes Turbo coding ideal for many communications applications with power or energy constraints. The objective of this study is to explore the potential of PCCC encoding scheme in enhancing the perfor-

mance of photon communication channels. Although fiber-optic channels enjoy large enough SNR levels that guarantee almost error free communication (i.e., BER $< 10^{-9}$), other photon channels, such as deep-space and inter-satellite laser channels, as well as wireless infrared channels, are often impaired by non-ideal channel/receiver conditions, and thus such systems can benefit from low complexity coding schemes with large performance gains. In particular, the aforementioned channels are often subjected to pulsewidth constraint, which in conjunction with energy constraint, imposes a lower limit on the error performance of such systems even in the absence of other performance-limiting impairments. Equivalently, the use of novel coding techniques with large coding gains will reduce the required signal energy to achieve a given bit or symbol error rate. This further implies that lower laser power levels will be needed, which ultimately enhances the life expectancy of the laser system. Moreover, a reduction in the required power level is of critical importance to a low-power wireless computing platform.

III. System Model: Shot Noise Limited Photon Channels

Although many different types of PCCC's have been proposed in the literature in recent years, without loss of generality, we limit our analysis to the rate $\frac{1}{n}$ codes where every source data bit is mapped to n coded symbols, respectively. The source provides the PCCC encoder with binary symbols (if a non-binary source is used, it is assumed that the source

symbols can be mapped to binary symbols). The source bits are independently interleaved and encoded by the $n-1$ constituent convolutional (CC) encoders¹. Hence, the encoder emits n bits for every input source bits. The sequence of channel bits provided by the encoder is then mapped to one of the M modulator symbols. We assume that $M = 2$. The modulator is assumed to be the type appropriate for intensity modulated photon communications. This includes PPM scheme commonly used for intensity modulated/direct-detection (IM/DD) optical communications. Let N denote the length of the interleavers. The bits from the input are interleaved before being fed into the convolutional encoders. The optical signal for PPM may be expressed as $\lambda_{PPM}(t) = \lambda_s \sum_{k=-\infty}^{\infty} \left[x_k P(t - kT_s) + \bar{x}_k P(t - kT_s - \frac{T_s}{2}) \right]$, where T_s is the duration of a coded symbol in sec, $P(t)$ is a unit amplitude non-return-to-zero (NRZ) pulse of duration $\frac{T_s}{2}$ sec, λ_s denotes the peak signal intensity in photons/sec, x_k is the binary symbol provided by the encoder to the modulator taking on $\{0, 1\}$, and \bar{x}_k denotes the inverted version of the binary symbol. We have elected this somewhat unusual representation to avoid introducing additional notation. In this analysis, we consider finite length sequences that are presented to the decoder for the recovery of the source symbols. Specifically, we consider a size nN sequence of received symbols. Since rate $1/n$ PCCC's are considered, N denotes the number of transmitted information bits. We consider a situation where the trellis of the encoders are terminated using a simple, yet effective technique proposed in [2]. Let $\underline{d} = [d_1, d_2, \dots, d_N]$ denote a binary data vector of length N taking on $\{0, 1\}$ that is fed into the encoder. The encoded sequence of symbols generated by the encoder may be represented by a size $L = nN$ vector $\underline{c} = [c_1, c_2, \dots, c_N]$ where c_i is a row vector of length n , representing the output of the PCCC in response to d_i . Note that we have not considered the terminating sequence as a part of the information block. This further implies that total number of symbols generated by the encoder in response to

a block of size N of data is a sequence of L coded symbols. Consequently, the modulator generates a sequence of signal intensities which may be represented by $\underline{\Delta} = [\Delta_1, \Delta_2, \dots, \Delta_N]$ where Δ_i is a vector of signal intensities representing c_i .

Since a direct-detection receiver is considered, the intensity of the received optical field is of interest. Let $\lambda_{rec} = \lambda_{PPM}(t) + \lambda_b$ denote the received signal intensity for the PPM system, with λ_b representing the noise intensity in photons/sec due to background radiation or dark current of the detector. For a shot noise limited channel, the detected signal may be modelled as a Poisson point process. One can show that the sufficient statistic for rendering decisions regarding the transmitted symbol here is the number of photons received in an observation interval [3]. Let $\underline{\Gamma} = [\underline{\Gamma}^{(1)}, \underline{\Gamma}^{(2)}, \dots, \underline{\Gamma}^{(n)}]$ where $\underline{\Gamma}^{(j)}$ ($j \neq 1$) is a row vector whose elements are selected from a set of observed photon counts [3] due to the j th constituent encoder. Obviously, $\underline{\Gamma}^{(1)}$ is a vector of photon counts associated with the uncoded data symbols. Since a shot noise limited IM/DD channel is considered², photon counts over non-overlapping intervals are independent random variables when conditioned on the generating data sequence. That is, $p(\underline{\Gamma}|\underline{d}) = \prod_{i=1}^n p(\underline{\Gamma}^{(i)}|\underline{d})$, where $p(\underline{\Gamma}|\underline{d})$ denotes the probability mass function (PMF) of $\underline{\Gamma}$ conditioned on \underline{d} . For a PPM channel, the code symbol time T_s is divided into two chip intervals, each of duration $\frac{T_s}{2}$ sec. In that event, the observed photon count vector $\underline{\Gamma}^{(j)}$ may be expressed as $\underline{\Gamma}^{(j)} = [\Gamma_{11}^{(j)}, \Gamma_{12}^{(j)}, \Gamma_{21}^{(j)}, \Gamma_{22}^{(j)}, \dots, \Gamma_{N1}^{(j)}, \Gamma_{N2}^{(j)}]$ where $\Gamma_{qm}^{(j)}$ is the photon count over the m th chip ($m = 1, 2$) of the q th ($q = 1, 2, \dots, N$) symbol generated by the j th ($j \neq 1$) constituent encoder. That is, in order to perform soft decoding, one requires to obtain $2N$ photon counts associated with each CC for the PPM case. This amounts to a total of $2L = 2nN$ (excluding any tail bits) photon counts for a rate $1/n$ PCCC encoder. Obviously, an optimum decoder compares the total photon count associated with a particular bit stream (or trellis) with other possible sequences' photon counts to render a decision in favor of that sequence. Motivated by this argument, one may

¹Although it is always possible to generate a rate $1/n$ PCCC code using n identical CC's, in general, one can generate such a code with less than n CC's, given that one can guarantee n independent interleaving operations.

²For a thermal noise-limited channel, a similar argument holds.

use the Transfer Function performance bounding technique to establish performance. To begin, let us consider the codeword generated by a constituent coder as a code fragment[4]. Moreover, let $T^{(k)}(L, I, D)$ denote the weight enumerating function (WEF) for the k th constituent encoder, defined as [5, 4] $T^{(k)}(L, I, D) = \sum_{l=1}^{\infty} \sum_{i=0}^{\infty} \sum_{d=0}^{\infty} t^{(k)}(l, i, d) L^l I^i D^d$, where $t^{(k)}(l, i, d)$ denotes the number of code fragments of length l with output Hamming weight d whose generating sequence is of Hamming weight i . Since we are interested in length N code fragments, hereafter $l = N$. Then, the conditional WEF for code fragments of length N may be defined as $T_{N,C}^{(k)}(i, D) = \sum_{d=0}^N t^{(k)}(N, i, d) D^d$, which implies that $T_N^{(k)}(I, D) = \sum_{i=0}^N \sum_{d=0}^N t^{(k)}(N, i, d) D^d I^i = \sum_{i=0}^N T_{N,C}^{(k)}(i, D) I^i$, where $T_N^{(k)}(I, D)$ is the WEF for all code fragments of length N . As has been observed by other investigators, the design of interleaver(s) is critical to the overall performance of PCCC coded systems. This, however, hampers any interleaver-independent analysis of a PCCC coded system. Fortunately, in [5], it is shown that one may consider a so-called "uniform interleaver" to carry out a performance analysis of PCCC coded systems. More importantly, in that study, it is demonstrated that the resulting performance measure provides an ensemble average of the overall bit or symbol error rate of PCCC coded systems when averaged over the choice of interleaver. We take a similar approach here and consider a rate 1/3 code. Then, the conditional WEF of a rate 1/3 PCCC coded system, $T_{N,C}^{PCCC}(i, D)$, may be expressed as[5] $T_{N,C}^{PCCC}(i, D) = \frac{T_{N,C}^{(k)}(i, D) T_{N,C}^{(k)}(i, D)}{\binom{N}{i}}$, where $\binom{N}{i} = \frac{N!}{(N-i)!i!}$. Hence, $T_{N,C}^{PCCC}(i, D) = \frac{1}{\binom{N}{i}} \sum_{d_1=0}^{\infty} \sum_{d_2=0}^{\infty} t^{(1)}(N, i, d_1) t^{(2)}(N, i, d_2) D^{d_1} D^{d_2}$. Finally, the WEF for the rate 1/3 PCCC encoder when code fragments of length N are only considered may be expressed as

$$T_N^{PCCC}(I, D) = \sum_{i=0}^{\infty} \frac{I^i}{\binom{N}{i}} \sum_{d_1=0}^{\infty} \sum_{d_2=0}^{\infty} t^{(1)}(N, i, d_1) t^{(2)}(N, i, d_2) D^{d_1} D^{d_2}. \quad (1)$$

Subsequently, we have

$$T_{N,C}^{PCCC}(i, D) = \sum_{d_1=0}^N \sum_{d_2=0}^N C(i, d_1, d_2) D^{d_1+d_2} = \sum_{d=0}^{2N} B(i, d) D^d \quad (2)$$

where $C(i, d_1, d_2) = \frac{1}{\binom{N}{i}} t^{(1)}(N, i, d_1) t^{(2)}(N, i, d_2)$ and $B(i, d) = \sum_{d_1=0}^N \sum_{d_2=0; d_1+d_2=d}^N C(i, d_1, d_2)$. One may view $B(i, d)$ as the number of codewords, generated by a weight i information sequence, whose redundant bit stream has a weight of d . Such codewords have a total Hamming weight of $i + d$, and are referred to as the (i, d) codewords hereafter.

For a PPM channel, then, the bit error rate (rate 1/3) PCCC coded system may be upper bounded as

$$P_b^{(PPM)} \leq \sum_{i=0}^N \sum_{d=0}^{2N} \frac{i}{N} B(i, d) \sum_{k_1=0}^{\infty} \sum_{k_2=k_1}^{\infty} \kappa(k_1, k_2) POS(k_2, (d+i)(K_s + K_b)) \times POS(k_2, (d+i)K_b) \quad (3)$$

where $POS(m, n) = \frac{n^m}{m!} \exp(-n)$, $K_b = \frac{\lambda_n T_n}{2}$, $K_s = \frac{\lambda_s T_s}{2}$, and $\kappa(k_1, k_2) = \begin{cases} 1 & k_1 \neq k_2 \\ 1/2 & k_1 = k_2 \end{cases}$. To elaborate, we consider an all-zero input information sequence, hereafter referred to as the $(0, 0)$ path, which will generate an all-zero output sequence (assuming that the CC's are all initialized). Now, let us consider a codeword generated by a weight i sequence. Then, i non-zero information bits will be incorrectly detected if the total photon count associate with the (i, d) path through the trellis for the corresponding codeword is less than that of the $(0, 0)$ sequence. Realizing that the photon counts over the chips associated with the desired path (which has a redundant weight of d , and hence a total weight of $d + i$) and those of the $(0, 0)$ path are only different in $(d + i)$ positions, the total photon count of the $(0, 0)$ path will exceed that of the correct path with the following probability:

$$\sum_{k_1=0}^{\infty} \sum_{k_2=k_1}^{\infty} \kappa(k_1, k_2) POS(k_2, (d+i)K_b) \times POS(k_2, (d+i)(K_s + K_b)).$$

The factor $\kappa(k_1, k_2)$ is included to account for the case where the two photon counts are identical, which with probability 1/2 will result in an error. This, in turn, will lead to an upper bound on the expected BER by first acquiring the expected number of bits in error and then dividing the resulting expression by the number of information bits (N). Since there are $B(i, d)$ codewords generated by weight- i input sequences, we arrive at (3). The remaining task is to obtain $B(i, d)$ for a specific CC. This task, however, is postponed to Section IV. Incidentally, one may define $p^{(m)}(N, i, d_m) = \frac{1}{\binom{N}{i}} t^{(m)}(N, i, d_m)$; $m = 1, 2$, as the probability of occurrence of a code fragment of weight d_m , generated by the m th constituent encoder, when the input information sequence has weight i . Then,

$$P_b^{(PPM)} \leq \sum_{i=0}^N \sum_{d_1=0}^N \sum_{d_2=0}^N \frac{i}{N} \binom{N}{i} p^{(1)}(N, i, d_1) \times \\ p^{(2)}(N, i, d_2) \sum_{k_1=0}^{\infty} \sum_{k_2=k_1}^{\infty} \kappa(k_1, k_2) \times \\ POS(k_2, (d+i)K_b) POS(k_2, (d+i)(K_s + K_b)).$$

This expression is quite similar to that observed in [4] for Gaussian channels. Finally, we can extend the previous results to the case of $1/n$ rate PCCC coded systems. Note that the key parameter for this extension is $B(i, d)$. That is, for a rate $1/n$ PCCC coded system,

$$P_b^{(PPM)} \leq \sum_{i=0}^N \sum_{d=0}^{(n-1)N} \frac{i}{N} B(i, d) \sum_{k_1=0}^{\infty} \sum_{k_2=k_1}^{\infty} \\ \kappa(k_1, k_2) POS(k_2, (d+i)K_b) \times \\ POS(k_2, (d+i)(K_s + K_b)). \quad (4)$$

Note that the total weight of the redundant bits may be as large as (in the absence of tail bits) $(n-1)N$. Finally, since BER less than 10^{-3} is of interest, one may approximate (4) further using an upper bound given in [6].

IV. Numerical Results

We consider a specific CC whose transfer function may be expressed as the ratio of two octal numbers

$5(1+D^2)$ and $7(1+D+D^2)$, i.e., 5/7 [4]. The reason for its selection is its low complexity and good gain as compared to other similar codes [4], although many other codes with appreciable coding gains have been introduced [7]. Without belaboring the details, $B(i, d)$ for 5/7 code may be obtained using a technique described in [4]. In the following, we consider a rate 1/3 PCCC encoder. First, we begin with the Poisson channel case. Figs. 1 and 2 depict the upper bound given by (4) (using the approximation given in [6]). For low to moderate (10–50) and large (100–200) background noise photon counts, similar to the Gaussian case, a substantial improvement in performance is obtained by simply changing the interleaver length N (code rate remains constant). This is a remarkable attribute of this coding technique. For example, for $K_b = 10$, one may achieve an upper bound on the system BER better than 10^{-7} using an interleaver of length $N = 100$ with $K_s = 5$. Note that, for the same K_b and K_s , an upper bound on the system BER of only 10^{-3} is achievable with $N = 10$. For an unusually high background noise count of 200 (signal completely merged in background noise), one may achieve an amazing upper bound on the system BER of 10^{-6} for $N = 200$ using only K_s of 15. In Fig. 1, we have also depicted the performance of an uncoded PPM system for comparison purposes. Since most laser systems operate under a fixed power level constraint (i.e., fixed λ_s), one has to scale K_s and K_b by a factor of 3 to account for the code rate of 1/3 in comparing coded and uncoded systems, since for a fixed power the energy must be scaled as a result of coding. To elaborate, the dash-dotted line in Fig. 1 represents the bit error rate for the uncoded PPM with a background radiation of 30 photons (for comparison to $K_b = 10$) and the signal photon count 3 times the K_s axis. From coding gain perspective, one can determine the magnitude of the laser power that can be saved for a given performance measure as a function of the interleaver length. As expected, a remarkable coding gain can be observed with an increase in N without any major modification in the coding structure. Interestingly enough, the performance of the uncoded system overlaps that of the coded system with $K_b = 50$ (i.e., a system with 5 times the λ_b of the uncoded system for identical laser power

levels) for a code with $N = 10$. Now, if one compares the systems with identical λ_s and λ_b (i.e., dashed and dash-dotted) for coded and uncoded scenarios, a remarkable improvement in performance is observed. In fact, when coded and uncoded systems are compared, and for an upper bound on the system BER of about 10^{-6} , a reduction by a factor of 4 in λ_s can be tolerated using PCCC coding with $N = 50$. This implies a saving of 6 dB in laser power using this coding scheme and $N = 50$. This number increases with N without altering the code rate.

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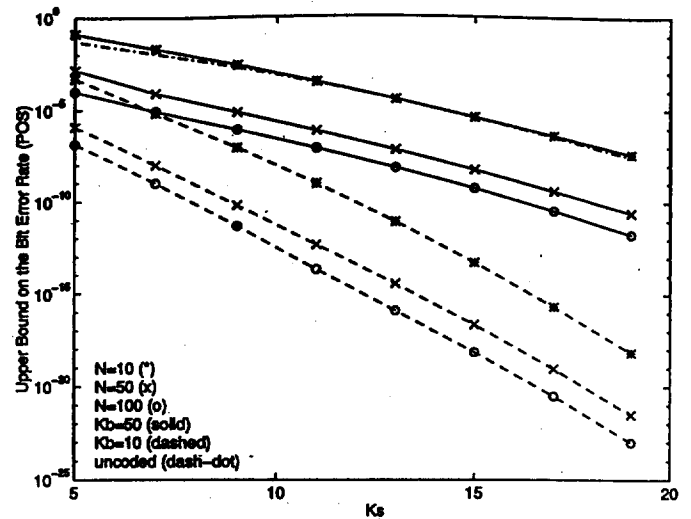


Figure 1 - Upper bound on the bit error rate of the PPM-modulated PCCC system versus the average number of signal photons K_s (shot noise-limited scenario).

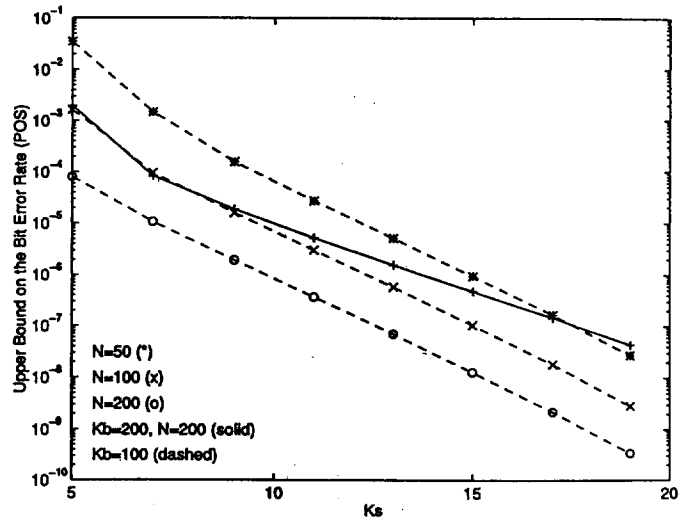


Figure 2 - Upper bound on the bit error rate of the PPM-modulated PCCC system versus the average number of signal photons K_s (shot noise-limited scenario)