MULTICAST TRANSMISSION WITH NONUNIFORM PHASE-SHIFT-KEY MODULATION AND CONVOLUTIONAL CODING OVER RAYLEIGH FADING CHANNELS

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Abstract— In this paper we present simple closed-form expressions for the probability of error for \( M \)-ary phase-shift-key (M-PSK) signaling over a channel with additive white Gaussian noise and nonselective Rayleigh fading. These expressions can be used to analyze the performance of standard uniform M-PSK constellations and nonuniform M-PSK constellations. Nonuniform M-PSK has recently been shown to be useful for multimedia communication, multicast transmission, and adaptive signaling. We analyze several systems that employ nonuniform M-PSK for multicast transmission over Rayleigh fading channels, and we determine the performance of nonuniform M-PSK with convolutional coding.

I. INTRODUCTION

Phase-shift-key (PSK) modulation has many advantages for use in wireless communication systems. PSK signals have a constant envelope and do not require amplitude information for demodulation. M-PSK has better spectral efficiency than binary PSK (BPSK), and nonuniform M-PSK can be used to send multiple messages simultaneously with different error probabilities [1], [2].

In [3], Craig developed a new expression for the probability of error for phase-shift-key signaling over a nonfading additive white Gaussian noise (AWGN) channel. In this paper, we use that expression to develop a simple closed-form expression for the probability of error for M-PSK transmitted over a nonselective Rayleigh fading channel. This expression is used to determine the performance of several systems that employ nonuniform M-PSK.

In many wireless communication systems, radios use broadcast or multicast signaling to transmit a message to multiple receivers. In [1] and [2] we show that nonuniform M-PSK can increase the throughput of multicast transmissions over nonfading AWGN channels by delivering additional information to more-capable receivers along with the multicast message. This additional information can be included with very little loss in performance for the reception of the basic message. This capability for additional throughput can be employed to transmit multimedia messages (e.g., voice and data) to more-capable receivers. In [1] we show that convolutional coding can be used with the nonuniform M-PSK constellations to provide more flexibility in performance tradeoffs and increased energy efficiency.

In this paper, results are presented for the performance of signaling schemes that use nonuniform M-PSK over Rayleigh fading channels. We determine the error probabilities for uncoded transmission of nonuniform M-PSK and for nonuniform M-PSK with convolutional coding. These schemes can be used in multicast transmissions to deliver two levels of information to receivers of differing capabilities, and we analyze the performance of several such schemes.

II. ERROR PROBABILITIES FOR UNCODED M-PSK

In this section, the results in [3] are applied to determine expressions for the error probabilities for coherent reception of nonuniform M-PSK on a channel with additive white Gaussian noise and nonselective Rayleigh fading. Consider the error probability for the signal illustrated in Figure 1 with decision region boundaries \( \theta_1 \) and \( \theta_2 \), \( \theta_1 > 0 \), \( \theta_2 > 0 \). The transmitted signal is labeled \( S \). The received signal is in error if the phase of the decision statistic is in either of the shaded regions \( R_1 \) or \( R_2 \) shown in Figure 1.

The probability of symbol error for the situation depicted in Figure 1 is given in [3] for the nonfading AWGN channel. Let \( E_i / N_0 \) denote the symbol energy-to-noise density ratio at the \( i \)th receiver. If the decision regions are specified by \( \theta_1 \) and \( \theta_2 \), \( 0 < \theta_1 < \pi, 0 < \theta_2 < \pi \), then the probability of symbol error is

\[
P_{i,s} = G \left( \theta_1, \frac{2E_i}{N_0} \right) + G \left( \theta_2, \frac{2E_i}{N_0} \right),
\]

where

\[
G(\theta, \alpha) = \frac{1}{2\pi} \int_{0}^{\pi} \exp \left\{ -\frac{\alpha^2 \sin^2(\theta)}{2\sin^2(\phi)} \right\} d\phi.
\]

For transmission of a single M-PSK symbol, the error probabilities are determined by integrating the product of the error

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expressions based on (1) and (2) with the Rayleigh density function. Let \( A_i \) represent the amplitude of the faded signal at receiver \( i \), where \( A_i \) is a Rayleigh random variable with parameter \( \tau_i \). The symbol energy-to-noise density ratio at receiver \( i \) is \( E_i/N_0 = A_i^2/(2\sigma^2) \), where \( \sigma^2 \) is the noise variance of the decision statistic. We can write \( G(\theta, 2E_i/N_0) = G(\theta, A_i^2/\sigma^2) \), which can be averaged over the Rayleigh density of \( A_i \). Let

\[
H_i(\theta) = \int_0^\infty \frac{a}{\tau_i^2} \exp\left\{-\frac{a^2}{2\tau_i^2}\right\} \times \frac{1}{2\pi} \int_0^{\pi-\theta} \exp\left\{-\frac{a^2 \sin^2 \theta}{2\sigma^2 \sin^2 \phi}\right\} da \, d\phi.
\]

By substituting in the value of \( A_i = N_0 \), and simplifying, \( H_i(\theta) \) may be written as

\[
H_i(\theta) = \frac{1}{2\pi} \int_0^{\pi-\theta} \frac{1}{\sin^2 \phi + \frac{\tau_i^2 \sin^2 \theta}{\sigma^2 \sin^2 \phi}} \, d\phi.
\]

Let \( E_i = E\{A_i^2/(2\sigma^2)\} = \tau_i^2/\sigma^2 \). Then

\[
H_i(\theta) = \frac{1}{2\pi} \int_0^{\pi-\theta} \frac{1}{E_i \sin^2 \theta \sin^2 \phi + \frac{\tau_i^2 \sin^2 \theta}{\sigma^2 \sin^2 \phi}} \, d\phi.
\]

Let \( E_i = E_i/N_0 \), \( e_i(\theta) = E_i \sin^2 \theta \), and \( w = 2\phi \). Then

\[
H_i(\theta) = \frac{1}{4\pi} \int_0^{2(\pi-\theta)} \frac{1 - \cos w}{2e_i(\theta) + 1 - \cos w} \, dw.
\]

By applying (2.554 2) and (2.553 3) of [4], a closed-form solution can be obtained,

\[
H_i(\theta) = \frac{\pi - \theta}{2\pi} - \frac{1}{2\pi} \sqrt{\frac{e_i(\theta)}{e_i(\theta) + 1}} \times \tan^{-1} \left\{ \frac{e_i(\theta) + 1}{e_i(\theta)} \tan (\pi - \theta) \right\}.
\]

For uniform \( M \)-PSK, \( \theta = \pi/M \), and the probability of symbol error at the \( i \)th receiver is

\[
P_{i,b} = 2H_i\left(\frac{\pi}{2}\right) = \frac{1}{2} \left[ 1 - \sqrt{\frac{E_{i}}{E_{i} + 1}} \right].
\]

which is identical to the expressions for BPSK in [5] and [6].

Nonuniform signal constellations can convey two or more types of messages, and it can be useful to use repetition for one type while not using repetition for the others. Consider first BPSK signaling with symbol repetition. The total average energy in \( n \) repetitions is \( nE_i \). If the fading is sufficiently slow and the repetition symbols are transmitted in consecutive signaling intervals, the fade level is constant over the duration of all repetitions of a given symbol. In this case, the probability of bit error is given by

\[
P_{i,b} = \frac{1}{2} \left[ 1 - \sqrt{\frac{nE_i}{nE_i + 1}} \right].
\]

If the symbols from the output of the repetition encoder are interleaved before transmission over the channel, diversity can be achieved. For the purpose of this analysis, the interleaver is assumed to be ideal, in that all of the repeated symbols representing a particular bit are assumed to experience independent fading. In addition, the receiver is assumed to have perfect channel state information; in other words, the receiver knows the exact fade level for each transmission interval. The receiver uses its knowledge of the fade level to perform maximal-ratio combining. The average energy per repetition of a given symbol is \( E_i \), and \( E_i = E_i/N_0 \). Then the probability of bit error is given by [5]

\[
P_{i,b} = \left[ 1 - e_i \sum_{k=0}^{L-1} \left( \frac{2k}{M-1} \right) \left( \frac{1 - e_i^2}{4} \right) \right].
\]

where \( e_i = \sqrt{E_i/(E_i + 1)} \).

Consider the situation in which the modulation is the nonuniform 4-PSK signal constellation described in [1] and [2]. This constellation consists of the points \( \{ e, -e, \pi + \theta, \pi - \theta \} \), where \( e \) is a parameter that controls the spacing of the points in the constellation and \( 0 \leq e \leq \pi/4 \). Two bits are transmitted with this constellation: a basic message bit and an additional message bit. The basic message chooses whether the transmitted signal is one of \( \{ e, -e \} \) or one of \( \{ \pi + \theta, \pi - \theta \} \), and the additional message chooses the individual point within the set selected by the basic message.

Repetition may be used to transmit one of the bits in multiple transmission intervals. In Section IV-B we consider a system that employs repetition for the additional message. One goal of this repetition is to increase the energy per bit of the additional message without increasing the degradation to the basic message. For fading channels, repetition can be used with interleaving to provide diversity.
The transmission of the additional message takes place on the quadrature portion of the signal, and the error probability can be determined from (8) by letting the average symbol energy-to-noise density per PSK symbol be $E_s \sin^2 \theta / N_0$. For general $M$-PSK constellations with repetition and ideal interleaving, the probability of error can be determined from the expressions in [5].

III. Multicast Transmission with Nonuniform PSK

Nonuniform $M$-PSK signaling can be used for multicast transmission to a set of neighbor radios in such a way that additional messages can be included for the benefit of receivers with extra capability. In [1] and [2], we show that these signaling schemes are effective for transmission over a channel in which the only disturbance is additive noise. In this section, we show that these schemes are also effective for transmission over a Rayleigh fading channel.

Consider a simple radio network consisting of one transmitter and two receivers that have different error probabilities for the receiving channel. In [1] and [2], we show that these signaling schemes are effective for transmission over a channel in which the only disturbance is additive noise. In this section, we show that these schemes are also effective for transmission over a Rayleigh fading channel.

For nonuniform 4-PSK, the probability of bit error for the basic message at the less-capable receiver is

$$P_{1,1} = H_1 \left( \frac{\pi}{2} - \theta \right) + H_1 \left( \frac{\pi}{2} + \theta \right) = \frac{1}{2} \left[ 1 - \sqrt{ \frac{E_1 \cos^2 \theta}{E_1 \cos^2 \theta + 1} } \right]. \quad (9)$$

The probability of bit error for the additional message at the more-capable receiver is

$$P_{2,2} = H_2 (\pi - \theta) + H_2 (\theta) = \frac{1}{2} \left[ 1 - \sqrt{ \frac{E_2 \sin^2 \theta}{E_2 \sin^2 \theta + 1} } \right]. \quad (10)$$

Note that (9) and (10) take the same form as (6), which is expected since the nonuniform 4-PSK signal may be treated as independent binary PSK signals on the inphase and quadrature components. Note that $(x/(x+1))$ is a strictly increasing function of $x$ for $x > 0$. Thus, (6) is a strictly decreasing function of $E_s / N_0$, and for $P_A = P_B$, the degradation and disparity can be calculated in terms of the received energies. These calculations are identical to those given in [1] and [2] for 4-PSK and an AWGN channel, so the equations for the degradation and disparity given in those papers also apply for nonuniform 4-PSK with nonselective Rayleigh fading.

The results in Figure 2, which is from [1], illustrate the degradation and disparity for nonuniform 4-PSK as a function of the phase offset $\theta$. For nonuniform 8-PSK, the disparity and degradation cannot be expressed as simple functions of $\theta$; these performance parameters are also a function of $P_A$ and $P_B$. The results in Figure 3 illustrate the performance of nonuniform 8-PSK transmitted over a Rayleigh fading channel with $P_B = P_A = 10^{-2}$.

One possible goal is to ensure that an additional message can be received by a radio that is half as far from the transmitter as the most distant radio. In many mobile communication systems, the propagation loss is approximately proportional to the fourth power of the distance from the transmitter to a receiver, in which case a factor of two in distance corresponds to a 12 dB difference in received power. Thus, the disparity must be less than 12 dB for the additional message to be received at the more-capable receiver. A degradation of less than 0.3 dB achieves this for nonuniform 4-PSK, and a degradation of 0.6 dB achieves this for nonuniform 8-PSK. Thus, very little additional power is required to include an additional message.

IV. Multicast Transmission with Convolutionally Encoded Nonuniform M-PSK

A. General Error Probabilities

In this section, we consider the performance of multicast transmission techniques that employ convolutionally encoded nonuniform M-PSK. Ideal interleaving is assumed, which means the fading amplitudes in any two intervals are independent and identically distributed. The results presented in this
chapter are for receivers that have perfect channel state information; that is, the decoder knows the fade level for each symbol. These results may be extended to receivers that have no channel state information, as in [7] and [8].

The error probability expressions for hard-decision decoding that are given in [9] can be applied for the channel with non-selective Rayleigh fading if the error probabilities at the output of the hard-decision demodulator are calculated from the expressions in Section II for Rayleigh fading. In this section, we focus on the performance of systems employing soft-decision decoding. The error probabilities for soft-decision decoding of convolutionally encoded \( M \)-PSK with Raleigh fading, ideal interleaving, and perfect channel state information are presented in [7] and [8]. For the transmission schemes considered in this paper, we assume that the transmitted path is the all-zeros sequence without loss of generality. Let \( \mathcal{P} \) denote the set of all non-zero paths, and suppose \( \varphi \in \mathcal{P} \). Let \( \eta_{\varphi} \) be a set consisting of the symbols in \( \varphi \) that differ from the symbols in the all-zeros path. If \( v \in \eta_{\varphi} \), then \( ||v||^2 \) denotes the squared Euclidean distance from symbol \( v \) to the symbol that represents all zero bits. The probability of bit error is

\[
P_{e,b} \leq \sum_{v \in \eta_{\varphi}} b(v) \prod_{v \in \eta_{\varphi}} \frac{1}{1 + E_v ||v||^2 / (4N_0)}, \tag{11}
\]

where \( b(\varphi) \) is the number of bit errors at the output of the Viterbi decoder that are caused by selecting \( \varphi \) instead of the all-zeros sequence.

### B. Nonuniform 4-PSK, Soft-Decision Decoding

In this section, results are presented for soft-decision decoding of convolutionally encoded nonuniform 4-PSK transmitted over a channel with additive white Gaussian noise and nonselective Rayleigh fading. We consider two possible signaling methods. In each method, the basic message is encoded with a convolutional code. For the results in this paper, the convolutional code is the rate 1/2, \( K = 7 \) code sometimes referred to as the NASA Voyager code. In each method, each bit at the input to the convolutional encoder results in two code symbols at the output of the encoder, and these code symbols are transmitted on separate 4-PSK symbols. The distinction between the two methods is in how the additional message is transmitted. In method 1, each bit of the additional message is repeated and transmitted on two 4-PSK symbols. In method 2, the additional message is encoded with the same convolutional code as the basic message, and no repetition is used.

For each signaling method, the bits that represent the basic message are interleaved before transmission over the channel. If method 1 is employed, the additional message bits are transmitted on multiple PSK symbols, so some diversity gain can be achieved by using interleaving. The performance of method 1 is evaluated with and without interleaving. If interleaving is not used for the additional message, we assume that the fade levels are the same for the two consecutive PSK symbols that convey one bit of the additional message. For method 2, the code symbols that represent the additional message are interleaved, and it is assumed that each PSK symbol experiences independent fading.

The results for channels with additive white Gaussian noise with and without Rayleigh fading are shown in Figure 4 for
The disparity for method 2 is the same for both the nonfading and Rayleigh fading channel. For method 1 without interleave of the additional message bits, the disparity at low error probabilities is much larger for the Rayleigh fading channel than for the nonfading channel. If interleave is employed for the additional message bits in method 1, the disparity for the Rayleigh fading channel is much less than the disparity if interleave is not employed. For the Rayleigh fading channel at most error rates of interest, method 2 produces a significantly smaller disparity than method 1. If the additional message to be sent without coding, then method 1 with interleave of the additional message achieves the lowest disparity.

C. Nonuniform 8-PSK, Soft-Decision Decoding

In this section, an example of a system that employs nonuniform 8-PSK is presented. The basic message is encoded with the rate 1/2, \( K = 7 \) convolutional code. The information rates for the basic and additional messages are constrained to be equal. Each nonuniform 8-PSK symbol conveys one bit of the basic message and one bit of the additional message. Thus, each PSK symbol conveys two code symbols generated by the basic message and one bit that represents the uncoded additional message. Interleave is employed, but each set of \( n \) code symbols generated by a bit of the basic message is kept together. Since no coding or repetition is used for the additional message, there is no diversity gain for the additional message.

The results in Figure 5 are for the standard convolutional code with a degradation of 0.5 dB and \( P_A = P_B \). The results show that the disparity is much larger for the Rayleigh fading channel than for the nonfading channel, especially at low error rates. For \( P_B = 10^{-2} \), the disparity for the nonfading channel is approximately 18 dB, while the disparity for the Rayleigh fading channel is greater than 25 dB. For \( P_B = 10^{-4} \), the disparity for the nonfading channel is approximately 20 dB, but the disparity for the Rayleigh fading channel is over 42 dB. These results suggest that if nonuniform 8-PSK signaling is employed at low error rates (\( P_B < 10^{-2} \)), some form of coding or repetition must be used for the additional message to keep the disparity from getting too large.

V. Conclusions

In this paper, we present a simple closed-form expression for the probability of error for uniform and nonuniform \( M \)-PSK signal constellations. This expression is used to analyze the performance of multicast transmission techniques that employ nonuniform \( M \)-PSK. These techniques improve the efficiency of communication during multicast transmissions by delivering a basic message to less-capable receivers and an additional message to more-capable receivers. The results indicate that for uncoded transmission, nonuniform 4-PSK constellations have the same disparity and degradation for a Rayleigh fading channel as they do for a channel with no fading. Consider a situation in which the more-capable receivers have a 12 dB higher signal-to-noise ratio than the less-capable receivers. In many mobile communication systems, this 12 dB difference corresponds to a factor of one-half in distance. For this 12 dB disparity, the additional energy required to include an additional message for more-capable receivers is less than 0.3 dB for nonuniform 4-PSK and 0.6 dB for nonuniform 8-PSK. The results for the systems that employ convolutional coding indicate that the multicast signaling techniques can be used successfully on Rayleigh fading channels, but some form of coding or repetition may be required for the additional message to keep the disparity from being too large.

REFERENCES


