Carrier Detection of Unbalanced QPSK Direct Sequence Signals

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Abstract—This paper reports on detecting the presence of direct sequence spread spectrum signals through detection of the 2f and 4f carrier components of the signals produced by nonlinear techniques. The modulation studied is a generic unbalanced quadrature phase-shift-keyed (UQPSK) modulation, i.e. a QPSK signal with unequal power in the two channels. The complete range of channel power ratio is covered, with equal emphasis on the generic unbalanced case and the two limiting cases of BPSK and (balanced) QPSK. Analytic expressions are presented for the detected signal-to-noise ratio of carrier harmonics 2f and 4f as a function of SNR, channel power ratio, and normalized input bandwidth. The interest is in detecting covert direct sequence spread spectrum signals, i.e. those with SNR < 0 dB. Measurements confirm every aspect of the theory. The least detectable signal type is balanced QPSK, which is detectable (at 4f) at a threshold SNR ranging from -2 to -13 dB as the detection process gain (chip rate/detection bandwidth) is varied from 40 to 80 dB.

Index Terms—Signal detection, pseudonoise code communication, quadrature phase shifting.

INTRODUCTION

When an UQPSK signal with noise is passed through a nonlinear device, detectable components of multiples of the carrier frequency (f) are created. Detection of these carrier components may be used to detect the presence of a covert direct sequence (DS) spread spectrum signal, i.e. one with SNR < 0 dB. The key variable is channel power ratio, p, defined as (Q channel power)/(I channel power); 0 ≤ p ≤ 1. The goal of this study is to find the channel power ratio which minimizes the detectability of both carrier components.

A survey of the unclassified peer-reviewed literature on carrier detection of PSK signals is found in a previous paper [1]. Gardner and Spooner [2] also present a useful comparison and discussion of the detection performance of the radiometer, chip-rate detector and 2f carrier detector used with BPSK, QPSK, SQPSK and MSK signals. Briefly, the situation is as follows: detection of the 2f carrier component of BPSK signals has been fairly well analysed (reviewed in [1]); there has been one published quantitative analysis of the detection of the 4f carrier component of QPSK signals [1]; both chip-rate and carrier detectors perform better than the radiometer in terms of discriminating against interfering signal power or varying noise levels [2]; both also accurately determine one characteristic signal frequency (the first step in signal identification and exploitation); and the only published measurements for PSK carrier detection at 2f or 4f are in [1]. Unbalanced quadrature phase-shift-keyed (UQPSK) signals have never been studied (to the best of our knowledge), either theoretically or experimentally.

Since practical carrier recovery is desirable for further signal analysis, carrier harmonics must be continuously recoverable, not merely detectable. This requirement precludes the use of purely digital detection techniques such as cyclostationary techniques [2]. Measurements confirming the theory are presented in [1] for BPSK and QPSK modulations, and here for UQPSK modulation with arbitrary p.

THEORY

The unbalanced QPSK DS signal of interest is

\[ s(t) = \sqrt{2P_1} \left[ c_I(t)d_I(t)\cos(2\pi f_0 t) + c_Q(t)d_Q(t)\sin(2\pi f_0 t) \right] \]  \hspace{1cm} (1)

where \( P_1 \) is the power in the I channel, \( c_I(t) \) and \( c_Q(t) \) are the I and Q channel chip sequences (± 1), and \( d_I(t) \) and \( d_Q(t) \) are the I and Q channel data sequences (± 1), respectively. The carrier frequency \( f_0 \) and chip rate \( R \) are common to both channels. Equation (1) includes the two limiting cases of BPSK (\( p = 0 \) or \( -\infty \) dB) and QPSK (\( p = 1 \) or 0 dB).

As illustrated in Fig. 1, the carrier detection system consists of a nonlinear device preceded by a wideband bandpass filter and followed by a superheterodyne spectrum analyser which provides narrowband filtering and envelope detection of the chosen carrier component. The signal, together with wideband gaussian noise of one-sided power spectral density \( N_0 \), is passed through the wideband bandpass filter of noise-equivalent bandwidth \( W \) centered at \( f_0 \); results are expressed in terms of the normalized bandwidth \( \eta = W/R \) (0.5 ≤ \( \eta \) ≤ 20). The signal-to-noise ratio at the input to the wideband filter is defined as SNR = (prefilter signal power) / \( N_0 R = E_C/N_0 \), where \( E_C \) is the chip symbol energy of the prefilter signal. After passing through the wideband filter and nonlinearity the resulting combination of signal and noise is passed through a narrowband detection filter (in the spectrum analyser) of bandwidth \( B << W \) centered at the detection frequency of 2\( f_0 \).

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or $4f_0$ and finally detected with the spectrum analyzer's envelope detector. The averaged signal-to-noise ratio at the output of the detection system is denoted $\text{SNR}_\eta$ with $n=2,4$. The theories for $\text{SNR}_2$ and $\text{SNR}_4$ for UQPSK signals with arbitrary $p$ and $\eta$ are developed in the companion paper [4].

The calculations assume that the unfiltered chips and data bits are statistically independent zero-mean random variables and that the wideband bandpass filter has rectangular shape with noise-equivalent bandwidth $W >> B$. Only a fraction $F(\eta)$ of the initial signal power passes through the wideband filter $[1]$. Our definition of $F(\eta)$ neglects passband attenuation, which affects signal and noise equally.

The theoretical $\text{SNR}_4$ for UQPSK signals with arbitrary $p$ and $\eta$, derived from $[4, (A4)]$, is

$$
\text{SNR}_4 = \frac{2\eta R B}{p} \frac{(1 - p) F^2 \text{SNR}^2}{F(\eta)^2 + 4(1 + p)^2 F^2 \text{SNR} + 2 F^2 \text{SNR}^2}
$$

(2)

where $\eta = \frac{1}{(1 + p)^2}$ and the dependence on $\eta$ is both explicit in (2) and implicit in $F = F(\eta)$, $\beta_4(\eta)$ and $\beta_{22}(\eta)$. $\beta_4$ and $\beta_{22}$ are two signal shape coefficients, defined by

$$
\beta_4 = \frac{1}{c_1} \left( \frac{c_2}{c_1} \right)^2 = \frac{1}{c_0} \left( \frac{c_2}{c_0} \right)^2
$$

and

$$
\beta_{22} = \frac{1}{c_1^2} \frac{1}{c_Q} \left( \frac{c_2}{c_Q} \right)^2
$$

(3)

They describe the distortions in baseband waveforms caused by bandpass filtering. Representative values of $F(\eta)$, $\beta_4(\eta)$ and $\beta_{22}(\eta)$ are presented in Table I.

For QPSK signals ($p=1$), (2) gives $\text{SNR}_4 = 0$, confirming that there is no power at $2f$ from a squared (balanced) QPSK signal. For BPSK signals $p=0$, the low-SNR limit of (2), which neglects the $\text{SNR}^2$ term in the denominator, is

$$
\text{SNR}_2 = \frac{(R/2B) F^2 \text{SNR}^2}{\eta + 2 F \text{SNR}}
$$

(4)

This formula, with suitable changes in the definition of input SNR, was previously derived $[5,(11)]$ for any weakly filtered constant envelope double-sideband signal and also more specifically derived $[6,(30)]$ for BPSK signals with arbitrary $\eta$. It is also inherent in the equations derived by Gardner's group $[2,(42)]$ using the cyclostationary analysis. For the most useful range of applications ($\eta \geq 1$ and $\text{SNR} \leq 0 \text{ dB}$), the maximum error in $\text{SNR}_2$ using this approximation is 0.2 dB (which occurs at $\eta = 1$ and $\text{SNR} = 0 \text{ dB}$). For BPSK signals at very low SNR, $\text{SNR}_2 \approx \text{SNR}_2^2$.

The theoretical $\text{SNR}_4$ for UQPSK signals with arbitrary $p$ and $\eta$, derived from $[4,(A4)]$, is

$$
\text{SNR}_4 = \frac{(3\eta R/2B) \gamma_4 F^4 \text{SNR}^2}{\text{Denom}}
$$

Denom = 24(1 + p)^4 \eta^4 + 96(1 + p)^4 \eta^4 F^2 \text{SNR}

$$
\text{SNR}_4 = \frac{72(1 + p)^2 \eta^2 F^2 \text{SNR}^2}{\text{Denom}} + \text{higher order terms}
$$

In (5),

$$
\gamma_4 = \left( \frac{1 + p^2}{1 + p} \right) \beta_4 - 6p \beta_{22}
$$

$$
\beta_4 = 1 + p^2 \beta_4 + 2p \beta_{22}
$$

and the higher order terms are defined and evaluated in [4]. A null in $\text{SNR}_4$ is predicted by (5) whenever $p$ is set to make $\gamma_4$ zero. That value is

$$
\beta_{null} = \left( \frac{3 \beta_{22} - \sqrt{9 \beta_{22}^2 - \beta_4^2}}{\beta_4} \right)
$$

(6)

The null is easily explained for the simplest case, the unfiltered case. For unfiltered UQPSK, the baseband chips are rectangular; they have no distortion. Hence, $\beta_{22} = \beta_4 = 1$ and (6) gives $\beta_{null} = 0.1716$ or $-7.66$ dB. This corresponds to a critical phase angle of $\pi/8$, as illustrated in Fig. 2. The four original phase states are $A(\pi/8), A2(\pi-\pi/8), B1(-\pi/8)$ and $B2(-\pi/8)$. After the phases are quadrupled, points A1 and A2 both move to point A$+\pi/2$ and points B1 and B2 both move to point B$-\pi/2$. That is, the four old states coincide at the two new states, $\pm \pi/2$, and the carrier of the quadrupled signal spends equal time in just two opposing phase states (i.e. 180° apart). This new signal is similar to a BPSK signal of carrier frequency $4f$, and thus has a "suppressed carrier", with no average power, at $4f$.

For QPSK signals ($p=1$) with arbitrary $\eta$ at low SNR, the higher order terms in (5) are dropped and the expression for $\text{SNR}_4$ reduces to

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>$F(\eta)$</th>
<th>$\beta_4$</th>
<th>$\beta_{22}$</th>
<th>$\beta_{null}$</th>
<th>$p_{null}$ (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>0.467</td>
<td>1.93</td>
<td>1.00</td>
<td>-4.38</td>
<td>-3.9</td>
</tr>
<tr>
<td>1.00</td>
<td>0.773</td>
<td>1.33</td>
<td>1.02</td>
<td>-6.42</td>
<td>-6.5</td>
</tr>
<tr>
<td>2.00</td>
<td>0.902</td>
<td>1.15</td>
<td>1.07</td>
<td>-7.32</td>
<td>-7.3</td>
</tr>
<tr>
<td>$\infty$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-7.66</td>
<td>-7.6</td>
</tr>
</tbody>
</table>

$^a$Calculated for a 3rd-order Butterworth bandpass filter.

$^b$Calculated from (6).
Fig. 2. Effect of quadrupling phase angles for UQPSK signals with the critical angle of $\pi/8$ between I and Q channels.

$$\text{SNR}_4 = \frac{(R/32B) \left[ 3 \beta_{A2} - \beta_{A4} \right]^2 F^2 \text{SNR}^4}{2 \eta^3 + 8 \eta^2 F \text{SNR} + 3 \eta \beta_{A2}^2 + \beta_{A4}^4 \eta F^2 \text{SNR}^2}$$ (7)

For the most useful range of applications ($\eta \geq 1$ and $\text{SNR} \leq 0$ dB), the maximum error in $\text{SNR}_4$ using this approximation is 0.3 dB (which occurs at $\eta = 1$ and $\text{SNR} = 0$ dB).

**EXPERIMENTAL CONFIRMATION**

A. Methods

The quadrature modulator, illustrated in Fig. 3, was composed of separate arms each with an attenuator after the corresponding BPSK modulator and before the two channels were combined. Attenuators with 0.1 dB steps were used to precisely adjust $p$ to define the sharp nulls in 2f and 4f power. Experiments showed that the two paths from the split of carrier power at the input to the modulator (point A) to the summing of the two channels at the combiner (point B) must be well matched in electrical length. For example, the removal of either one attenuator or 30 cm of cable (38° of carrier phase at 70 MHz) severely reduced the depth of the resonant null in 4f and changed $p_{\text{null}}$. The removal of only 3 cm of cable length (a 3° phase change) reduced the strength of the 2f null at $p=1$ by 25 dB.

A carrier frequency $f_0 = 70$ MHz was chosen to make use of available equipment. A detection bandwidth of $B = 10$ Hz was used. The results in this paper and [1] were all obtained with a set of tubular LC bandpass filters. Here we present results from one filter set to $\eta = 1$ (which is optimum [1]) i.e. $W = R = 5.2$ MHz. ($R/B = 5.2 \times 10^5 = 57$ dB). The non-

Fig. 3. Generation of Unbalanced QPSK DS signals.

linearity and SNR measurement techniques are illustrated in Fig. 1 and described in [1].

B. Results

The previous paper [1] showed that for BPSK and QPSK signals with $\text{SNR} < 0$ dB, the optimum value of $\eta$ is about 1.0 ± 0.3. Since low SNR values are of greatest interest, $\eta = 1$ was used in most measurements and discussions in this report.

The detectability of a typical UQPSK signal ($p = -3$ dB) was measured at 2f and 4f as a function of SNR for $\eta = 1$. The results are presented in Fig. 4 where they may be compared with those for 2f detection of BPSK and 4f detection of QPSK. For $\text{SNR} < 0$ dB, theory and all measurements agree within 1 dB. The 4f component of UQPSK is only 2 dB less detectable than that of QPSK. The 2f component of UQPSK is 10 dB less detectable than that of BPSK. The slopes of the curves reach the theoretical 2.0 dB/dB for both 2f curves, but only reach 3.6 dB/dB for the 4f curves (compared to the very-low-SNR limit of 4.0 dB/dB). The highest reasonable interceptor's process gain of 80 dB gives a calculated slope of 3.9 dB/dB at $\text{SNR}_4 = 10$ dB.

For $\text{SNR} > 10$ dB, the four curves asymptotically approach the self-noise limits, for which signal x signal pattern noise is the only important noise component [4].

The effect of UQPSK channel power ratio on the detection of 2f power is shown in Fig. 5 for $\eta = 1$ and $\text{SNR} = 10$ dB. The predicted null in 2f is clearly visible. (A high SNR value was used to define the null very well). The dominant factor is $(1 - p)^3$ in the numerator of (2). The sharpness of the null shows how a typical channel unbalance
Fig. 4. SNR$_n$ of detected carrier components ($n = 2$ or $4$) vs input SNR. Detection of $2f$ for BPSK ($B2f$), $4f$ for QPSK ($Q4f$) and both components for UQPSK ($U2f$ and $U4f$) with $p = 0.5$ or $-3$ dB. $\eta = 1$, $R/B = 57$ dB.

of about 0.3 dB can dramatically increase the detectability of the $2f$ component of QPSK signals.

The effect of UQPSK channel power ratio on the detection of $4f$ power is also shown in Fig. 5. The sharp null in $4f$ power (with $\eta = 1$) is clearly seen at $P_{null} = -6.5$ dB. The measured channel power ratios which extinguish the $4f$ power at other values of $\eta$ are presented in Table I. The nulls have a depth >30 dB for all $\eta$. For virtually unfiltered signals ($\eta = 20$), $P_{null} = -7.6 \pm 0.1$ dB, in excellent agreement with the theoretical prediction, -7.66 dB.

**DISCUSSION**

Theoretical detection performance boundaries, functions of both SNR and $p$, are presented in Fig. 6 for the case of $\eta = 1$ and $R/B = 60$ dB. They consist of the detection thresholds for the $2f$ and $4f$ components and the line defining equal detectability for those two components. The detection threshold for $2f$ approaches infinity at $p = 0$ dB, but lies below the detection threshold for $4f$ for all $p < -0.4$ dB. The detection threshold line for $4f$ approaches infinity at $p = -6.4$ dB, the location of the null in $4f$ power with $\eta = 1$. The third performance boundary, that of equal detectability of $2f$ and $4f$, is also marked. The three lines define four regions of signal detectability: only $2f$ detectable; only $4f$ detectable; both detectable but $2f$ more detectable (denoted "$2f > 4f$"); and both detectable but $4f$ more detectable (denoted "$4f > 2f$"."). One region of non-detectability (denoted "neither") is also defined. From Fig. 6, the value of $p$ which minimizes carrier detection is seen to be any value in the narrow range of [-0.4, 0.0] dB. That is, balanced, or nearly balanced, QPSK signals are less detectable by nonlinear carrier detection techniques than any other type of UQPSK signal. While Fig. 6 applies to the specific case of $R/B = 60$ dB, $\eta = 1$, and detection SNR = 10 dB, similar performance diagrams drawn for other values of those parameters would not differ qualitatively from Fig. 6.

The minimum detectable SNR of BPSK and balanced QPSK signals detected by both carrier and chip-rate detectors over the complete practical range of interceptor's process gain ($R/B$) is illustrated in Fig. 7. Carrier detection boundaries are calculated from (4) and (7); the chip-rate detection boundary (for both BPSK and QPSK) is calculated from [3], with a shift of 2 dB in input SNR for the same detected SNR. As the process gain ($R/B$) is varied from 40 to 80 dB, balanced QPSK is detectable at a threshold SNR ranging from -2 to -13 dB; over the same range, BPSK is detectable at a threshold SNR varying from -12 to -32 dB.
CONCLUSIONS

In this paper, analytic expressions were presented for the detected SNR or SNR as a function of input SNR, channel power ratio, p, and normalized input bandwidth, η. Measurements presented here and in [1] confirm every aspect of the theory.

For unfiltered UQPSK there is a sharp null in 4f power at a channel power ratio of \( p_{\text{null}} = -7.66 \) dB, the point at which the phase angle between the two channels is \( \pi/8 \). As bandpass filtering becomes narrower (i.e. η decreases) \( p_{\text{null}} \) moves closer to unity, as illustrated in Table I. The 4f null cannot be used to hide the signal from carrier detection because at \( p = p_{\text{null}} \), the 2f carrier component is easily detected.

Regions of signal detectability were mapped as a function of SNR and p in Fig. 6. Balanced (or nearly balanced) ideal QPSK signals are less detectable by carrier detection techniques than either BPSK or UQPSK signals. Note, however, that the 1f or 2f component of non-ideal QPSK signals may possibly be easily detected through modulator imperfections [1], and that the ideal QPSK signal is more strongly detected with the chip-rate detector (see [3] and Fig. 7).

REFERENCES