PERFORMANCE OF NONLINEARLY AMPLIFIED CODED MULTICARRIER SPREAD SPECTRUM SYSTEMS IN THE PRESENCE OF MULTIPATH FADING

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ABSTRACT

In this paper, we analyze the effect of a nonlinearity on the bit error rate performance of convolutionally coded multicarrier spread spectrum (MCSS) systems in the presence of multipath fading. This is an extension of the work in [1] to include the effect of a nonlinearity on coded MCSS systems.

1 INTRODUCTION

Orthogonal multicarrier modulation techniques, also known as orthogonal frequency division multiplexing (OFDM), have drawn considerable interest due to its potential for high speed transmission and due to its effectiveness in an interference limited environment such as a multipath fading channel [2]. This technique has already been used in digital audio broadcasting [2] and has been chosen for European digital terrestrial broadcasting [3].

The major disadvantage of OFDM is its high peak to mean envelope power ratio (PMEPR) [4], due to the multiplexing of multiple carriers before transmission. When linear amplification is desired, this high PMEPR requires high output backoff [3] resulting in low DC to RF conversion efficiency. This is especially undesirable for mobile portable transmitter where power is limited resource. Nevertheless, when the amplifier is operated near or in the saturation (nonlinear) region to achieve high power efficiency, nonlinear amplification causes signal distortion and spectrum re-growth due to inter-modulation products.

In this paper, we extend the analysis in [1] to the performance of a convolutionally coded MCSS systems in the presence of a nonlinear amplifier and multipath fading. We use bit error rate (BER) as our system performance measure.

2 SYSTEM AND CHANNEL MODEL

2.1 MODULATION

The convolutionally encoded information sequence is interleaved and converted from a serial stream to M parallel streams. The binary code symbol stream for the q-th carrier is denoted by \{d_{q,i}\}. The signal on the q-th carrier is given by

\[ d_q(t) = \sum_{i=-\infty}^{\infty} d_{q,i} P_T(t - iT_s) \]

where \(T_s = T_b R_c\) is a symbol duration and \(P_T\) is a unit rectangular pulse in [0, T]. \(R_c\) is the code rate and \(T_b\) is the information bit duration after the serial to parallel process in the absence of coding. The signal on q-th carrier is then multiplied by a pseudo random spreading code \(a_q(t)\), where

\[ a_q(t) = \sum_{i=-\infty}^{\infty} \sum_{i=0}^{N-1} a_{q,i} P_T(t - iT_c - iT_s) \]

and the chip sequence \(\{a_{q,i}\}\) is binary sequence of i.i.d. random variables with equal probability. Each spreading code has a chip duration of \(T_c = T_s / N\), where \(N\) is the spreading gain of the system. Finally, signals on each carrier are summed before amplification. The output signal \(x(t)\) of the modulator is then given by

\[ x(t) = \sqrt{2P} \sum_{q=1}^{M} a_q(t)d_q(t) \cos(w_c t + w_q t + \theta_q) \] (1)

where \(P\) is the power per carrier, \(w_c\) is the carrier frequency and \(\{\theta_q\}\) is sequence of i.i.d. random variables uniformly distributed over [0, 2\(\pi\)]. The separation between q-th carrier frequency and center frequency \(w_c\) is denoted by \(w_q = (2\pi q)/T_c\).

2.2 CHANNEL MODEL

The modulated signal is first nonlinearly amplified, then distorted by multipath fading and corrupted by additive white Gaussian noise. For the channel noise, we consider AWGN with a two-sided spectral density \(N_0/2\). For multipath fading channel, we assume a slowly changing Rayleigh flat fading for each carrier. The low pass equivalent channel response for q-th carrier is as follows

\[ h_q(t) = \beta_q \exp(j\varphi_q) \delta(t) \]

where \(\varphi_q\) is uniformly distributed over [0, 2\(\pi\)] and \(\beta_q\) is an i.i.d. Rayleigh r.v. with \(E[\beta_q^2] = 2\sigma_f^2\). \([\cdot]\) denotes the
expectation and $\delta(t)$ is the Dirac delta function. For the nonlinear amplifier model, we use a 5th order polynomial model for an amplitude nonlinearity (AM/AM) and assume the phase nonlinearity (AM/PM) is negligible. The output envelope $g(A(t))$ of the amplifier is related to the input envelope $A(t)$ of the signal as

$$g(A(t)) = A(t) + \alpha_1 A^3(t) + \alpha_2 A^5(t)$$

where $\alpha_1$ and $\alpha_2$ determine the amount of nonlinearity. Figure 1 shows $g(A(t))$ when $\alpha_1 = -0.1952$ and $\alpha_2 = 0.0162$. The coefficients $\alpha_1$ and $\alpha_2$ are obtained from curve fitting (least square) the solid state amplifier model used in [3, equation (7)]. In Figure 1, the input signal undergoes nonlinear amplification in the input envelope exceeds 0.4.

We can express modulator output in terms of the envelope $A(t)$ and the phase $\phi(t)$ as

$$z(t) = A(t) \cos(\omega_c t + \phi(t)),$$

where $A(t)$ and $\phi(t)$ is listed in [1, eq.(3)]. The maximum value of the envelope $A(t)$ is $M\sqrt{2P}$ and the root-mean-square (rms) value of $A(t)$ is $\sqrt{\frac{1}{T} \int_0^T A^2(t) dt}$.

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These two quantities, max$(A(t))$ and rms values are to be used in Section 4 when the signal operation point of the amplifier is determined. The output $y(t)$ of the amplifier is given by [1],

$$y(t) = g(A(t)) \cos(w_c t + \phi(t)) = \{1 + \alpha_1 A^2(t) + \alpha_2 A^4(t)\} x(t) = x(t) + x_3(t) + x_5(t),$$

where

$$x_3(t) = 2\alpha_1 P \sqrt{2P} \sum_{i=1}^M \sum_{j=1}^M \sum_{l=1}^M a_{i1} a_{i2} d_{j1} d_{j2} d_{j3} \cos((w_c + w_j) t + \theta_j),$$

$$x_5(t) = 4\alpha_2 P \sqrt{2P} \sum_{i=1}^M \sum_{j=1}^M \sum_{l=1}^M a_{i1} a_{i2} d_{j1} d_{j2} d_{j3} \cos((w_c + w_\ell) t + \theta_\ell).$$

For the sake of convenience, we omit the variable $t$ of $a_{i1}(t)$ and $d_{i1}(t)$. In the preceding equations, the index $j$ and $\ell$ are $l_1 + l_2 - l_3$ and $l_1 + l_2 - l_3 + l_4 - l_5$, respectively; and $\theta_j = \theta_{i1} + \theta_{i2} - \theta_{i3}, \theta_\ell = \theta_{l1} + \theta_{l2} - \theta_{l3} + \theta_{l4} - \theta_{l5}$. We have to note that the terms from 3rd order and 5th order nonlinearity, i.e. $x_3(t)$ and $x_5(t)$ in (5) and (6) can be divided into two parts, a desired term and an interference term. For $x_3(t)$ when $(l_3 = l_1) \cup (l_3 = l_2)$, which we call event $A$, the signals become $2\alpha_1 P x(t)$, which is just a scaled replica of the original signal before the amplification; for example when $(l_3 = l_1)$, $x_3(t)$ equals $2\alpha_1 P \sqrt{2P} \sum_{i=1}^M a_{i1} d_{i2} \cos((w_c + w_{i2}) t + \theta_{i2})$. There are $M$ scaled replicas of $x(t)$ from the $(l_3 = l_1)$ and $M$ scaled replicas of $x(t)$ from the $(l_3 = l_2)$, which makes $2M - 1$ scaled replicas of $x(t)$ from the event $A$. Similarly, for $x_5(t)$ when $(l_2 = l_3 \cap l_4 = l_5) \cup (l_2 = l_5 \cap l_3 = l_4)$ $(l_3 = l_5 \cap l_4 = l_5) \cup (l_1 = l_3 \cap l_2 = l_5) \cup (l_1 = l_5 \cap l_2 = l_3)$, which we call event $B$, the signals become $4\alpha_2 P \sqrt{2P} x(t)$. There are $6M^2 - 9M + 4$ scaled replicas of $x(t)$ from event $B$. $6M^2 - 9M + 4$ can be easily calculated from the following equality $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$ where $|.|$ denote the size of set. Thus $x_3(t)$ and $x_5(t)$ can be rewritten as

$$x_3(t) = 2\alpha_1 P \sqrt{2P} \sum_{i=1}^M \sum_{j=1}^M \sum_{l=1}^M a_{i1} d_{j1} d_{j2} d_{j3} d_{j4} \cos((w_c + w_j) t + \theta_j),$$

$$x_5(t) = 4\alpha_2 P \sqrt{2P} \sum_{i=1}^M \sum_{j=1}^M \sum_{l=1}^M a_{i1} d_{j1} d_{j2} d_{j3} \cos((w_c + w_\ell) t + \theta_\ell),$$

where $A^C$ and $B^C$ are the complements of events $A$ and $B$, respectively.

### 3 PERFORMANCE ANALYSIS

In this section, we first analyze the demodulator output statistics, and we then derive the upper bound on the average bit error probability for coded systems.

#### 3.1 DEMODULATOR AND ITS OUTPUT STATISTICS

The received signal $r(t)$ after the multipath fading and AWGN channel is as follows

$$r(t) = n(t) + F \sqrt{2P} \sum_{q=1}^M \beta_q a_q d_q \cos((w_c + w_q) t + \theta_q) +$$

$$2\alpha_1 P \sqrt{2P} \sum_{i=1}^M \sum_{j=1}^M \sum_{l=1}^M I_{(A^C)} \beta_i a_{i1} d_{j1} d_{j2} d_{j3} d_{j4} \cos((w_c + w_j) t + \theta_j) +$$

$$4\alpha_2 P \sqrt{2P} \sum_{i=1}^M \sum_{j=1}^M \sum_{l=1}^M \sum_{k=1}^M I_{(B^C)} \beta_i a_{i1} d_{j1} d_{j2} d_{j3} d_{j4} d_{j5} \cos((w_c + w_\ell) t + \theta_\ell),$$

where $F = \{1 + 2\alpha_1 P (2M - 1) + 4\alpha_2 P (6M^2 - 2M + 4)\}$, $\theta_q = \theta_\ell + \varphi_q$, $\theta_\ell = \theta_i + \varphi_\ell$ and $\theta_i = \theta_j + \varphi_i$. The $I(\cdot)$ is indicator function, which equals one if the event $\{\cdot\}$ occurs and zero otherwise. The demodulated symbol $Z_q$ after the integrate-and-dump filter of the $q$th subcarrier is

$$Z_q = \int_0^T r(t) a_q (t) \cos((w_c + w_q) t + \theta_q) dt$$

$$= n + D_q + I_{(A^C)}$$
where \( \eta \) is normally distributed with mean 0 and variance \( N_0 T_s/4 \) and

\[
D_q = F \sqrt{2P} \beta_q d_{q,0} T_s/2,
\]

\[
I_{\text{non},q} = \alpha_1 \beta_q P \sqrt{2P} \sum_{i_1=1}^{M} \sum_{i_2=1}^{M} \sum_{i_3=1}^{M} I_{\{A_j^i\}} d_{i_1} d_{i_2} d_{i_3}
\]

\[
\cos(\theta_j - \theta_q) \int_0^{T_s} a_{i_1} a_{i_2} a_{i_3} d\theta + 2\alpha_2 \beta_q
\]

\[
P^2 \sqrt{2P} \sum_{i_1=1}^{M} \sum_{i_2=1}^{M} \cdots \sum_{i_3=1}^{M} I_{\{B_j^i\}} d_{i_1} d_{i_2} \cdots d_{i_3}
\]

\[
\cos(\theta_i - \theta_q) \int_0^{T_s} a_{i_1} a_{i_2} \cdots a_{i_3} d\theta.
\]

The events \( \{A'\} \) and \( \{B'\} \) are defined as \( \{A^c \cap (l_1 + l_2 + l_3 = q)\} \) and \( \{B^c \cap (l_1 + l_2 + l_3 + l_4 + l_5 = q)\} \), respectively. The conditional mean and variance of \( Z_q \) are

\[
E[Z_q|\beta_q] = F \sqrt{2P} \beta_q d_{q,0} T_s/2,
\]

\[
\text{Var}[Z_q|\beta_q] = \frac{N_0 T_s}{4} + \frac{P^2 \beta_q^2 T_s^2}{4N} (G_q^2 + H_q^2)
\]

where

\[
G_q = 2\alpha_1 P \sum_{i_1=1}^{M} \sum_{i_2=1}^{M} \sum_{i_3=1}^{M} I_{\{A_j^i\}}, H_q = 4\alpha_2 P^2 \sum_{i_1=1}^{M} \sum_{i_2=1}^{M} \cdots \sum_{i_3=1}^{M} I_{\{B_j^i\}}.
\]

We have used the fact that \( E[I_{\text{non},q}|\beta_q] = 0 \) and \( \text{Var}[I_{\text{non},q}|\beta_q] = P^2 \beta_q^2 T_s^2 (G_q^2 + H_q^2)/4N \). The conditional SNR(\( \beta_q \)) is defined as

\[
\text{SNR}(\beta_q) = \frac{E^2[Z_q|\beta_q]}{\text{Var}[Z_q|\beta_q]} = \frac{2F_s \beta_q^2}{1 + \bar{c} + \bar{s}_q \beta_q^2 c_q/N}
\]

where

- \( \bar{F}_q = [F^2 + G_q^2 + H_q^2] T_s / 2 \): average received symbol-energy-to-noise power density ratio \( (E_s/N_0) \) on a \( q \)-th carrier in the absence of multipath fading.

- \( \bar{r}_q = \frac{1}{M} \sum_{q=1}^{M} \bar{F}_q \): average received \( E_s/N_0 \) per carrier in the absence of multipath fading.

- \( \bar{c}_q = (G_q^2 + H_q^2)/F^2 \): normalized nonlinear interference variance on a \( q \)-th carrier.

- \( \bar{c}_q = \frac{1}{M} \sum_{q=1}^{M} \bar{c}_q \): average normalized nonlinear interference variance per carrier.

Since \( I_{\text{non},q} \) are linear combination of independent random variables, providing that \( M \) is sufficiently large, the probability density function (pdf) of \( I_{\text{non},q} \) becomes normal by the central limit theorem. Based on this, we analyze the bit error rate performance.

### 3.2 BIT ERROR RATE FOR CODED SYSTEM

For the coded system, we consider the bit error probability of convolutional codes with maximum likelihood decoding. We assume that a perfect channel estimate is available so that we can weight the output of the integrate-and-dump filter of \( q \)-th carrier by a weighting factor \( g_q \) which is defined as \( g_q = E[Z_q|\beta_q] \text{Var}[Z_q|\beta_q] \). We assume that we have a sufficiently large size of interleaver that code symbols in the same carrier are independent to each other. Bounds on the bit error probability for a code can be expressed as follows

\[
P_{b,e} \leq \sum_{d=0}^{\infty} w_d P_2(d)
\]

where \( w_d \) is the number of paths with information weight \( d \), and \( P_2(d) \) is the error probability between two codewords which differ in \( d \) symbols and \( d_f \) is the free distance of the code. We consider a convolutional code of rate \( R_c = 1/2 \) with constraint length 7. We use truncated \( w_k \) up to approximately weight 30 which are tabulated in [5]. The pairwise error probability \( P_2(d) \) is given as follows, assuming we send all-zero message,

\[
P_2(d) = P\left( \sum_{i=1}^{d} g_i Z_i \leq 0 \right) = P\left( Z(d) \leq 0 \right).
\]

Since we approximate the statistics of demodulator output values given \( \beta_i \) as Gaussian random variables, the statistics of \( Z(d) \) given \( \beta_i \)'s are also Gaussian. The conditional pairwise error probability is then

\[
P\left( Z(d) \leq 0 | \beta_1, \beta_2, \ldots, \beta_d \right) = Q\left( \sqrt{\sum_{i=1}^{d} \text{SNR}(\beta_i)} \right)
\]

where

\[
Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp(-u^2/2) du
\]

\[
\text{SNR}(\beta_i) \geq \text{SNR}_{\text{min}}(\beta_i) = \frac{2\bar{r}_q \beta_i^2}{1 + \bar{c} + 4\bar{r}_q c_q^2 / N}.
\]

The \( c \) in the above equation is taken from the maximum value of \( c_q \) among \( M \) carriers, i.e. \( c = \max_{1 \leq q \leq M} c_q \). Since \( Q(\cdot) \) function is monotonic, we obtain the following inequality

\[
Q\left( \sqrt{\sum_{i=1}^{d} \text{SNR}(\beta_i)} \right) \leq Q\left( \sqrt{\sum_{i=1}^{d} \frac{2\bar{r}_q \beta_i^2}{1 + \bar{c} + 4\bar{r}_q c_q^2 / N}} \right).
\]

Now we need to average above equation over the random parameters, \( \{\beta_i\} \)'s. For the numerical convenience, we use the alternative representation for \( Q(x) \) [6, 7], which is given by

\[
Q(x) = \frac{1}{\pi} \int_{0}^{\pi/2} \exp\left(-x^2 / 2 \sin^2 \theta \right) d\theta.
\]

With this representation, (11) is upper bounded by

\[
P_2(d) \leq \frac{1}{\pi} \int_{0}^{\pi/2} I_{\text{mn}}^d(\theta) d\theta
\]

where
\[ I_{in}(\theta) = \int_0^{\infty} \exp\left\{ -\frac{1}{2\sin^2 \theta} \text{SNR}_{\text{min}}(\beta_i) \right\} f_{\beta_i}(\beta_i) d\beta_i \quad (14) \]

and \( f_{\beta_i}(\beta_i) \) is a pdf of Rayleigh Random variable. By change of variable, (14) becomes

\[ I_{in}(\theta) = \int_0^{1} \exp\left\{ -\frac{y}{2\sin^2 \theta} \right\} \frac{a}{(1- by)^2} \exp\left\{ -\frac{ay}{1- by} \right\} dy \]

where \( a = (1+\epsilon)/2\tilde{R}_s \), \( b = e/2N \), \( \tilde{R}_s = \tilde{R}_b R_e = 2\sigma_0^2 \tilde{R}_s \), \( \tilde{R}_b \) is the average received signal-energy-to-noise power spectrum density ratio per bit \( (E_b/N_0) \) per carrier. Note that we have defined \( N = R_e T_b/T_c \). If we let the spreading gain of the uncoded system \( N_u = T_b/T_c \), then \( N = R_e N_u \).

4 NUMERICAL RESULTS

We start by normalizing the root mean square value of the input envelope to have 3 different values so that it lies in strong, medium, and weak nonlinear regions. In order to do that we set \( P = e/2M \). The rms value of \( A(t) \) is then \( \sqrt{e} \) and the maximum value of \( A(t) \) becomes \( \sqrt{M} \). Larger value of \( e \) means more nonlinear amplification, in other words, the rms value of envelope is close to saturation (nonlinear) region. Notice that the peak envelope value increases as \( M \) becomes large even though the mean square value of the envelope is constant.

In this paper, we constrain our interest to eight carriers \( (M = 8) \). The reason for this is to make the Gaussian approximation of the interference from the nonlinearity \( I_{\text{non},q} \) more accurate, \( M \) should be reasonably large, but the maximum value of envelope \( (\sqrt{M} \sqrt{e}) \) should also fit in the input range of Figure 1. With 5th order polynomial model, the curve fitting of saturation region diverges if the input range becomes large. A higher polynomial order is needed to represent the saturation region over a large input range. The rms value and maximum envelope value corresponding to \( e \) are listed in Table 1.

In Figure 2-4, we plot the upper bound on the bit error rate versus average received \( E_b/N_0 \) per carrier for \( N_u = 1, 4, 16 \) and for \( e = 1/2, 1/4, 1/8 \) in AWGN channel. The upper bound on the bit error rate is calculated from the fact that \( P_2(d) \leq Q \left( (d \text{SNR}_{\text{min}}(\beta_i))^{0.5} \right) \). We see that the nonlinear effect on the bit error rate reduces as the spreading gain increases. The bit error rate also decreases as we operate in more linear region, which is an expected result.

In Figure 5-7, we show the upper bound on the bit error rate with AWGN and multipath fading environments. These bounds are obtained by numerically integrating (13). We see that bit error rate decreases as we operate our signal in more linear region and with a larger spreading gain in all three cases similar to the previous results in AWGN channel alone.

5 CONCLUSION

In this paper, we have shown the performance of nonlinearly amplified coded MCSS systems in the presence of additive white Gaussian noise and multipath fading. The performance is evaluated in terms of bit error rate and is analyzed based on the Gaussian assumption of the interference from the nonlinearity. It is also shown that the bit error rate degradation from the nonlinear amplifier can be reduced significantly even with a small size of spreading factor.

![Figure 1: Transfer Characteristic of Amplifier Model](image)

Table 1. rms and max\( \{A(t)\}\) correspond to \( e \)

<table>
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<th>( e )</th>
<th>rms</th>
<th>max( {A(t)}) for ( M=8 )</th>
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<td>1/2</td>
<td>0.7071</td>
<td>2</td>
</tr>
<tr>
<td>1/4</td>
<td>0.5</td>
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REFERENCE

Figure 2: Upper bound on BER in AWGN, $e = 1/2$

Figure 3: Upper bound on BER in AWGN, $e = 1/4$

Figure 4: Upper bound on BER in AWGN, $e = 1/8$

Figure 5: Upper bound on BER in fading, $e = 1/2$

Figure 6: Upper bound on BER in fading, $e = 1/4$

Figure 7: Upper bound on BER in fading, $e = 1/8$