COMPLEX PROCESSING IN QUATERNARY DIRECT-SEQUENCE SPREAD-SPECTRUM RECEIVERS

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ABSTRACT
The processing that is often employed in receivers for quaternary direct-sequence spread-spectrum communications operates separately on the inphase and quadrature components of the spread-spectrum signal. Recent results on quaternary complex sequences, which are also referred to as four-phase sequences, provide a strong motivation to employ processing methods that permit the receiver to benefit from the correlation properties of the complex sequences. This can be accomplished through the use of complex processing in the receiver. Receiver architectures are developed to take advantage of the improved periodic correlation properties of the quaternary complex sequences, and an analysis is presented which relates the performance of these receiver architectures to the complex correlation functions for the spreading sequences.

I. INTRODUCTION
Direct-sequence (DS) spread spectrum is employed in a wide range of applications including the Global Positioning System [1] and the IS-95 code-division multiple-access (CDMA) mobile cellular system [2]. In each of these two systems the spread-spectrum modulation is quaternary DS (QDS) spread spectrum with different binary spreading sequences on the inphase and quadrature components. Such a QDS spread-spectrum signal can be generated by adding two binary DS spread-spectrum signals that are modulated on orthogonal carriers. The binary signals may be binary phase-shift-key spread-spectrum signals or binary amplitude-shift-key spread-spectrum signals. Among the varieties of QDS spread spectrum that can be obtained by combining inphase and quadrature binary DS spread-spectrum signals are the quadrature-phase-shift-key (QPSK), offset QPSK, and minimum-shift-key spread-spectrum signals [3]. For each of these examples of QDS spread spectrum, the receiver may employ separate inphase and quadrature processing as is shown in Figure 1. Most of the performance evaluations of QDS spread-spectrum multiple-access communications are based on this architecture [3-5].

Recent results [6] suggest there are good QDS spread-spectrum signals that are not constructed from pairs of binary DS spread-spectrum signals. Boztas, Hammons, and Kumar [6] constructed quaternary complex sequences that are asymptotically optimum with respect to the bounds of Welch and Sidelnikov [8]. These sequences have better periodic correlation than quaternary sequences that are derived from pairs of standard binary sequences, and so the new quaternary sequences have the potential to provide better performance in multiple-access communication systems. Whether this potential is realized for an asynchronous spread-spectrum multiple-access system depends on the architecture of the spread-spectrum system, particularly the receiver, and on the aperiodic correlation that is achieved by the new quaternary sequences. In this paper we describe a receiver architecture that takes advantage of the improved periodic correlation properties, and we investigate the effects of the aperiodic correlation on the performance of the receiver. The proposed architecture also has the advantage that it exploits the complex processing capabilities of modern digital signal processing hardware.

II. QUATERNARY CORRELATION FUNCTIONS
In the vast majority of applications, DS spread-spectrum signals that are not generated at the same location are not synchronous in time or phase at any of the receivers. As a result the crosscorrelations that arise in the receiver span portions of two consecutive data symbols of an interfering signal [7]. For systems with binary data and binary signature sequences, this motivates the use of the periodic crosscorrelation function and the odd crosscorrelation function [3,8]. The periodic crosscorrelation function corresponds to two
adjacent data symbols taking on the same value, and the odd crosscorrelation function corresponds to the two data symbols taking on different values.

The DS spread-spectrum systems considered in this paper employ quaternary data modulation and use quaternary complex sequences as signature sequences. The quaternary sequences are composed of elements from the set \{1, j, -1, -j\}. For quaternary data there are four possible transitions between consecutive data symbols, and therefore there are four complex crosscorrelation functions of interest [9]. The quaternary complex crosscorrelation functions can be computed from two aperiodic crosscorrelation functions. For complex sequences \(x\) and \(y\) with period \(N\), the aperiodic crosscorrelation function is defined by

\[
C_{x,y}(l) = \begin{cases} 
N^{-1-l} \sum_{j=0}^{N-l} x_j y_{j+l}, & 0 \leq l \leq N-1, \\
N^{-1+l} \sum_{j=0}^{N-l} x_{j-l} y_j, & 1 - N \leq l < 0, \\
0, & |l| \geq N.
\end{cases}
\]

The quaternary complex crosscorrelation functions are defined [9-11] by

\[
\theta_{x,y}^{(n)}(l) = C_{x,y}(l) + C_{x,y}(l - N) \exp(jn\pi/2)
\]

for \(0 \leq l \leq N - 1\) and \(n \in \{0, 1, 2, 3\}\). The phase shift between two consecutive data symbols is \(n\pi/2\), where \(n\) is 0, 1, 2, or 3. The periodic crosscorrelation function corresponds to \(n = 0\). The quaternary complex autocorrelation functions can be derived from the quaternary complex crosscorrelation functions by setting \(x = y\).

III. QUATERNARY RECEIVER ANALYSIS

The receivers of interest in this paper are those that exploit the correlation properties of quaternary complex sequences. Block diagrams for such a transmitter and receiver are shown in Figure 2. The quaternary data signal is given by

\[
b(t) = \sum_{\ell=-\infty}^{\infty} b_\ell p_T(t - \ell T).
\]

The data sequence \(b = (b_\ell)\) is composed of elements of the form \(b_\ell = \exp(j\varphi_\ell)\), where \(\varphi_\ell \in \{0, \pi/2, \pi, 3\pi/2\}\) for each integer \(\ell\). The function \(p_T(t)\) is the rectangular pulse of duration \(T\).

The complex spreading signal is

\[
a(t) = \sum_{m=-\infty}^{\infty} a_m \psi(t - mT_c).
\]

The spreading sequence \(a = (a_m)\) is also composed of elements of the form \(a_m = \exp(j\varphi_m)\), where \(\varphi_m \in \{0, \pi/2, \pi, 3\pi/2\}\) for \(0 \leq m \leq N-1\). The spreading sequence has period \(N\), so \(a_{m+N} = a_m\) for each integer \(\ell\). The function \(\psi(t)\) is the chip waveform, which can be any time-limited function (limited to \([0,T_c]\)). A common choice for the chip waveform is \(p_T(t)\).

![Fig. 2. A QDS System with a Complex Modulator and a Complex Correlator](image)

For the results presented in this paper, there is one period of the spreading sequence per data symbol, so \(T = NT_c\).

The product of the quaternary data sequence and the quaternary spreading sequence is a complex baseband spread-spectrum signal \(v(t) = a(t)b(t)\). The baseband spread-spectrum signal is modulated onto the complex carrier signal \(A \exp[j(\omega_c t + \vartheta)]\), as shown in Figure 2. The transmitted signal is

\[
s(t) = \text{Re} \left\{ A v(t) \exp[j(\omega_c t + \vartheta)] \right\}.
\]

An asynchronous QDS spread-spectrum multiple-access system with \(K\) transmitters is shown in Figure 3. The transmitted signal from the \(k\)th transmitter is\n
\[
s_k(t) = \text{Re} \left\{ A_k v_k(t) \exp[j(\omega_c t + \vartheta_k)] \right\}.
\]

for \(1 \leq k \leq K\). The complex baseband spread-spectrum signal for the \(k\)th transmitter is \(v_k(t) = a_k(t)b_k(t)\), where

\[
b_k(t) = \sum_{\ell=-\infty}^{\infty} b^{(k)}_\ell p_T(t - \ell T)
\]

is the \(k\)th data signal. The \(k\)th signature sequence is \(a^{(k)} = (a^{(k)}_m)\), and thus the \(k\)th spreading signal is

\[
a_k(t) = \sum_{m=-\infty}^{\infty} a^{(k)}_m \psi(t - mT_c).
\]

In general the transmitters are not synchronous in time or phase. Signals arriving at a receiver therefore have different time offsets, as represented by the delay elements in Figure 3. The received signal is

\[
r(t) = \sum_{k=1}^{K} s_k(t - \tau_k) + n(t),
\]

where \(n(t)\) is additive white Gaussian noise with spectral density \(N_0/2\). As in [10], the received signal can be written as

\[
r(t) = \sum_{k=1}^{K} \left[ \text{Re} \left\{ A_k v_k(t - \tau_k) \exp[j(\omega_c t + \gamma_k)] \right\} \right] + n(t),
\]
where $\gamma_k = \theta_k - \omega_c \tau_k$. Consider a correlation receiver matched to the $i$th signal. As is customary [3], we let $\tau_i = 0$ and $\gamma_i = 0$, so that all time delays and phase shifts are measured relative to those of the $i$th signal. All time delays are defined modulo $T$ and all phase angles are defined modulo $2\pi$, (i.e. $0 \leq \tau_k < T$ and $0 \leq \gamma_k < 2\pi$ for each $k$ ) [3].

For the analysis of a correlation receiver matched to the $i$th signal, the random variable $Z_i$ is defined by

$$Z_i = \int_0^T r(t) a_i^*(t) \exp(-j\omega_ct)dt.$$  \hspace{1cm} (4)

Combining (3) and (4), we find that

$$Z_i = \sum_{k=1}^K \left[ \int_0^T s_k(t-\tau_k) a_i^*(t) \exp(-j\omega_ct)dt \right] + \eta_i,$$  \hspace{1cm} (5)

where the random variable $\eta_i$ is defined by

$$\eta_i = \int_0^T n(t) a_i^*(t) \exp(-j\omega_ct)dt.$$  

As shown in Figure 3, the product of the received signal and the complex exponential is the input to an ideal low-pass filter. The low-pass filter is included to remove double-frequency components. In practice if $\omega_C \ll \frac{1}{T}$, the integrator removes the double-frequency components, and the low-pass filter is not needed. Substituting (1) into (5) we have

$$Z_i = \sum_{k=1}^K \left[ \int_0^T a_k(t) a_i^*(t-\tau_k) \exp[j(\omega_c(t+\gamma_k))] \right]$$

$$\times a_i^*(t) \exp(-j\omega_ct)dt + \eta_i.$$  

Notice that $\text{Re}\{z\} = (z + z^*)/2$ implies

$$Z_i = \sum_{k=1}^K \left[ \int_0^T \frac{1}{2} a_k(t) a_i^*(t-\tau_k) \exp[j(\omega_c(t+\gamma_k))] \right]$$

$$\times a_i^*(t) \exp(-j\omega_ct)dt + \int_0^T \frac{1}{2} (a_k(t-\tau_k) a_i^*(t) \exp[-j(\omega_c(t+\gamma_k))]$$

$$\times a_i^*(t) \exp(-j\omega_ct)dt + \eta_i.$$  

Ignoring the double frequency components of $Z_i$, we see that the output of the $i$th receiver is

$$Z_i = \frac{1}{2} \sum_{k=1}^K \left[ \int_0^T (a_k(t-\tau_k) a_i^*(t) \exp(j\gamma_k)) dt ight]$$

$$+ \frac{1}{2} \int_0^T (a_k(t-\tau_k) a_i^*(t) \exp[-j(2\omega_c t + \gamma_k)]) dt + \eta_i.$$  

The data signal from the $k$th transmitter is given by (2). Thus $b_k(t-\tau_k) = b_{k1}^{(k)}$ for $0 \leq t < \tau_k$, and $b_k(t-\tau_k) = b_{01}^{(k)}$ for $\tau_k \leq t < T$. Also note that for $0 \leq t < T$, $b_i(t) = b_{01}^{(i)}$ and $a_i(t) a_i^*(t) = 1$ for all $i$.

$$Z_i = \frac{1}{2} \sum_{k} \left[ \exp(j\gamma_k) A_k \int_0^T b_k(t-\tau_k) a_i(t-\tau_k) a_i^*(t) dt ight]$$

$$+ \frac{1}{2} A_i \int_0^T b_i(t) a_i(t) a_i^*(t) dt + \eta_i.$$  

Next consider the continuous-time partial crosscorrelation functions [3]

$$R_{k,i}(\tau) = \int_0^T a_k(t-\tau) a_i^*(t) dt, \quad 0 \leq \tau \leq T,$$

and

$$\tilde{R}_{k,i}(\tau) = \int_\tau^T a_k(t-\tau) a_i^*(t) dt, \quad 0 \leq \tau \leq T.$$  

It is clear that $Z_i$ can be written in terms of these correlation functions. We define

$$I_{k,i}(\bar{z}, \tau, \gamma) = \frac{A_k}{T} \left[ b_{-1} \tilde{R}_{k,i}(\tau) + b_0 R_{k,i}(\tau) \right] \exp(j\gamma),$$  \hspace{1cm} (7)

which represents the normalized complex multiple-access interference from the $k$th signal. In (7) $\bar{b}$ is the vector $(b_{-1}, b_0)$, which represents two consecutive symbols.
Expressing (6) in terms of (7), we have

\[ Z_i = \frac{T}{2} \sum_k I_k,i(b_k, \tau_k, \gamma_k) + \frac{T}{2} A_i b_i^{(0)} + \eta_i. \]

The total normalized multiple-access interference is given by

\[ I_i(b, \tau, \gamma) = \sum_k I_k,i(b_k, \tau_k, \gamma_k). \]  
(8)

The vector of the data symbols is

\[ b = \left( b_0^{(1)}, b_1^{(1)}, b_0^{(2)}, \ldots, b_0^{(K)}, b_0^{(K)} \right) \]

where \( b_i^{(k)} \in \{1, j, -1, -j\} \) for all \( L \) and \( 1 \leq k \leq K \). The vector of time delays is

\[ \tau = (\tau_1, \tau_2, \ldots, \tau_K), \]

and the vector of phase angles is

\[ \gamma = (\gamma_1, \gamma_2, \ldots, \gamma_K). \]

The final expression for the output of the \( i \)th receiver is

\[ Z_i = \frac{T}{2} \left[ I_i(b, \tau, \gamma) + A_i b_i^{(0)} \right] + \eta_i. \]  
(9)

IV. MULTIPLE-ACCESS INTERFERENCE AND THE QUATERNARY CORRELATION FUNCTIONS

The second term in (9) is \( A_i b_i^{(0)} T/2 \), which corresponds to the desired signal. To minimize the multiple-access interference it is therefore necessary to minimize (8), which is minimized by minimizing (7) for each triple \((b_k, \tau_k, \gamma_k)\). For a given value of \( \tau \) in the range \( 0 \leq \tau < T \), there exists an integer \( l \) in the range \( 0 \leq l \leq N - 1 \) such that \( \ell T_c \leq \tau < (\ell + 1) T_c \). As shown in [3], the multiple-access interference can be written as

\[ I_{k,i}(b, \tau, \gamma) = \frac{A_k b_0}{T} \left[ C_{k,i}(\ell - N) \hat{R}_\psi(\tau - \ell T_c) \right. \]

\[ + C_{k,i}(\ell + 1 - N) \hat{R}_\psi(\tau - \ell T_c) \]

\[ + \left. b_0 \left( C_{k,i}(\ell) \hat{R}_\psi(\tau - \ell T_c) + C_{k,i}(\ell + 1) - N \hat{R}_\psi(\tau - \ell T_c) \right) \exp(j\gamma) \right] \]

\[ = \frac{A_k b_0^{(k)}}{T} \left[ C_{k,i}(\ell) + \frac{b_0 - b_0}{b_0 - b_0} C_{k,i}(\ell + N) \right. \]

\[ \times \hat{R}_\psi(\tau - \ell T_c) \]

\[ + \left. \left[ C_{k,i}(\ell + 1) + \frac{b_0 - b_0}{b_0 - b_0} C_{k,i}(\ell + 1 - N) \right] \times \hat{R}_\psi(\tau - \ell T_c) \right] \exp(j\gamma), \]

where \( \hat{R}_\psi(t) \) and \( \hat{R}_\psi(t) \) are the continuous-time partial autocorrelation functions for the chip waveform defined in [3].

The final step is to express the multiple-access interference as a function of the complex crosscorrelation functions, and the result is

\[ I_{k,i}(b, \tau, \gamma) = \frac{A_k b_0}{T} \left[ \theta_{k,i}^{(0)}(\ell) \hat{R}_\psi(\tau - \ell T_c) \right. \]

\[ + \left. \theta_{k,i}^{(n)}(\ell + 1) \hat{R}_\psi(\tau - \ell T_c) \right] \exp(j\gamma), \]  
(10)

where the integer \( n \) is defined by \( \exp(jn\pi/2) = b_{-1}/b_0 \).

Notice that some components of (10) depend on the chip waveform, and some components depend on the quaternary complex correlation functions of the signature sequences. In practice the selection of signature sequences to reduce the crosscorrelation functions has more effect on the performance than the selection of the chip waveform.

The optimal periodic correlation properties of the quaternary complex sequences in Family A are desirable for certain applications. However, in asynchronous DS spread-spectrum systems the effects of multiple-access interference cannot be described completely in terms of the periodic correlation function \( \theta_{k,i}^{(0)} \) only. In typical scenarios only one-fourth of the crosscorrelations in the receiver are periodic crosscorrelations, and the remaining crosscorrelations involve \( \theta_{k,i}^{(n)} \) for \( n \neq 0 \).

V. WORST-CASE MULTIPLE-ACCESS INTERFERENCE

For a correlation receiver matched to the \( i \)th signal the interference from the \( k \)th signal (i.e., (10)) is maximized for \( \tau = \ell T_c \). A detailed proof of this fact for binary sequences is given in [3], and the verification for quaternary complex sequences follows this proof closely.

For \( \tau = \ell T_c \), (10) reduces to

\[ I_{k,i}(b, \tau, \gamma) = \frac{A_k b_0}{T} \left[ \theta_{k,i}^{(0)}(\ell) \hat{R}_\psi(0) \right. \]

\[ + \left. \theta_{k,i}^{(n)}(\ell + 1) \hat{R}_\psi(0) \right] \exp(j\gamma). \]

In [3] it is shown that \( \hat{R}_\psi(0) = 0 \) and \( \hat{R}_\psi = \mathcal{E}_\psi \), where \( \mathcal{E}_\psi \) is the energy of the chip waveform, thus

\[ I_{k,i}(b, \tau, \gamma) = \frac{A_k b_0}{T} \left[ \theta_{k,i}^{(n)}(\ell) \mathcal{E}_\psi \right] \exp(j\gamma). \]

An upper bound for the interference for each \( b, \tau, \) and \( \gamma \) is therefore

\[ |I_{k,i}(b, \tau, \gamma)| \leq \frac{A_k}{T} \mathcal{E}_\psi c_{\text{max}}, \]

(11)

where \( c_{\text{max}} = \max\{\theta_c, \hat{\theta}_c\} \). The parameters \( \theta_c \) and \( \hat{\theta}_c \), which are measures of the maximum crosscorrelation for a set of quaternary complex sequences \( \chi \), are defined by

\[ \theta_c = \max\{|\theta_{x,y}^{(0)}(\ell)| : 0 \leq \ell < N, x \in \chi, y \in \chi, x \neq y \} \]

and

\[ \hat{\theta}_c = \max\{|\theta_{x,y}^{(n)}(\ell)| : 0 \leq \ell < N, x \in \chi, y \in \chi, x \neq y, 1 \leq n \leq 3 \}. \]
The parameter $c_{\text{max}}$ is the maximum value among the four quaternary complex crosscorrelation functions for the set of signature sequences. If $c_{\text{max}} \neq \theta_c$, the value of $c_{\text{max}}$ for a set of sequences depends on the phases of the sequences. The set of phases that minimize $c_{\text{max}}$ are referred to as the cross-optimal (CO) phases. Finding the CO phases for a large set of sequences, especially if the period is large, requires a prohibitive amount of computation. As a result, we employ suboptimal search routines to find phases of the sequences that have good aperiodic crosscorrelation properties.

Use of auto-optimal (AO) phases [8] for sequences has been suggested for communications over multipath channels. It is much simpler to find the AO phases for a set of sequences than to find the CO phases. Results on AO phases are given in [8] for binary sequences and in [9] for quaternary sequences. The quaternary sequences investigated in this paper are the quaternary complex sequences of Family $A$ introduced in [6]. The sequences of Family $A$ are of interest due to their optimal periodic correlation properties.

The set of quaternary complex sequences of period 31 from Family $A$ corresponding to the generator polynomial 100323 has been shown [9] to have $c_{\text{max}} = 14.32$ for the AO phases of the sequences. These sequences have $\theta_c = 6.40$ for all phases of the sequences, but $\theta_c = 14.32$ for the AO phases.

Our results show for some of the sets of sequences of interest there are phases that result in smaller values of $c_{\text{max}}$ than the AO phases. For example, consider the set of 33 quaternary complex sequences of period 31 that is generated by the polynomial 113123. The value of $c_{\text{max}}$ is 15.26 for the AO phases [9]. Using a suboptimal search routine we found a set of phases for which $c_{\text{max}}$ is 12.53. Further reductions in $c_{\text{max}}$ can be obtained for smaller sets of sequences. For example, we found a set of eleven of the quaternary complex sequences from Family $A$ of period 31 for which $c_{\text{max}}$ is 11.70.

Consider a multiple-access DS system with 11 transmitters. Assign as a signature sequence to each transmitter one of the quaternary complex sequences from the set of sequences in Family $A$ that has $c_{\text{max}} = 11.70$. Let each of the transmitted signals be received with equal power, (i.e., $A_k = \mathcal{A}$ for all $k$). The chip waveform is the rectangular pulse of duration $T_c$ with energy $E_\phi = T_c$. The maximum normalized multiple-access interference for this system is the maximum of $A^{-1}|I_{k,i}(\mu, \tau, \gamma)|$, maximized over all values for $\mu, \tau, \gamma, k,$ and $i$. Using the bound of (11), we see the maximum normalized multiple-access interference is less than $c_{\text{max}}/N = 11.70/31 = 0.377$. If the set of signature sequences and the phases of these sequences are not selected carefully, the maximum normalized multiple-access interference $c_{\text{max}}/N$ may be as large as 0.492.

In contrast to the quaternary complex sequences of period 31 for which $c_{\text{max}}$ is 14.32 for the AO phases, $c_{\text{max}} = 21$ for the AO phases of a set of binary Gold sequences of period 31 [8]. Using a suboptimal search routine phases can be found that reduce $c_{\text{max}}$ to 17 for the binary Gold sequences [8]. The smaller value of $c_{\text{max}}$ for the quaternary complex sequences demonstrates that for the receiver structure examined in this paper the quaternary complex sequences from Family $A$ give a smaller value for the worst-case multiple-access interference than Gold sequences. Reduced multiple-access interference means increased capacity for the multiple-access system.

VI. CONCLUSIONS

An analysis of a DS spread-spectrum receiver that exploits the periodic correlation properties of certain quaternary complex sequences is given in Section 3. It is shown that the multiple-access interference is directly proportional to various complex linear combinations of aperiodic cross-correlation functions for the quaternary complex sequences. The multiple-access interference in the receiver cannot be determined from periodic correlations alone, so it is necessary to evaluate the quaternary aperiodic correlation functions. The proposed receiver architecture exploits the capabilities of modern digital signal processing hardware through the use of complex operations in the despreading and down-conversion of the received signal. Optimization of the phases of the spreading sequences can result in a significant reduction in the maximum multiple-access interference, as illustrated in Section 5.

REFERENCES


