MAXIMUM-LIKELIHOOD MODULATION CLASSIFICATION FOR PSK/QAM

J.A. Sills

Signal Exploitation and Geolocation Division
Southwest Research Institute
San Antonio, Texas 78238-5166

ABSTRACT
This paper addresses automatic modulation classification for PSK and QAM signals under coherent and noncoherent conditions. In particular, the paper extends previous results by treating the classification of higher-state QAM signals. A maximum-likelihood algorithm is presented for coherent classification of PSK and QAM signals. We evaluate the algorithms performance for various PSK and QAM modulation types including 64-state QAM and then compare it with a psuedo maximum-likelihood noncoherent classification technique in terms of error rate, false alarm rate, and computational complexity. The application of these results to the design and performance of an automatic signal recognizer is discussed throughout the paper.

1. INTRODUCTION
Automatic modulation recognition is a rapidly evolving area of signal exploitation with applications in DF confirmation, monitoring, spectrum management, interference identification, and electronic surveillance. Generally stated, a signal recognizer is used to identify the modulation type (along with various parameters such as baud rate) of a detected signal for the purpose of signal exploitation. For example, a signal recognizer could be used to extract signal information useful for choosing a suitable counter measure, such as jamming.

In recent years interest in modulation recognition algorithms has increased with the emergence of new communication technologies. In particular, there is growing interest in algorithms that treat quadrature amplitude modulated (QAM) signals, which are used in the HF, VHF, and UHF bands for a wide variety of applications including FAX, modem, and digital cellular.

Many techniques for modulation identification have been published in the literature. Early work in modulation identification is found in a report by Weaver, Cole, and Krumland [1] in which frequency-domain parameters were used to distinguish between six candidate modulation types. One of the well-known early papers treating digital modulation types was by Liedtke [2] in which he presents results based on a statistical analysis of various signal parameters to discriminate between amplitude shift keying (ASK), FSK, and PSK. Other techniques using signal parameters have been reported in [3], [4], [5], [6], and [7]. A combination of techniques including pattern recognition are used in [8] and [9]. Several authors have applied techniques from higher-order statistics that exploit cyclostationarity to identify modulation [10]. Still others have applied neural networks to the problem [11, 12]. A recent book by Azzouz and Nandi [13] gives more details on these and other recent techniques for modulation identification.

Another group of authors have applied techniques from maximum-likelihood (ML) decision theory to modulation identification. Kim et al. use a truncated series approximation of the likelihood ratio function for distinguishing a BPSK from an MPSK (M ≥ 4), but these results apply in low SNR only [14]. Extensions to high SNR and 16-state QAM are presented in [15]. Sapiano presents a PSK classification technique with improved sensitivity to parametric degradation [16]. Most recently, Boiteau presented a comprehensive review of the literature on signal classification and provided a generalized framework that does not require any restriction on the baseband pulse [17].

In this paper we extend previous results on maximum-likelihood classification for PSK/QAM by developing general solutions for coherent and noncoherent classification of PSK/QAM signals with an arbitrary number of signal states. Performance curves are presented for both the coherent and noncoherent cases for various modulation types including 64-state QAM.

The paper begins with an introduction in Section 1. Section 2 presents the signal model for PSK/QAM communications. The coherent classifier is presented in Section 3 along with performance curves showing error rates and false alarm rates. The noncoherent case is treated in Section 4. Performance curves are presented and then noncoherent performance is compared with coherent performance. Section 5 contains conclusions and recommendations.

2. SIGNAL MODEL FOR PSK/QAM
We receive a signal \( r(t) = s(t) + n(t) \), \( 0 \leq t \leq T \) where \( s(t) \) is a signal emitted from a non-cooperative transmitter and \( n(t) \) is additive white gaussian noise (AWGN) with a two-sided power spectral density (PSD) of \( \frac{E_n}{2} \). The signal \( s(t) \) is represented by

\[
s(t) = A(t) \cos(\omega_c t + \theta_c + \phi(t)) = \text{Re} \left\{ A(t) e^{j\phi(t)} e^{j(\omega_c t + \theta_c)} \right\}
\]

where \( A(t) \) and \( \phi(t) \) are the modulated amplitude and phase, \( \omega_c \) is the carrier frequency, and \( \theta_c \) is an unknown phase offset.

The received signal is one of \( N \) candidate modulation types. Let the integers \( i = 0, 1, \ldots, N - 1 \) enumerate the candidate modulation types, such that \( m_i \) for \( i = 0, 1, \ldots, N - 1 \) denotes the event that the intercepted signal belongs to the \( i^{th} \) modulation type. We will assume equal \textit{a priori} probabilities \( P[m_i] \).

Let \( \tilde{s}(t) = A(t) e^{j\phi(t)} \) denote the complex envelope of \( s(t) \). The complex envelope of a PSK/QAM signal can be expressed in terms of

\[
\tilde{s}(t) = \sqrt{\frac{2E_s}{E_p \Delta f}} \sum_n a_n p(t - nT_s - t_d)
\]

This work was supported by the Advisory Committee for Research and Development at Southwest Research Institute.
where \( a_n \) is a sequence of symbols taken from a set of \( M(i) \) complex numbers \( I(m_i) = \{ \mu_1, \mu_2, \ldots, \mu_{M(i)} \} \). \( R_s \) is the symbol rate, and \( t_d \) is an unknown timing offset. The pulse shape \( p(t) \) is any of the standard symmetric types such as a square-root raised cosine or a square pulse. The symbol energy is \( E_s \) provided that (1) we define

\[
E_p = \int_{-\infty}^{\infty} p^2(t) dt
\]

and (2) we model \( a_n \) as independent random variables with equally likely assignment from the set \( I(m_i) \) such that

\[
E_a = E[|a_n|^2] = \frac{1}{M(i)} \sum_{n=1}^{M(i)} |\mu_n|^2
\]

where \( E[\cdot] \) denotes the expected value operator. It is customary to normalize the set \( \{ \mu_n \} \) such that \( E_a = 1 \). The input signal-to-noise ratio (SNR) is defined by \( \gamma = \frac{E_s}{N_0} \).

Each modulation type is characterized by its symbol configuration in the complex plane, which defines the amplitude and phase values for the set \( I(m_i) \).

3. COHERENT ML CLASSIFICATION

Automatic signal classification is a rather difficult problem in composite hypothesis testing since so many parameters are unknown: symbol rate \( R_s \); carrier frequency \( \omega_c \); carrier phase \( \theta_c \); pulse shape \( p(t) \); SNR \( \gamma \); and timing offset \( t_d \). A common approach is to first estimate the unknown parameters and then attempt to classify the signal according to modulation type. Although estimating these parameters is nontrivial, it is not impractical. There are a wide variety of techniques for estimating the signal parameters some of which are given in [18].

In this section we evaluate the performance of coherent ML classification in which all of the signal parameters are known. In this case, the signal is classified by forming likelihood ratios from the demodulated matched-filter output

\[
\hat{r}_n = \int_{-\infty}^{\infty} r(t) e^{-j(\omega_c t + \phi)} \sqrt{E_p} p(t - nT_s - t_d) dt
\]

\[
= r_{1,n} + j r_{Q,n}
\]

It follows that \( r_{1,n} = \sqrt{E_s} \alpha_{1,n} + n_{1,n} \) and \( r_{Q,n} = \sqrt{E_s} \alpha_{Q,n} + n_{Q,n} \), where \( \alpha_{1,n} = \text{Re}\{a_n\} \) and \( \alpha_{Q,n} = \text{Im}\{a_n\} \). The noise components \( n_{1,n} \) and \( n_{Q,n} \) are independent, zero mean, with variance \( \frac{N_0}{2} \).

Given that the demodulation type is \( m_i \), the probability density function (PDF) governing the demodulated symbols \( \hat{r}_n \) can be expressed in the form

\[
p_i(r_{1,n}, r_{Q,n}|m = m_i) = \frac{1}{M(i)} \sum_{k=1}^{M(i)} e^{-\frac{(r_{1,n} - \sqrt{E_s} \alpha_{1,k})^2 + (r_{Q,n} - \sqrt{E_s} \alpha_{Q,k})^2}{2\sigma^2}}
\]

where \( \mu_{1,k} = \text{Re}\{\mu_k\} \) and \( \mu_{Q,k} = \text{Im}\{\mu_k\} \) and \( \sigma = \frac{N_0}{2} \). It is worth noting that \( r_{1,n} \) and \( r_{Q,n} \) are not necessarily independent—for example consider 8-PSK and QAM-32 from the V.32 standard [19].

We represent \( N \) demodulated symbols in vector form:

\[
\tilde{r} = r_1 + j r_Q = \begin{bmatrix} \tilde{r}_1 \\ \tilde{r}_2 \\ \vdots \\ \tilde{r}_N \end{bmatrix}
\]

The PDF governing \( \tilde{r} \) is given by

\[
p_i(r_1, r_Q|m = m_i) = \prod_{n=1}^{N} p_i(r_{1,n}, r_{Q,n}|m = m_i)
\]

The coherent maximum likelihood (ML) classifier is simply a rule for choosing among the candidate modulation types given \( \tilde{r} \). Choose \( m = m_k \) if and only if

\[
p_i(r_1, r_Q|m = m_i)
\]

is maximum for \( i = k \).

We investigate the performance of the coherent classifier by evaluating its error rate as a function of SNR for the following PSK/QAM modulation types: (\( m_1 \)) BPSK; (\( m_2 \)) QPSK; (\( m_3 \)) 8-PSK; (\( m_4 \)) QAM-16; (\( m_5 \)) QAM-32; and (\( m_6 \)) QAM-64. For PSK, the symbol configurations are well known. For QAM however, there are many possibilities including rectangular and circular configurations. We consider the rectangular configurations defined by the V.32 and V.33 standards [19] and shown in Figure 1.

![Figure 1: PSK/QAM symbol configurations.](image-url)

Figure 2 shows the performance in terms of probability of error and false-alarm rate for the coherent classifier resulting from 1000 Monte-Carlo simulations. The performance indicates that the coherent ML classifier makes less than one error in ten across all six modulation types provided the SNR is greater than or equal to 10 dB. This performance represents the best possible error rate that can be achieved. This level of performance is unlikely in practice due primarily to phase incoherence between the transmitter and receiver; that is, the parameter \( \theta_c \) is rarely known when the SNR is 10 dB. In fact coherent carrier acquisition for high-state QAM...
requires SNR levels much larger than 10 dB. Nevertheless, Figure 2 provides a benchmark for classification performance from which to compare noncoherent techniques.

In this case, the signal is classified by finding the amplitude $|\tilde{r}_n|$ to compare noncoherent techniques. Figure 2 provides a benchmark for classification performance from which requires SNR levels much larger than 10 dB. Nevertheless, Figure 2 provides a benchmark for classification performance from which to compare noncoherent techniques.

![Diagram](image1)

(a) Error rate.

![Diagram](image2)

(b) False alarm rate.

Figure 2: Coherent performance using $N = 256$ symbols.

4. NONCOHERENT PSUEDO-ML CLASSIFICATION

In this section we evaluate the performance of noncoherent ML classification in which all of the signal parameters are known except the carrier phase $\theta_c$. In this case the demodulated symbol is rotated by an unknown carrier phase

$$\tilde{r}_n = \sqrt{E_s} a_n e^{j\theta_c} + \tilde{n}_n$$

In this case, the signal is classified by finding the amplitude $|\tilde{r}_n|$ and the phase difference

$$\Delta \psi_n = |\psi_n - \psi_{n-1}| \mod 2\pi$$

where

$$\psi_n = \tan^{-1} \frac{r_{Q,n}}{r_{I,n}}.$$ 

Given the transmitted symbol $a_n$, the amplitude $|\tilde{r}_n|$ is a Ricean-distributed random variable with PDF

$$p_{|\tilde{r}_n|}(r_n | a_n) = \frac{r_n}{\sigma^2} e^{-\frac{(r_n^2 - 1)}{2\sigma^2}} I_0 \left( \frac{r_n \sqrt{E_s} a_n}{\sigma^2} \right)$$

where $I_0(\cdot)$ is the 0th-order modified Bessel function and $\sigma^2 = \frac{N_0}{2}$.

The exact expression for the PDF of the phase difference $\Delta \psi_n$ is very complicated, but for sufficiently large SNR, it can be approximated by a Gaussian:

$$p_{\Delta \psi_n}(\Delta \psi_n | a_n, a_{n-1}) \approx \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{(\Delta \psi_n - \Delta \phi)^2}{2\sigma^2_n}}$$

where $\sigma_n^2 = \frac{N_0}{2} \left( \frac{1}{\sigma_n^2} + \frac{1}{\sigma_{n-1}^2} \right)$ and $\Delta \phi = \tan^{-1} \frac{\alpha_{Q,n}}{\alpha_{I,n}} - \tan^{-1} \frac{\alpha_{Q,n-1}}{\alpha_{I,n-1}}$. The approximation given by (1) follows from the approximation $\tan^{-1} \delta \approx \delta$ for small $\delta$.

For sufficiently large values of SNR, $|\tilde{r}_n|$ and $\Delta \psi_n$ are nearly independent, hence we can approximate their joint PDF by

$$p_{|\tilde{r}_n|, \Delta \psi_n}(r_n, \Delta \psi_n | a_n, a_{n-1}) \approx p_{|\tilde{r}_n|}(r_n | a_n) p_{\Delta \psi_n}(\Delta \psi_n | a_n, a_{n-1})$$

We next apply the law of total probability to find the conditional density function $p_{|\tilde{r}_n|, \Delta \psi_n}(r_n, \Delta \psi_n | m = m_i)$. There are efficient ways to perform this calculation, but as a general expression

$$p_{|\tilde{r}_n|, \Delta \psi_n}(r_n, \Delta \psi_n | m_i) = \sum_{n=1}^{M(i)} \sum_{k=1}^{M(i)} p_{\delta_{n}(n_1, m)}[r_n] p_{\delta_{n}(n_1, m)}[\Delta \psi_n]$$

Given $N$ demodulated symbols, the PDF for the $(N-1)$-dimensional decision vectors

$$R = \begin{bmatrix} |\tilde{r}_1| \\ |\tilde{r}_2| \\ \vdots \\ |\tilde{r}_{N-1}| \end{bmatrix} \quad P = \begin{bmatrix} \Delta \psi_1 \\ \Delta \psi_2 \\ \vdots \\ \Delta \psi_{N-1} \end{bmatrix}$$

is

$$p_{R, P}(R, P | m = m_i) = \prod_{n=1}^{N-1} p_{|\tilde{r}_n|, \Delta \psi_n}(r_n, \Delta \psi_n | m_i)$$

We select the modulation type that corresponds to the largest of the $p_{R, P}(R, P | m = m_i)$.

The performance of the pseudo-maximum-likelihood modulation classifier is illustrated in Figure 3. This figure shows the performance in terms of probability of error and false-alarm rate for the noncoherent classifier resulting from 1000 Monte-Carlo simulations. These results indicate that the noncoherent pseudo-ML classifier makes less than one error in ten across the tested modulation types provided the SNR is greater than or equal to 13 dB. Comparing Figures 2 and 3, it is evident that the noncoherent classifier exhibits a performance loss of approximately 3 dB.

A critical parameter in the design of a PSK/QAM recognizer is the number of symbols that are used to decide between modulation types. Using a large number of symbols in the likelihood-ratio test reduces the probability of error and probability of false alarm; hence, the one-error-in-ten performance can be sustained down to lower SNR. When fewer symbols are used, we require higher SNR for this performance level.
5. CONCLUSIONS

In this paper we investigated automatic modulation classification for PSK and QAM signals. We presented a maximum likelihood framework for both the coherent and noncoherent cases. This general approach treated PSK and QAM up to an arbitrary number of signal states. The performance of both the coherent and noncoherent classifier was investigated for various modulation types including 64 state QAM. Noncoherent performance exhibited a 3 dB loss compared to coherent performance.

6. REFERENCES


