FAST JOINT SPACE-TIME ADAPTIVE PROCESSING
FOR DS/SS ANTENNA ARRAY SYSTEMS

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ABSTRACT

Under a maximum cross-correlation criterion the familiar space-time RAKE processor is equipped with a single orthogonal space-time auxiliary vector for blind multiple access interference (MAI) and noise suppression with one complex space-time degree of freedom. This approach is readily extended to cover blind processing with multiple auxiliary vectors and any desired number of complex degrees of freedom below the space-time product. A sequential procedure for conditional auxiliary-vector weight optimization is shown to lead to superior BER performance when rapid system adaptation through limited training data is sought.

I. INTRODUCTION

For a DS/CDMA system with M antenna elements, system processing gain \( L \), and \( N \) resolvable multipaths (usually \( N \) is between 2 and 4 including the direct path if any), joint optimum processing under minimum-mean-square criteria requires processing in the \( M(L + N - 1) \) space-time product space. As an example, if \( M = 5 \), \( L = 64 \), and \( N = 3 \), filter optimization needs to be carried out in the complex \( C^{330} \) vector space. Adaptive sample-matrix-inversion (SMI) implementations of the MMSE/MVDR filter solution are known to require data samples several times the space-time product \( M(L + N - 1) \) to approach the performance characteristics of their ideal counterparts [1]. In fact, theoretically, system optimization with less than \( M(L + N - 1) \) data samples is not even possible due to the singularity of the underlying sampled autocovariance matrix. With CDMA chip rates at 1.25 MHz, \( L = 64 \), and typical fading rate measurements between 4.5 Hz for mobiles on foot and 70 Hz for vehicle mobiles (carriers are assumed at 900 MHz) [2], the fading channel may fluctuate decisively as fast as every 280 data symbols. In this context adaptive SMI filter optimization in the \( C^{330} \) vector space becomes an unrealistic goal.

This paper focuses on fast adaptive joint space-time optimization through small training sets that can “catch-up” with multipath fading communications channels. In this direction, first we extend the auxiliary-vector framework -originally introduced in [3] for time-domain processing of DS/CDMA signals- to cover joint space-time processing in complex data vector spaces. In this context, the familiar joint space-time RAKE filter is equipped with a single orthogonal auxiliary vector -selected under a maximum cross-correlation criterion- for blind space-time MAI and noise suppression with one degree of freedom. Further generalization leads to adaptive generation of multiple, orthogonal to each other and to the space-time RAKE filter, auxiliary vectors for blind space-time MAI and noise suppression with any desired number of degrees of freedom below the space-time product. Interestingly enough, this generalization parallels well known “blocking matrix” processing techniques that have been used successfully for radar and antenna array applications in the past [4] and recently for time-domain-only CDMA signal processing [5]. However, to avoid sample-matrix-inversion altogether even for reduced \( P \)-dimension \( (P < M(L + N - 1)) \) blocking matrix processing, we generate a data dependent sequence of \( P \) auxiliary vectors through successive conditional optimization. Performance-wise, we maintain that in small sample support situations conditional optimization outperforms \( P \)-dimension blocking matrix processing without ever inverting the underlying \( P \times P \) covariance matrix.

II. SIGNAL MODEL

The contribution of the \( k \)-th user, \( k = 0, \ldots, K - 1, \) to the transmitted signal is denoted by

\[
u_k(t) = \sum_i b_k(i) \sqrt{E_k} s_k(t - iT)e^{j2\pi f_c t + \phi_k},
\]

where \( b_k(i) \in \{-1, 1\} \) is the \( i \)-th transmitted data (information) bit, \( E_k \) denotes energy, \( \phi_k \) is the carrier phase with carrier frequency \( f_c \), and \( s_k(t) \) is the assumed normalized user signature given by

\[
s_k(t) = \sum_{l=0}^{L-1} \delta_k(l) \psi(t - lT_c).
\]

In (2), \( \delta_k(l) \in \{-1, 1\} \) is the \( l \)-th bit of the spreading sequence of the \( k \)-th user, \( \psi(t) \) is the chip waveform, \( T_c \) is the chip period, and if \( T \) in (1) denotes the information bit duration then \( T/T_c = L \) is the system processing gain.

After multipath fading channel “processing”, the total signal due to all users received at the input of a narrowband uniform linear array of \( M \) elements is given by

\[
x(t) = \sum_{k=0}^{K-1} \sum_{n=0}^{N-1} \alpha_{k,n}(\tau) u_k(t - n/B - \tau) a_{k,n} + n(t),
\]

where \( N \) is the total number of resolvable multipaths (without loss of generality assumed to be the same for all users) and \( \alpha_{k,n}(\tau), k = 0, \ldots, K - 1, n = 0, \ldots, N - 1, \) are independent zero-mean complex Gaussian random variables that remain constant within a bit interval and model the fading process.
signal is defined by phenomena. Moreover, \( r_k \) in (3) denotes the relative transmission delay of user \( k \) with respect to user 0 with \( r_0 = 0 \), and with \( x_c(t) \) bandlimited to \( B = 1/T_c \) the tap-delay line channel model has taps spaced at chip intervals \( T_c \). The \( M \times 1 \) array response vector \( a_{k,n} \) for the \( n \)-th path of the \( k \)-th user signal is defined by

\[
a_{k,n}[m] = e^{j2\pi(m-1) \frac{a_{k,n}x_n}{A}}, \quad m = 1, \ldots, M, \quad (4)
\]

where \( x_n \) identifies the angle of arrival of the \( n \)-th multipath of the \( k \)-th user, \( \lambda \) is the carrier wavelength, and \( d \) is the element spacing (usually \( d = \lambda/2 \)). Finally, \( n(t) \) in (3) denotes an \( M \)-dimension complex Gaussian noise process that is assumed white both in time and space.

After carrier demodulation

\[
x(t) = \sum_{k=0}^{K-1} b_k(i) \sum_{n=0}^{N-1} c_{k,n}(i) \sqrt{E_x} s_k(t - iT - \frac{\theta_k}{2})
\]

(5)

where \( c_{k,n}(i) = \alpha_{k,n}(i)e^{-j2\pi c_0 \frac{n}{N} + \gamma_k} \) with \( \gamma_k \) being the carrier total phase absorbed into the channel coefficient. Assuming synchronization at the reference antenna element \( (m = 1) \) with the signal of the user of interest, say user 0, direct sampling of \( x(t) \) at the chip rate \( 1/T_c \) (or chip-matched filtering and sampling at the chip rate) over the multipath extended \( L+N-1 \) chip period prepares the data for one-shot detection of the \( i \)-th information bit of interest \( b(i) \). We visualize the space-time data in the form of a \( M \times (L+N-1) \) matrix

\[
X_{M \times (L+N-1)} = [x_{M \times 1}(0) \ x_{M \times 1}(T_c) \ldots \ x_{M \times 1}((L+N-2)T_c)]
\]

(6)

To avoid in the sequel cumbersome 2-D filtering operations and notation, we decide at this time to "vectorize" \( X_{M \times (L+N-1)} \) by sequencing all matrix columns in the form of a vector:

\[
X_{M \times (L+N-1)} = vec\{X_{M \times (L+N-1)}\}
\]

(7)

From now on \( X \) denotes the joint space-time data in the \( C^{M \times (L+N-1)} \) complex vector domain.

The cornerstone for any form of joint space-time filtering is the space-time RAKE filter that we define compactly for user 0 as the cross-correlation between the received space-time data \( X \) and the desired information bit \( b_0 \):

\[
V_0 \triangleq E_x \{ X b_0 \}
\]

(8)

where the statistical expectation operation \( E \{ \cdot \} \) is taken with respect to \( b_0 \) only.

### III. AUXILIARY-VECTOR PROCESSING

With respect to a specific DS/CDMA user of interest (user 0), we assume that we have available the joint Space-Time (S-T) RAKE filter \( V_0 \) as defined in the previous section and a finite set of joint S-T data \( X_j, j = 1, 2, \ldots, J \). No knowledge of the information bits of the user of interest in \( X_j, j = 1, \ldots, J \), is assumed, so only "blind" system optimization may be pursued. We wish to determine a detection strategy for a DS/CDMA system with \( M \) antenna elements, system processing gain \( L \), and \( N \) resolvable multipaths per user, given the S-T RAKE vector \( V_0 \) and a total number of \( J \) training data.

Without loss of generality, let us assume that the S-T RAKE filter \( V_0 \) is normalized, that is \( V_0^H V_0 = 1 \). Next, we consider an arbitrary fixed "auxiliary" vector \( G \) that is orthonormal with respect to \( V_0 \):

\[
G^H V_0 = 0 \quad \text{and} \quad G^H G = 1.
\]

(9)

We propose S-T adaptive processing with a single joint S-T degree of freedom in the form of \( w_{AV}^H X \), where [3]

\[
w_{AV} \triangleq V_0 - \mu G, \quad \mu \in C.
\]

(10)

Decision making is carried out as seen in Fig. 1:

\[
\hat{b}_0 = sgn(Re\{w_{AV}^H X\}).
\]

(11)

With an arbitrary but fixed auxiliary vector \( G \) that satisfies the constraints in (9), the filter in (10) can be optimized with respect to the complex scalar \( \mu \) only. The MS-optimum value of \( \mu \) can be identified from two different -yet equivalent- points of view. Since \( w_{AV} \) is by construction distortionless in the \( V_0 \) direction (\( w_{AV}^H V_0 = 1 \)), we may seek the value of \( \mu \) that minimizes the output variance (energy) \( E\{ |w_{AV}^H X|^2 \} \). An equivalent interpretation motivated by Fig. 1 is to look for the value of \( \mu \) that minimizes the MSE \( E\{ (V_0^H X - \mu^* G^H X)^2 \} \) between the S-T main-beam processed data \( V_0^H X \) and the orthogonal auxiliary-vector processed data \( \mu^* G^H X \). The solution to the first minimum output variance problem can be obtained by direct differentiation of the variance expression. The solution to the second MSE problem can be obtained by direct application of the optimum linear MS estimation theorem. The solution is, of course, identical in both cases and it is given by the following proposition.

**Proposition 1** The value of the complex scalar \( \mu \) that minimizes the output variance of the filter \( w_{AV} \triangleq V_0 - \mu G \) or minimizes the mean square error between the main beam processed data \( V_0^H X \) and the auxiliary-vector processed data \( \mu^* G^H X \) is

\[
\mu = \frac{G^H R V_0}{G^H R G},
\]

(12)

where \( R = E\{XX^H\} \) is the S-T data covariance matrix.

Proposition 1 identifies the optimum value of \( \mu \) for any fixed auxiliary vector \( G \). We now turn our attention to the selection of an auxiliary vector subject to an appropriately chosen criterion. The selection criterion for the auxiliary vector \( G \) that we propose in this work is motivated by the MSE interpretation of the filter in Fig. 1 and it is the maximization of the magnitude of the crosscorrelation between the \( V_0 \) processed data \( V_0^H X \) and the auxiliary-vector processed data \( G^H X \), subject to the constraints in (9):

\[
G = \arg \max \{ |E\{V_0^H X (G^H X)^*\}| \} = \arg \max \{ |V_0^H R G| \},
\]

subject to \( G^H V_0 = 0 \) and \( G^H G = 1 \).

(13)

We note that both the criterion function \( |V_0^H R G| \) and the constraints are phase invariant. Therefore, without loss of generality, we may identify the unique auxiliary vector that is
a solution to the constraint optimization problem in (13) and places \( V_0^H R G \) on the non-negative real line \( (V_0^H R G \geq 0) \). The following proposition identifies this optimum auxiliary vector according to our criterion. The proof is omitted due to lack of space.

**Proposition 2** The auxiliary vector

\[
G = \frac{RV_0 - (V_0^H RV_0) V_0}{\|RV_0 - (V_0^H RV_0) V_0\|} \tag{14}
\]

maximizes the magnitude of the crosscorrelation between the main-beam processed data \( V_0^H X \) (w.l.o.g. \( V_0^H V_0 = 1 \)) and the auxiliary vector processed data \( G^H X \), \( |V_0^H RG| \), subject to the constraints \( G^H V_0 = 0 \) and \( G^H G = 1 \). ❑

Proposition 2 completes the design of the joint S-T auxiliary vector detector. As a brief summary, the sequence of calculations is as follows: \( G \) is given by (14), then \( \mu \) is calculated by (12), the filter structure takes the form of (10), and decisions are made according to (11). The S-T data autocovariance matrix \( R \) that appears in the expression for the ideal \( G \) and \( \mu \) may be sample average estimated, \( \hat{R} = \frac{1}{J} \sum_{j=1}^{J} X_j X_j^H \), as usual.

Straightforward algebraic manipulations show that the output variance of the auxiliary vector filter as a function of the auxiliary vector \( G \) is

\[
VAR_{AV} = E\{[w_B^H X]^2\} = V_0^H R V_0 - \frac{|G^H R V_0|^2}{G^H R G}. \tag{15}
\]

It is interesting to observe that, as it turns out, the auxiliary-vector selection criterion that we proposed corresponds to the maximization of the numerator of the second term in the \( VAR_{AV} \) expression subject to the constraints in (9). Maximize the second term as a whole, that is including the denominator, with respect to \( G \) with \( G^H V_0 = 0 \) and unconstrained norm leads to optimization with full \( M(L + N - 1) \) degrees of freedom that takes us back to the optimum MVDR solution. Joint S-T optimum MVDR filtering was considered in [6], for example.

In the following section we extend Propositions 1 and 2 to cover joint S-T processing with multiple auxiliary vectors.

**IV. MULTIPLE AUXILIARY VECTORS**

We consider a set of \( P \), \( 1 \leq P \leq M(L + N - 1) - 1 \), auxiliary vectors \( G_1, G_2, \ldots, G_P \) that are orthonormal with respect to each other and to the normalized S-T RAKE vector \( V_0 \). We may organize the auxiliary vectors in the form of an \( A(L + N - 1) \times P \) matrix \( B \) where \( G_p, p = 1, \ldots, P \), constitutes the \( p \)-th column of \( B \):

\[
B_{M(L+N-1)xP} = [G_1, G_2 \ldots G_P]. \tag{16}
\]

With this setup the orthonormality conditions translate to

\[
B^H V_0 = 0_{P \times 1} \quad \text{and} \quad B^H B = 1_{P \times P}. \tag{17}
\]

We call \( B \) a “blocking” matrix because it blocks signal components in the S-T direction of interest \( V_0 \). Next, we consider S-T processing of DS/CDMA signals with \( P \) joint S-T degrees of freedom in the form of a vector \( \mu_{P \times 1} \). In this context, the overall linear filter can be written compactly as

\[
w_B \triangleq V_0 - \sum_{i=1}^{P} \mu_i G_i \quad = V_0 - B \mu, \tag{18}
\]

and decisions are made according to

\[
\hat{b}_0 = \text{sgn}(\Re\{w_B^H X\}). \tag{19}
\]

Blocking matrix processing techniques have been of considerable interest in antenna and radar array processing applications in the past [4]. In fact, some recent research efforts focus on the selection of the blocking processor based on eigenvalue metrics [7]-[9] or data independent Discrete-Fourier (DFT) or Walsh-Hadamard transforms (WHT) [5]. In this present work, instead of eigen analysis or data independent processing, we wish to extend the design criterion introduced in the previous section for processing with a single auxiliary vector to cover blocking matrix processing with \( 1 < P \leq M(L + N - 1) - 1 \) auxiliary vectors.

For an arbitrary but fixed blocking matrix \( B \) that satisfies the constraints in (17), the linear filter \( w_B \) can be optimized with respect to the \( P \)-dimension complex vector \( \mu \). Since the overall blocking matrix filter \( w_B \) in (18) is by construction distortionless in the \( V_0 \) direction of interest \( \langle w_B^H V_0 \rangle = 1 \), we may search for the weight vector \( \mu \) that minimizes the output variance \( E\{[w_B^H X]^2\} \). This would suppress all signal components that are not in the \( V_0 \) S-T direction of interest. Equivalently, we may instead identify the vector \( \mu \) that minimizes the MSE \( E\{(V_0^H X - \mu^H B^H X)^2\} \) between the main-beam processed data \( V_0^H X \) and the blocking-matrix processed data \( \mu^H B^H X \). Both approaches lead, of course, to the same solution and the optimum weight vector is identified by Proposition 3 below. This result is a generalization of Proposition 1 of the previous section.

**Proposition 3** The \( P \)-dimension, \( 1 \leq P \leq M(L + N - 1) - 1 \), complex weight vector \( \mu \) that minimizes the output variance of the filter \( w_B \triangleq V_0 - B \mu \) or minimizes the mean square error between the main-beam processed data \( V_0^H X \) and the blocking-matrix processed data \( \mu^H B^H X \) is

\[
\mu = (B^H R B)^{-1} B^H V_0. \tag{20}
\]

With this result, the output variance of the *ideal* filter \( w_B \) as a function of the blocking matrix \( B \) can be seen to be

\[
VAR_B \triangleq E\{[w_B^H X]^2\} = V_0^H R V_0 - V_0^H R B (B^H R B)^{-1} B^H R V_0. \tag{21}
\]

Blocking matrix processing with \( P \) joint S-T degrees of freedom requires inversion of the \( P \times P \) blocked data autocovariance matrix \( B^H R B \) as seen by (20). To avoid matrix inversion operations altogether, we may consider recursive *conditional* optimization of the \( P \) weighted auxiliary vectors. The anticipated benefits are significantly lower computational optimization complexity and superior BER performance for small sample support optimization. In parallel to the blocking-matrix filter definition in (18), let us introduce a new filter \( w_c \) consisting of \( P \), \( 1 \leq P \leq M(L + N - 1) - 1 \), weighted auxiliary vectors that are orthonormal with respect to each other and to the normalized S-T RAKE vector \( V_0 \):

\[
w_c \triangleq V_0 - \sum_{i=1}^{P} c_i G_i. \tag{22}
\]

\( G_1 \) and \( c_1 \) are optimized exactly as in (14) and (12). Next, we upgrade the “main-beam” to \( V_0 - c_1 G_1 \) and we seek an
auxiliary vector $G_2$ that maximizes the crosscorrelation magnitude $\left[ (V_0 - c_1 G_1)^H R G_2 \right]$ subject to the orthonormality constraints $G_2^H V_0 = 0$, $G_2^H G_1 = 0$, and $G_2^H G_2 = 1$. Arguing as in Proposition 2 we find

$$G_2 = \frac{R(V_0 - c_1 G_1) - [V_0^H R(V_0 - c_1 G_1)]V_0 - [R(V_0 - c_1 G_1)]V_0 - \left[ G_1^H R(V_0 - c_1 G_1) \right] G_1}{G_1^H R(V_0 - c_1 G_1) G_1}.$$  \hfill (23)

The conditionally MS-optimum value of $c_2$ given $G_1$, $c_1$, and $G_2$ is

$$c_2 = \frac{G_2^H R(V_0 - c_1 G_1)}{G_2^H G_2}.$$  \hfill (24)

To complete the recursive construction of the filter $W_c$ in (22) let us assume that $G_i$, $c_i$ are defined for $i = 1, \ldots, p$, $1 \leq p \leq (P - 1)$. Then, with "main-beam" $V_0 = \sum_{i=1}^p G_i$, the auxiliary vector $G_{p+1}$ that maximizes the crosscorrelation magnitude $\left[ (V_0 - \sum_{i=1}^p G_i)^H R G_{p+1} \right]$ subject to the orthonormality constraints is

$$G_{p+1} = \frac{R(V_0 - \sum_{i=1}^p c_i G_i) - [V_0^H R(V_0 - \sum_{i=1}^p c_i G_i)]V_0 - \left[ \sum_{i=1}^p c_i G_i \right] V_0 - \left[ \sum_{i=1}^p c_i G_i \right] V_0 - \left[ \sum_{i=1}^p G_i \right] V_0 - \left[ \sum_{i=1}^p G_i \right] V_0 - \left[ \sum_{i=1}^p c_i G_i \right] G_i}{\sum_{i=1}^p c_i G_i} V_0 - \sum_{i=1}^p c_i G_i G_i}.$$  \hfill (25)

The conditionally MS-optimum coefficient $c_{p+1}$ becomes

$$c_{p+1} = \frac{G_{p+1}^H R(V_0 - \sum_{i=1}^p c_i G_i)}{G_{p+1}^H G_{p+1}}.$$  \hfill (26)

In the following section we compare the plain S-T RAKE filter $V_0$, the full-scale MVDR filter, the single auxiliary-vector filter $W_{AV}$ in (10), the blocking matrix filter $W_B$ in (18), and the conditionally optimized sequence of auxiliary vectors in the form of the filter $W_c$ in (22). The comparisons focus on the induced BER as a function of the required sample support.

V. NUMERICAL & SIMULATION STUDIES

We consider the DS/CDMA signal model in (5) for a system with $M = 5$ antenna elements and processing gain $L = 15$. We assume the presence of $K = 6$ active users with fixed signature assignments and the normalized synchronous signature cross-correlations between the user of interest and the five interferers are chosen between 0.2 and 0.3. Each user signal experiences $N = 3$ independent paths with angles of arrival uniformly distributed in $( -\pi/2, \pi/2 )$.

In Figs. 2-6 we examine the BER performance of all filters discussed in this work as a function of the total SNR of the user of interest (the sum of the received SNRs over all paths) and the sample support. All BERs for all filters are analytically evaluated and the results that we present are averages over 100 independent S-T channels and 10 independent filter realizations per S-T channel.

In Fig. 2 we study the BER characteristics of the full-scale MVDR filter. The total SNR of the interferers is fixed at 7dB, 8dB, 9.5dB, 10.5dB, and 12dB, respectively. Fig. 2 shows the BER of the user of interest as a function of its total SNR over the 0-12dB range for various training set sizes. The BERs of the ideal S-T RAKE and the ideal MVDR filter are included as reference points.

Fig. 3 replicates the studies of Fig. 2 for the blocking-matrix filter $W_B$ in (18) with $P = 5$ auxiliary vectors selected according to recursion (25), (26). Some minimal improvement with respect to small sample support BERs can be seen.

Fig. 4 presents the same studies for the single auxiliary-vector filter $W_{AV}$ in (10). Significant BER improvement is achieved for training set sizes equal to a few times the space-time product $M(L + N - 1) = 85$.

In Fig. 5 we maintain the same set-up and we study the BER behavior of a conditionally optimized sequence of $P = 10$ auxiliary vectors in the form of the filter $W_c$ in (22). These results indicate superior performance by orders of magnitude compared with the previously examined filters for sample support between $M(L + N - 1) = 85$ and $10M(L + N - 1) = 850$.

Finally, Fig. 6 assumes fixed total SNR for the user of interest at 12dB and merges the results of Figs. 2-5 in the form of BER plots versus required number of training samples. The superiority of the conditionally optimized sequence of auxiliary vectors and the light-weight single auxiliary vector filter in comparison with their blocking-matrix optimized and full-scale MVDR counterparts is apparent over the whole $M(L + N - 1) = 85$ to $20M(L + N - 1) = 1,700$ data support range.

REFERENCES


Figure 1: Joint space-time auxiliary-vector detection.

Figure 2: BER versus total SNR for the space-time MVDR filter with varying sample support.

Figure 3: BER versus total SNR for a space-time blocking matrix filter with five auxiliary vectors and varying sample support.

Figure 4: BER versus total SNR for the single auxiliary-vector filter with varying sample support.

Figure 5: BER versus total SNR for a sequence of ten conditionally optimized auxiliary vectors with varying sample support.

Figure 6: BER versus number of training samples for the filters examined in Figs. 2-5 (total SNR of the user of interest is fixed at 12dB).