PERFORMANCE ANALYSIS OF LOW EARTH ORBIT (LEO) LAND MOBILE SATELLITE USING MOMENT TECHNIQUE

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ABSTRACT

Performance evaluation of the signal over the LEO satellite channel usually involves great degree of difficulty due to the system complexity and time varying elevation angle (EL). Especially, it is hardly possible to evaluate error performance in a direct manner using closed form analysis. In this study, we propose a moment technique based on known moments in the presence of adjacent channel interference (ACI) and Gaussian noise. The Doppler frequency shift and Raleigh multipath channel characterized by the various fading factors are also considered.

I. INTRODUCTION

Mobile Communication using Low Earth Orbit (LEO) satellite is in practice these days. In this application, the channel is mainly affected by time-varying elevation angle, in addition to the conventional disturbances. However, performance analysis considering all these factors is hardly possible in reality. In this paper, we proposed a moment based error performance approximation of PSK signals over the LEO channel.

This evaluation can be analyzed using a two-dimensional Classical Moment Technique (CMT). The CMT is an effective and powerful tool in dealing with numerical evaluation of the expectation of a complicated function with respect to random variables whose moments are known [1-2]. Since the fading factor considered in this study including amplitude and phase distortions, two-dimension CMT, which is derived from one-dimension CMT, is needed to construct the discrete joint probability density function. Rayleigh fading is an example in our case. Although ultimate application of this study is to design a LMS system over LEO channel, the approximate error performance can be used as a basis for the link budget calculation initially. In Section II, the system and performance of the considered system has been derived by analytic approach. Section III discusses the moment technique to construct the discrete density function as well as the accuracy of this approximation. The moment calculation for Intersymbol Interference (ISI), Adjacent Channel Interference (ACI), and Ricean distribution is given in section IV.

II. SYSTEM AND ITS ERROR PROBABILITY

In a LEO satellite channel under multiple user application, the adjacent channel interference should be considered. In addition to the ACI, additive Gaussian noise corrupts the signal both on uplink and downlink transmission. Consider for a fixed point of LEO communication, the system modeled as Fig. 1.

![System model for a fixed point of LEO](image)

Let’s consider a PSK signal sequence,

\[ s(t) = \sqrt{2A} \sum_k p(t - kT) \cdot \cos(\omega_c t + \theta_k) \]  

(1)

where \( A \) is the transmitted signal power, \( p(t) \) is the unity pulse over the symbol time, \( T \) is the symbol duration, \( \omega_c \) is the angular frequency of the carrier and \( \theta_k \) is the transmission phase taken from one of \( M \)-phases \( \{ \frac{2\pi}{M} i | i = 0, 1, \ldots, M-1 \} \). The equal probability of the transmitted signal phase is assumed. Also, we assume the adjacent channel interference is

\[ I_u(t) = \sum_i I_{u,i} \cdot \cos( (\omega_c + \Delta\omega_{u,i}) t + \theta_{u,i} ) \]

(2)

where

\[ \Delta\omega_{u,i} = \Delta\omega_{u,i,t} + \theta_{u,i} \]

(3)

\( (\omega_c + \Delta\omega_{u,i}) \) is the carrier frequency for the ith uplink adjacent channel, \( \theta_{u,i} \) is the transmission phase of the ith uplink adjacent channel, and \( \Delta\omega_{u,i} \) is the frequency deviation from center carrier frequency \( \omega_c \). Then, the signal arrives at the front end at the satellite repeater is

\[ x(t) = s(t) + I_u(t) + n_u(t) \]

(4)

where \( n_u(t) \) is a uplink narrow-band Gaussian noise. To simplify the analysis, the signal \( x(t) \) can be expressed as

\[ x(t) = \sqrt{2A} \cdot S_1 \cdot \cos(\omega_c t + \theta_0 + \theta_1(t)) + n_u(t) \]

(5)

where

\[ S_1(t)e^{j\theta_1(t)} = p(t) + \sum_{k \neq 0} p(t - kT)e^{j(\theta_k - \theta_0)} \]

\[ + \sum_i \left( \frac{I_{u,i}}{A} \right)^{1/2} e^{j(\theta_{u,i}(t) - \theta_0)} \]

(6)
On the right side of the equation, first term is the desired signal, second term belongs to intersymbol interference (ISI) and the third term is from the adjacent channel. At the input of TWTA, the signal is

\[ x(t) = R(t) \cos(\omega_c t + \theta_0 + \eta(t)) \]  

with the conditional probability density function

\[ P_r(R, \eta | S_1, \theta_1) = \frac{R}{2\pi \sigma_u^2} \exp \left[ -\frac{R^2 + 2AS_1^2 - 2\sqrt{2}ARS_1 \cos(\eta - \theta_1)}{2\sigma_u^2} \right] \]  

and the output signal of the transponder becomes

\[ y(t) = f(R) \cos(\omega_c t + \theta_0 + \eta(t) + g(R)) \]  

where \( f(R) \) is AM/AM conversion and \( g(R) \) represents AM/PM nonlinearity at the satellite transponder. These two functions are assumed to be memoryless.

The output signal, \( y(t) \), is attenuated and faded on the downlink path to be the fading signal, \( z(t) \), with envelope \( v \cdot f(R) \) and phase distortion \( \psi \), i.e.

\[ z(t) = v \cdot f(R) \cdot \cos(\omega_c t + \theta_0 + \eta(t) + g(R) - \psi) \]  

where \( v \) is the amplitude distortion factor and \( \psi \) is the phase distortion factor. Finally, the received signal is written

\[ r(t) = z(t) + I_d(t) + n_d(t) \]  

Then, quadrature detector coherently demodulates the signal. The sample taken at an optimal instant \( t_0 \) generates a maximum signal energy \( p(t_0) = \max[p(t)] \). The detector will determine the transmitted phase based on the observation of the sample, \( r_R = Z_R + n_{d,c} \) and \( r_I = Z_I + n_{d,s} \), where the components \( n_{d,c} \) and \( n_{d,s} \) are statistically independent in-phase and quadrature phase of the downlink Gaussian noise, and \( n_d \) is the Gaussian noise with zero mean and variance \( \sigma^2 \). Based on location of the pair \((r_R, r_I)\) on phase decision zone, signal detector determine which one of \( M \) phases, \( \theta_0 \), was, as is depicted in Fig. 2, where the received signal is distorted on both amplitude and phase. Without loss generality, we assume that the transmitted signal with \( \theta_0 = 0 \). From Fig. 2, an error occurs if and only if \((r_R, r_I)\) falls in error region, outside of the decision area of \( 2\pi/M \) radian centered at \( \theta = 0 \). Then, the conditional error probability is given

\[ P_e(r | R, \eta, \nu, \psi, \xi_d) = \frac{1}{2} \cdot \text{erfc}\left(\frac{Z}{\sqrt{2\sigma_d}}\right) \]  

where

\[ Z = v \cdot f(R_0) \cdot \cos(\eta + g(R_0) - \psi) + \sum_i \sqrt{2I_{d,i}} \cos(\xi_{d,i}(t_0)) \]  

(12-1)

and

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(12-1)
\[
P_u'(R', \eta | S_i, \theta_i) = e^{-x} dx \cdot d\eta \cdot P(v, w) dv \cdot d\psi
\]

where
\[
a = \frac{\sqrt{A}}{\sigma_u} \cdot S_i \cos(\eta - \theta_i)
\]

Then, using two-dimensional central moment technique (we will discuss later), Eq(16) can be expressed as
\[
P_E(r) = \sum_i C_i \cdot E_{S_i, \eta_i} \left[ \sum_j C_j \cdot E_{\xi_i} \left[P_u'(r | X_j, \eta_k, \psi_i, \xi_d) \right] \right]
\]

\[
P_u'(X_j, \eta_k | S_i, \theta_i) \cdot u(X_j + \frac{\sqrt{A}}{\sigma_u} \cdot S_i \cdot \cos(\eta_k - \theta_i))
\]

\[
= E_{S_i, \theta_i, \xi_d} \left[P_u'(r | S_i, \theta_i, \xi_d) \right]
\]

where \((C_i, \xi_i, \psi_i)\) is the cubature rule for fading factor; \((C_j, X_j)\) is the Gauss-Hermite quadrature rule, and \((C_i, \eta_i)\) is the Chebyshev first-Kind quadrature rule[3].

We find the probability of \(R'\) and \(\eta\) depends on \(S_i\) and \(\theta_i\), i.e. \(S_i \cdot \sin(\eta - \theta_i)\). From Eq(6)
\[
S_i \cos \theta_i + jS_i \sin \theta_i
\]

\[
= p_0 \cdot \sum_{k=0}^1 p_k \cdot \cos \theta_k + j \cdot \left( \sum_{i=0}^1 \frac{I_{i,j}}{A} \right) \cdot 2 \cdot \cos \xi_{u,i}(t)
\]

\[
+j \cdot \left( \sum_{k=0}^1 p_k \cdot \sin \theta_k + \sum_{i=0}^1 \frac{I_{u,i}}{A} \right) \cdot 2 \cdot \sin \xi_{u,i}(t)
\]

\[
= (p_0 + U_1 + V_{u,0}) + j(U_2 + V_{u,1})
\]

where
\[
p_0 = p(t_0) \quad \text{and} \quad p_k = p(t_0 - kT)
\]

\[
U_i = \sum_{k=0}^1 p_k \cdot \cos \theta_k
\]

\[
V_{u,0} = \sum_{k=0}^1 p_k \cdot \sin \theta_k
\]

and
\[
V_{u,1} = \sum_{i=0}^1 \frac{I_{u,i}}{A} \cdot 2 \cdot \cos \xi_{u,i}(t_0)
\]

\[
V_{u,0} = \sum_{i=0}^1 \frac{I_{u,i}}{A} \cdot 2 \cdot \sin \xi_{u,i}(t_0)
\]

Then, we have,
\[
S_i \cdot \sin(\eta - \theta_i)
\]

\[
= (p_0 + U_1 + V_{u,1}) \cdot \sin \eta - (U_2 + V_{u,0}) \cdot \cos \eta
\]

Hence, to analyze the error probability, we can employ the CMT again to find the joint probability function for \(U_i\) and \(U_2\), and \(V_{u,0}\) and \(V_{u,1}\). Then, the expected value of error probability, Eq(18), will becomes
\[
P_E(r) = \sum_{i,j} C_i \cdot C_j \cdot E_{\xi_d} \left[P_E(r | U_{i,j}, U_{Q,j}, V_{U,j}, V_{U,Q,j}, \xi_d) \right]
\]

\[
= E_{\xi_d} \left[P_E^n(r | \xi_d) \right]
\]

where \((C_i, U_{i,j}, U_{Q,j})\) and \((C_j, V_{U,j}, V_{U,Q,j})\) are the cubature rule of ISI and uplink ACI respectively. It depends not only on \(U_i\), \(U_Q\), \(V_{U,j}\) and \(V_{U,Q,j}\) but also on downlink ACI. For downlink ACI, according to [4]
\[
E_{\xi_d} \left[P_E(r | \xi_d) \right]
\]

\[
= \frac{1}{2} \cdot \text{erfc}(\rho_1) + \frac{1}{\sqrt{\pi}} \cdot \exp(-\rho_1^2) \cdot \sum_i H_{2i-1}(\rho_1) \cdot \rho_2^{2i} \cdot \mu_{2i} \cdot (2i)!
\]

for BPSK

\[
E[\rho_2^2] = \frac{1}{\sqrt{2\pi}} \cdot \frac{(v \cdot f(R))^2}{\sigma_d^2}
\]

\[
\mu_{2i} = E_{\xi_d} \left[V_{d,2i} \right]
\]

\[
V_{d,i} = \sum_{i=0}^1 \beta_i \cdot \cos \xi_{d,i}(t_0)
\]

\[
\beta_i^2 = \frac{I_{d,i}}{\frac{1}{2} \cdot (v \cdot f(R))^2}
\]

\[
H_i(\cdot) \text{ is the Hermite polynomial.}
\]

For QPSK,
\[
E_{\xi_d} \left[P_E(r | \xi_d) \right] = \text{erfc}(\rho_2) - \frac{1}{4} \cdot \text{erfc}^2(\rho_2)
\]

\[
+ \frac{1}{\sqrt{\pi}} \cdot \exp(-\rho_2^2) \cdot \sum_i \left[ 2 - \text{erfc}(\rho_2) \right] \cdot \sum_j H_{2i-1}(\rho_2) \cdot \rho_2^{2j} \cdot \mu_{2i} \cdot (2i)! \cdot (2j)!
\]

\[
\cdot (\rho_1)^{2(i+j)} \cdot \mu_{2i+2j}
\]

where
\[
\rho_2 = \rho_1 \cdot \sin \frac{\pi}{4}
\]

\[
\mu_{2i+2j} = E_{\xi_d} \left[V_{d,2i}^2 \cdot V_{d,Q}^{2j} \right]
\]

\[
V_{d,i} = \sum_{i=0}^1 \beta_i \cdot \cos \xi_{d,i}(t_0)
\]

\[
V_{d,Q} = \sum_{i=0}^1 \beta_i \cdot \sin \xi_{d,i}(t_0)
\]

Hence, based on downlink ACI point, the overall average error probability depends on the moment of downlink, \(\mu\); it depends on carrier power to downlink noise ratio, \(\rho\), and downlink ACI power to carrier power ratio, \(\beta\). Also, so, the error probability can be found as
\[
P_E(r) = P_E^n(\rho, \beta, \mu)
\]

After we get average error rate \(P_E(r)\) from equation (24) for fixed point of LEO, we should consider the Doppler frequency shift as well as the time varying...
elevation angles in LEO satellite communication systems. From [5], the Doppler frequency shift is considered as a function of elevation angles. Hence, the average error rate over the LEO channel can be expressed as

\[ P_E = E_a[P_E(r | \alpha)] \]

for continuous case (25-1)

\[ P_E = \sum P_E \cdot p_a(\alpha) \]

for discrete case (25-2)

Table I reported the probability density function of ELs for the different constellations considered. To get the average system BER, integration is required for EL of [0, 2\pi] for an individual BER.

<table>
<thead>
<tr>
<th>ELs</th>
<th>0-10</th>
<th>10-20</th>
<th>20-30</th>
<th>30-40</th>
<th>40-50</th>
<th>50-60</th>
<th>60-70</th>
<th>70-80</th>
<th>80-90</th>
</tr>
</thead>
<tbody>
<tr>
<td>GlobalNet</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>21.9</td>
<td>44.0</td>
<td>22.7</td>
<td>8.4</td>
<td>2.6</td>
<td>0.4</td>
</tr>
<tr>
<td>Iridium</td>
<td>0.0</td>
<td>16.5</td>
<td>32.1</td>
<td>26.3</td>
<td>11.6</td>
<td>6.0</td>
<td>4.2</td>
<td>1.6</td>
<td>0.8</td>
</tr>
<tr>
<td>Odyssey</td>
<td>0.0</td>
<td>0.0</td>
<td>5.2</td>
<td>15.1</td>
<td>19.0</td>
<td>19.9</td>
<td>16.9</td>
<td>4.8</td>
<td></td>
</tr>
</tbody>
</table>

Table I. Discretized Probability Density Function for the Elevation Angle [6].

III. Moment Technique

One-Dimension CMT: Eqs(14) and (16) can be expressed as Eq(18) based on the two-dimension moment technique. The two-dimension moment technique can be easily developed from one-dimension moment technique. The average of function \( g(x) \) based on the observation can be expressed as

\[ E[g(x)] = \frac{1}{b-a} \int_a^b g(x) \cdot f_x(x) \cdot dx \]

where \( f_x(x) \) denotes the probability density function of the random variable \( x \). Evaluation of \( E[g(x)] \) is equivalent to compute the integrals of equation (26). Whenever, \( f_x(x) \) is not available, we can resort to numerical techniques to calculate the approximate value of the integral. The set of \( \{x_i \}_{i=1}^N \) are called the abscessas and the set of \( \{\omega_i \}_{i=1}^N \) are called weights. The set of abscessas and weights is usually referred to as a quadrature rule. Although some of the quadrature rule \((\omega_i, x_i)\) corresponding to weight function \( g(x) \) are available in tabulated form [3], mostly it is not. Hence, the algorithms to generating the Gauss quadrature rule based on the known moments of weight function are developed.

Given a random variable \( x \) with range \([a,b]\) and all of whose moments exist, it is possible to define a sequence of polynomials \( p_0(x), p_1(x), \ldots, p_m(x) \), where the degree of \( p_m(x) \) is \( m \), that are orthonormal with respect to \( x \). The three term recurrence relation is known for the orthogonal polynomials, \( p_m(x) \), i.e.

\[ x \cdot p_{j+1}(x) = \beta_{j+1} \cdot p_{j-2}(x) + \alpha_j \cdot p_{j-1}(x) + \beta_j \cdot p_j(x) \]

\[ j = 1, 2, \ldots, m \]

The polynomial \( p_n(x) = k_n \cdot \Pi_{i=1}^n (x - x_i), k_n > 0, \) has \( n \) real roots \( a < x_0 < x_1 < \ldots < x_n < b \). These roots of the orthogonal polynomials play an important role as abscessas in Gauss quadrature rule. By defining

\[ \omega_n = \frac{1}{k_n} \cdot \frac{1}{p_{n+1}(x_i) \cdot p_n'(x_i)} \]

\[ l=1, 2, \ldots, N \]

the set of \( \{\omega_n, x_i \}_{i=1}^N \) is called quadrature rule with degree of precision \( 2N-1 \). This is the highest degree of precision that can be obtained by any quadrature rule with \( N \) weights and abscessas.

Based on the one-dimension CMT, the cubature rule theory for computing the average \( E[g(x,y)] \) be

\[ E[g(x,y)] = \sum_{i=1}^N \omega_i \cdot g(x_i, y_i) \]

Two-Dimension CMT: From the known joint moment \( \mu_{x,y} \) to compute the average of \( g(x,y) \),

\[ E[g(x,y)] = \int \int g(x,y) \cdot f_{x,y}(x,y) \cdot dx \cdot dy \]

\[ = \sum_{i=1}^N \sum_{j=1}^N \omega_{ij} \cdot g(x_i, y_j) \]

Since

\[ \mu_{x,y} = \sum_{i=1}^N \omega_i \cdot x_i \cdot \int y^k \cdot f_{x,y}(y \mid x_i) \cdot dy \]

\[ = \sum_{i=1}^N \omega_i \cdot x_i \cdot \mu_{x,y} \]

Since the quadrature rule \((\omega_i, x_i)\) are constructed from \( \mu_{x,y} \), \( \{\mu_{x,y} \}_{i=1}^N \) can be solved if the \( N_y \) sets of the joint moments are known. Therefore, based on above solutions of \( \mu_{x,y} \) for a fixed \( i \), one can apply the moment technique again for each \( i \) to construct the conditional density function \( f_{y \mid x}(y \mid x_i) \) with \( N_y \) points

\[ f_{y \mid x}(y \mid x_i) = \sum_{i=1}^{N_y} f_{y \mid x}(y \mid x_i) \cdot \delta(y - y_j) \]

Then,

\[ f_{x,y}(x,y) = \sum_{j=1}^{N_y} \sum_{i=1}^{N_y} \omega_{ij} \cdot \omega_j \cdot \delta(x - x_i) \cdot \delta(y - y_j) \]

if \( x \) and \( y \) are independent, the procedure will be simplified

\[ f_{x,y}(x,y) = \sum_{j=1}^{N_y} \sum_{i=1}^{N_y} \omega_{ij} \cdot \delta(x - x_i) \cdot \delta(y - y_j) \]

Hence, the Eq(18) becomes

\[ P_E(r) = \sum_{i=1}^N \sum_{j=1}^N C_k \cdot C_j \cdot E_{\xi_0, \theta_0} \left[ \sum_{k=1}^{N} \sum_{l=1}^{N} C_k \cdot C_l \cdot E_{\xi_0, \theta_0} \left[ P_r(X_k, \eta_l, \psi_j, \xi_d) \cdot u(X_k + \sqrt{A} \cdot S_j \cdot \cos(\eta_l - \theta) \right] \right] \]

\[ \left( 34 \right) \]
where \((C_i, v_i)\) is the quadrature rule of amplitude distortion for fading factor and \((C_j, \psi_j)\) is the quadrature rule of phase distortion for fading factor.

**IV. Numerical Analysis for Moment Calculation**

According to the theory that we derived, we have to know the moments of ISI, uplink ACI and downlink ACI, to construct the discrete probability density function for Eqs(18) and (21) and for Eqs(22) and (23). The moment calculation of ISI can be found in [2] From the obtained moment of ISI, using two-dimension moment technique, we can find the cubature rule for equation (21). For the moment calculation of ACI, we set the complex random variable as 

\[
V = V_1 + jV_Q \sim b \times \cos(i(t_0) + \psi(t_0)) \sim b_i \sim \mathcal{N}^i(0)
\]

where \(b_i\) can be either \(b_i = \left(\frac{I_{uid}}{A}\right)^{1/2}\) in Eq(28), or \(b_i = \beta_i\) in Eqs(22) and (23).

The calculation procedure is similar to ISI. However, the phase of downlink ACI is assumed to be uniformly distributed. Hence, the characteristic function will be

\[
\Phi_i(\omega) = E_{\xi_i}\{\exp\left(\frac{j}{2}(b_i e^{j\xi_i(t_0)} \omega + b_i e^{-j\xi_i(t_0)} \omega)\right)\}
\]

\[
= \frac{1}{2\pi} \int_0^{2\pi} \exp\left(\frac{j}{2}(b_i(e^{j\xi_i}(u - j\nu) + e^{-j\xi_i}(u + j\nu))) \cdot d\xi_i\right)
\]

Then, moment of ACI can be obtained by

\[
\mu_{qq'} = \frac{1}{(j)^{q+r}} \frac{\partial^q}{\partial u^q} \frac{\partial^r}{\partial v^r} \Phi(u, v) \bigg|_{uv=0}
\]

For normalized Ricean process [16], \(v\), it has a density function as

\[
p_v(v) = 2 \cdot (K + 1) \cdot v \cdot \exp[-K - (1 + K) \cdot v^2] \cdot I_0(2v \cdot \sqrt{K(K + 1)})
\]

where \(K\) is the Ricean parameter depends on different ELs [5]. The moment of Ricean process be

\[
\mu_N = \int_0^\infty v^n \cdot p_v(v) \cdot dv
\]

\[
= 2 \cdot (K + 1) \cdot \exp(-K) \cdot \sum_{m=0}^{n} \frac{(K(K + 1))^m}{m! \Gamma(m + 1)} \cdot \frac{1}{(1 + N + 2m + 1)} \cdot \Gamma\left(\frac{2 + N + 2m + 1}{2}\right) \cdot (1 + K)^{N + 2m + 2}/2
\]

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\]

**CONCLUSION**

We discuss the method using moment technique to analyze the error probability for a LEO system. Since there are a lot of the degradation factors in the system, the closed form solution for performance is hardly found. In our study, in order to avoid the difficulty in calculating the expectation of the complicated error-related functions, we use the moment technique to construct the discrete probability density function of each degradation factor. Although the evaluation method via moment technique can not provide the exact solution of the error performance of system, it can be used as an important reference to design a system considering the multiple degrading factors mentioned above.

**REFERENCE**