Minimum Mean Square Error (MMSE) Receiver Employing 16-QAM in CDMA Channel with Narrowband Gaussian Interference

Prasad Shamain and Laurence B. Milstein
ECE Dept., University of California at San Diego, La Jolla, CA 92093-0407.

Abstract: It has been shown that employing a higher order constellation in conjunction with CDMA can improve the performance of the multiuser system in severe multipath channels. We verify that the performance improvement gained by the above interference suppression technique in the multiuser case continues to hold in the multiuser environment with narrowband Gaussian interference.

I. INTRODUCTION

Out of various novel structures of a CDMA receiver, the adaptive MMSE receiver [1] has been, perhaps, the most attractive due to its simplicity and ease of implementation. We showed that a system employing 16-QAM in conjunction with CDMA can outperform one employing QPSK when severe multipath causes significant ISI [2]. We study here the advantage of employing 16-QAM instead of QPSK in a severe multipath multiuser environment which also includes narrowband interference.

Pateros and Saulnier [3] studied an adaptive correlator receiver for a DS-SS system employing BPSK signalling in a single user, time-invariant, multipath environment that has a continuum of multipaths but causes no significant ISI. They found that an MMSE receiver detected the transmitted data, removed interference and coherently combined multipaths in the presence of interference undergoing a static single-path channel. We generalize the above channel model to a multiuser, frequency selective channel, where the delay spread may exceed the symbol duration. Since the users and the interferer may be colocated, we extend the multipath channel model to the interfering signal. In fact, we compare two equal data-rate, equal bandwidth, equal $E_s/N_0$ (bit-energy/noise-power) systems, namely - the 16-QAM and the QPSK systems. Some of the delay path profiles are such that their duration exceeds the symbol duration of the QPSK system, but not that of the QAM system. This condition introduces severe ISI for the former system. We compare both systems in the following three scenarios: single user, single path; single user, multipath; and multiuser, multipath. We derive an upper bound on the performance of an MMSE receiver in a multiuser, flat fading scenario. We apply the same technique to a multiuser, frequency selective fading scenario with negligible or mild ISI.

II. SYSTEM MODEL

We assume a direct-sequence CDMA system with two users active at any time. This is a likely scenario in a wireless LAN (WLAN) which caters to bursty data communication. Even though the MMSE receiver is known for its robustness against mismatched power, we will not try to exploit that aspect in this paper, since, at pedestrian speeds, the power control loop is expected to compensate for such power mismatch. Hence, we assume perfect power control. We also assume a multipath Rayleigh fading channel with $L$ resolvable paths, and we use the standard WSSUS fading model with a flat multipath intensity profile (MIP) [4]. We compare the performance of the 16-QAM and QPSK systems in the following manner: We keep transmitted symbol power, data rate and system bandwidth the same. Due to the latter constraint, we use length 31 Gold codes with the 16-QAM system and length 15 Gold-like codes for the QPSK system [5].

A. Channel Model

We use the two-cluster multipath model of [6]. In this model, we assume that the paths appear in two clusters such that the paths in any one cluster are separated by exactly one chip duration. The beginning of each cluster is governed by a delay random variable and is described in [7]. We consider various scenarios described by differing number of paths in the two clusters, as well as differing relative delays between the paths. Each additional path contributes primarily to the diversity gain of the QAM system. However, in the case of the QPSK system, each additional path may contribute primarily to either diversity gain or intersymbol interference (ISI), depending on its delay from the first path. The extent of either the ISI or the diversity depends on the partial autocorrelation properties of the spreading sequences. Table 1 summarizes the relevant information for all four patterns of multipaths under consideration. We assume the narrowband interference is transmitted over a similar channel.

B. Receiver Structure

In order to facilitate the analysis, we assume rectangular chip shapes. The receiver consists of a chip matched filter followed by a tapped delay line [Fig. 1 in [8]]. The 16-QAM constellation with 46 (=31+15) taps and the QPSK constellation with 30 (=15+15) taps (number of taps being equal to the processing gain plus the delay spread) are described by the following two equations, respectively:

$$d_1(t) = p(t)\pm(2k_1 - 1)\pm j(2k_2 - 1)\quad \forall \quad k_1 = 1, 2, \quad k_2 = 1, 2;$$

$$d_2(t) = \sqrt{p(t)}(1 \pm j1),$$

where $p(t)$ is a rectangular pulse.

We borrow all the notation from [2]. The contents of the tapped delay line due to user 1, user 2, narrowband interference and AWGN are denoted by $s_1$, $s_2$, $J$ and $n$, respectively. Mathematically, the received vector is represented as

$$r = s_1 + s_2 + J + n.$$ 

Since the chip matched filtering and mathematical formulation of $s_1$, $s_2$ and $n$ are described in detail in [2], we describe the formulation of the narrow-band interference vector –

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where

\[ \Phi(f) = \begin{cases} N_0 & \text{if } |f| \leq \frac{B}{2}, \\ 0 & \text{if } |f| > \frac{B}{2}. \end{cases} \]

Alternately, it has the following autocorrelation function:

\[ \phi(\tau) = N_0 \frac{\sin(\pi B \tau)}{\pi \tau}. \]

The complex impulse response of the multipath channel is represented by

\[ \beta_J(\tau) = \sum_{l=1}^{L} (\alpha^*_l + j \alpha^*_l) \delta(\tau - \tau_{jl}), \]

where \((\alpha^*_l + j \alpha^*_l)\) and \(\tau_{jl}\) are the complex Gaussian amplitude and the time-delay of the \(l^{th}\) multipath, respectively. The continuous-time signal at the output of this channel is given by

\[ J(t) = \sum_{l=1}^{L} (\alpha^*_l + j \alpha^*_l) n_{NB}(t - \tau_{jl}). \]

We carry out all signal processing in discrete time. Hence chip matched filtered samples are given by

\[ J[n] = \int_{(n-1) T_c}^{n T_c} J(t) \, dt \quad \forall n = 1, \ldots, N, \]

where \(N\) is the number of taps and \(T_c\) is the chip duration. Let us construct \(J\) as shown below:

\[ J = \begin{bmatrix} J[1], \ldots, J[N] \end{bmatrix}. \]

We assume that we have some means of estimating fades of the desired user (i.e., user 1) accurately and these fades remain constant during a symbol duration. In order to get the optimum coefficient vector, we solve the following Wiener Hopf equation:

\[ c = R^{-1} p, \]

where \(R = E[p^H p]\) and \(p = E[p^H d_e(0)]\), where \(d_e(0)\) is the desired data signal. Exploiting the statistical independence among various r.v.s, we can write

\[ R = R_1 + R_2 + R_J + R_n, \]

where \(R_1, R_2, R_J\) and \(R_n\) are autocorrelation matrices of \(s_1, s_2, J\) and \(n\), respectively.

The formulation of \(R_1, R_2\) and \(R_n\) is described in detail in [2]; and the elements of \(R_J\) are given by [7]

\[
[R_J]_{ik} = \frac{N_0}{\pi} \left[ T_c (k - i + 1) \sin(\Omega (k - i + 1)) + 2 T_c (k - i) \sin(\Omega (k - i)) - T_c (k - i - 1) \sin(\Omega (k - i - 1)) + \frac{1}{\pi B} \cos(\Omega (k - i + 1)) - \cos(\Omega (k - i)) - \cos(\Omega (k - i - 1)) \right],
\]

where \(\Omega = \pi B T_c\) and \(\sin(\cdot) \triangleq \int_0^T \frac{\sin(t)}{t} \, dt\).

**B. Test Statistics**

The test statistic, \(g\), is formed by taking the real part of the inner product of the tap-coefficient vector, \(c\), and the received vector, \(r\):

\[ g = \Re[r^H c] = \Re[(s_1 + s_2 + J + n) c]. \]

We form the test statistics under two sets of assumptions that are designated as “fast fading” and “slow fading”, respectively. In either case, the time-delays associated with the user as well as with the interferer change slower than do the fades. Hence they are assumed to be accurately known. For fast fading, the fades of the undesired user and the interference are assumed not to be tracked by the receiver, and hence treated as r.v.s. It has been shown in [7] that for this scenario, conditioned on the data of the desired and interfering users and the delay r.v.s described above, the component of test statistic due to the narrowband Gaussian interference is a conditional Gaussian r.v. as the number of multipaths increases asymptotically. Note that, strictly speaking, we have non-Gaussian test statistic, as the contents of the delay elements are given by the product of two Gaussian distributed r.v.s, namely, the complex Gaussian fades and the chip matched samples of the narrowband Gaussian interference. We bypass this difficulty by applying the above asymptotic result.

As the dynamics of fading processes are the same for the desired user as well as for the interfering user, the assumption about “fast fading” cannot be justified per se. However, it simplifies the analysis and provides an upper bound on the performance. In “slow fading”, we take a realization of the fading, multi-input channel and assume that the fades remain constant throughout the transmission of a given data-frame.

We will denote the mean of test statistic by \(m\). By exploiting the fact that the fades and noise are zero-mean r.v.s, we have

\[ m = E[g] = E[\Re(r^H c)] = \Re[s_1 c]. \]

We denote the variance of the test statistic by \(\text{var}\). By exploiting the statistical independence among r.v.s, we have

\[ \text{var} = E[g^2] = E[\Re(r^H c)]^2 = \frac{1}{2} \left( c^H (R_2 + R_J + R_n) c \right). \]

**IV. Performance Evaluation**

The metric for performance is \(P_{	ext{err}}\), defined in [2] as the error rate in detecting a voltage level in either the sine or the
cosine channel. Mathematically,
\[ P_{ES} = 2P_{eL} - P_{eL}^2, \]  
(16)
where is \( P_{ES} \) is the probability of symbol error. We approach this problem under two sets of assumptions:

1. **Analytical Approach**: We assume here that all the delay r.v.s in the channel are identical to zero. This makes the multiuser system synchronous and all the multipaths appear at integer multiples of \( T_c \) spacing. We employ the “fast fading” assumption and can analytically average over the fades of the desired user.

2. **Semi-analytical Approach**: We follow here the channel model described in Section II that undergoes “slow fading.” This approach is summarized by
   - taking a particular realization of the multi-input channel,
   - finding the optimum tap weights,
   - finding the probability of error by averaging over all possible data patterns,
   - repeating the same procedure for a number of realizations of the channel.

We take up to 2000 realizations of channels. Each realization (fades as well as time delays) includes a realization of the channel for the desired user and the interfering user, as well as for the narrowband interferer. In the semi-analysis that is used for “slow fading”, we assume that all the fades are constant and the optimum coefficients are obtained using this knowledge about the fades.

**A. Flat Fading Single User**

i. **Analytical Approach**

We can obtain the following lower and upper bounds on \( P_{eL} \) [see [7] for the derivation]:
\[ \frac{1}{\pi} \prod_{l=1}^{2L} \frac{1}{1 + \lambda_l} \int_0^{\pi/2} \sin^{2L}(\theta) d\theta \leq P_{eL}(QPSK) \leq \frac{1}{2} \prod_{l=1}^{2L} \frac{1}{1 + \lambda_l}, \]  
(17)
where \( \lambda_l \), \( l=1 \) to \( 2L \), are the eigenvalues of a matrix that depends on \( R^{-1} \). Note that \( R = R_2 + R_3 + R_n \) and \( R_2 \) is a zero matrix here as it is a single user system. Note also that the above \( P_{eL} \) is derived for QPSK signalling, which can be then extended to higher level signalling by using the Eqn.(5-2-42) in [9]. Specifically, for the 16-QAM (treated as two 4-PAM systems), we have
\[ P_{eL}(16 - QAM) = \frac{3}{2} P_{eL}(QPSK). \]  
(18)

ii. **Semi-analytical Approach**

Since \( R_3 \) now depends on the delay r.v.s, we resort to the sample averaging referred to above.

**B. Frequency Selective Fading Single User**

i. **Analytical Approach**

We assume that all multipaths appear at integer multiples of \( T_c \). Under this assumption, we can obtain lower and upper bounds using Eqn. (17). As described in [7], the upper bound is really an approximation, since it only holds when ISI is negligible.

ii. **Semi-analytical Approach**

Since \( R_3 \) again depends on the delay r.v.s, we resort to sample averaging.

**C. Frequency Selective Fading Multiuser**

i. **Analytical Approach**

With the assumption of a synchronous CDMA system, and with the same assumptions as in the single user case, we can extend the analysis to the multiuser scenario. However, due to the presence of the interfering user \( \mathbf{R}_2 \) in (3), we cannot assume that the test statistic (conditioned on data and fades of desired user) to be conditionally Gaussian. We show in [7] that for a flat multipath intensity profile, the component of the test statistic due to this additional user is an asymptotically Gaussian r.v. in terms of the number of multipaths.

ii. **Semi-analytical Approach**

Again, \( R_2 \) and \( R_3 \) depend on the delay r.v.s. Hence we resort to sample averaging.

**V. Results**

In this section, we compare the results obtained by the analytical and the semi-analytical approaches. The narrowband Gaussian interferer occupies a fraction (denoted as BW in the figures) of the system bandwidth and it has a power spectral density (PSD) \( N_o \). The relevant values of these quantities are given in the figures. In the case of the semi-analysis, we assume that the receiver has been synchronized to the first path of the desired user. A point to be noted here is that the bounds are obtained for a synchronous CDMA system. In an asynchronous CDMA system, the sampling time may not be optimized for the subsequent paths. This may reduce the diversity gain achievable in the asynchronous system. We have, in the following plots, the lower bound (LB) and the upper bound (UB) for the synchronous system, as well as results from the semi-analyses for both the synchronous (SA-Synch) and the asynchronous (SA-Asynch) systems. We verify the validity of the bounds by comparing them with the semi-analysis for the synchronous system.

**A. Single User Flat/Multipath Channel**

We compare the QPSK and the QAM systems for narrowband Gaussian (NBG) interference. We show in Fig. 1, the validity of the bounds (the QAM system) for flat fading, for selective fading with profile 2, and for selective fading with profile 4 (see Table [1]), that result in no ISI, negligible ISI and mild ISI, respectively. The bounds are obtained under the assumption of
fast fading. Hence, it may happen that the semi-analysis, under the assumption of slow fading (thereby exploiting the MAI suppression capability), results in a better performance than the lower bound. Empirically, we find that the lower bounds hold except when $P_{e_L}$ is lower than $10^{-5}$.

We compare the performance of the QPSK and the QAM systems in the single-user system in Fig.2. We observe that in severe ISI, the QAM system outperforms the QPSK system at high SNR. In our channel models, we have discrete multipaths, and the sampling time of the receiver is optimized only for the first incoming path. Due to random delays of the subsequent paths, such a receiver will suffer diversity loss due to the non-optimal sampling time. However, the advantage of this design is that it simplifies the receiver structure so that we do not need a separate finger for each path being demodulated. We assess the impact of this arrangement for narrowband Gaussian interference in Fig.3. We observe that the performance loss is negligible.

B. Multiuser Multipath Channel

We find that, as in the single-user case, the bounds hold in the mild/negligible ISI scenario, as shown in Fig.4. We observe in Fig. 5 the validity of the assumption about the Gaussian nature of the MAI component of the test statistic. Finally, we compare the performance of the QPSK and the QAM systems for the two-user system in Fig.6, and observe that in a severe ISI as well as mild ISI situations, the QAM system outperforms the QPSK system at high SNR.

VI. CONCLUSION

We derived lower and upper bounds on the performance of MMSE receivers employing the QPSK and the QAM constellations in the presence of narrowband Gaussian interference when the signals experienced flat fading. For the single user scenario with mild ISI, these bounds are in agreement with the semi-analysis. For the multiuser scenario, the bounds also hold when ISI is negligible, and confirm our assumption that the contribution to the decision statistic of an MMSE receiver due to an interfering user in a multipath channel can be modeled as a Gaussian r.v.

Using a higher order constellation with the CDMA system results in a larger number of taps in MMSE receivers, which, in turn, affords us better diversity combining and improved interference suppression. In the single-user scenario, with narrowband interference, the QAM system outperforms the QPSK system when ISI is severe. In the multiuser scenario, with narrowband Gaussian interference, the QAM system outperforms the QPSK system, even with mild ISI. Therefore, as the MAI increases in the presence of the ISI, the extra degrees of freedom obtained by using a higher order constellation can help us to improve the performance of a CDMA system.

REFERENCES


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Validity of bounds: single user QAM system in NBG

Figure 1: Bounds for single-user QAM system in NBG

Validity of bounds: two-user QAM system in NBG

Figure 4: Bounds on two-user QAM system in NBG

Validity of Gaussian assumption: two-user QAM system in NBG

Figure 5: Gaussian assumption for QAM system in NBG

QPSK v/s QAM: Two user system in NBG

Figure 6: QPSK v/s QAM two-user system in NBG system