Multipath Tracking and Combining in Direct-sequence Spread-spectrum Packet-radio Communications

Roman J. Bednarek and Daniel L. Noneaker
Department of Electrical and Computer Engineering, Clemson University
102 Riggs Hall, Box 340915
Clemson, SC 29634-0915

ABSTRACT

Multipath fading in wide-area radio communications motivates the use of rake reception in a packet-radio system that employs direct-sequence (DS) spread-spectrum modulation. The acquisition header in each packet is used to derive initial tap delays for multipath combining in the rake receiver. However, fast fading can result in differences in the optimal tap delays during the reception of different portions of a given packet. The receiver obtains best performance if it tracks changes in the channel response during reception of a packet and makes corresponding changes to the tap delays, but the improved performance is achieved only at the expense of the additional signal energy and complexity required to implement the tracking.

In this paper, two strategies for tracking and combining multipath signal components are examined for a DS packet-radio system. Analytical and simulation results are presented for the probability of error for each approach under consideration. In particular, the performance of a simple scheme using fixed tap delays for the duration of the packet is compared with the performance of more complex multipath-tracking receivers. The effects of the channel characteristics, the mobile velocity, and the number of taps is considered in the comparison.

I. Introduction

Direct-sequence (DS) spread-spectrum has been proposed for use in military packet-radio systems [1],[2] and in emerging personal communications systems [3]. It is also used in mobile cellular code-division multiple-access (CDMA) systems such as the IS-95 standard [4]. In the battlefield environment, the presence of threats to individual network elements and the mobility of the network as a whole dictate a robust network design, and a DS network employing packet-data transmission is widely viewed as a means to achieve this robustness. The design of the receiver is a key to obtaining acceptable link performance and, indirectly, acceptable network performance in a DS packet-radio network, and it is the design of the receiver that is addressed in this paper.

Transmissions in a mobile communications system are often subjected to multipath fading [5], and direct-sequence spread-spectrum modulation allows the receiver to resolve distinct multipath components of the received signal. A correlation receiver can be used to reject signal energy from all but a single desired path, or a rake receiver can be used to combine signal energy from several propagation paths [6]. In rake reception, each of several correlators is synchronized to a distinct multipath component of the received signal, and the outputs of the correlator taps are combined to yield a decision statistic for the received data symbol.

In a hardware implementation of a rake receiver, only a fixed number of correlators are available, and it is desirable to adapt the tap-delay settings of the correlators to changes in the channel response in order to optimize performance. The best performance is obtained if the receiver can measure the changes in the strength of the multipath components and adapt the tap delays to track the strongest components. The fade rate of the channel limits the accuracy of tracking that is possible at the receiver, and it determines the rate at which tap delays must be updated in order to effectively exploit adaptive rake reception. In contrast, if a rake receiver with fixed tap delays is used, the system is unable to adapt to the changes in the channel once the initial tap delays are set.

The focus of this paper is a DS spread-spectrum packet-data communications system and the comparison of two techniques for rake reception: reception in which the tap delays are set during packet acquisition and remain fixed for reception of the entire packet (fixed-tap rake reception), and reception in which the tap delays are adapted periodically during reception of the packet in order to compensate for changes in the channel (adaptive rake reception). It is assumed that adaptation in the latter approach is based on retraining sequences that are placed at periodic intervals in the data transmission. Binary differential phase-shift-keyed (DPSK) modulation is considered with noncoherent rake reception using equal-gain square-law combining. The probability of bit error is determined over a multipath, fading channel for several mobile velocities and tap-delay update rates. The performance of the adaptive rake receiver is compared with the performance of fixed-tap rake receivers, and the minimal update rate for effective use of adaptive taps is determined for different channels and fade rates.

II. Doubly Selective Fading Channels

The channel is characterized by an impulse response that is given by

\[ h(t, r) = \sum_{i=0}^{M-1} h_k \delta(r - iT_c), \]

for \( t \in [(k-1)T, kT) \),

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where $T$ is the symbol duration, $T_c$ is the chip duration, and $M$ is the number of paths. The amplitude of each path is represented by a zero-mean complex-valued Gaussian random variable $h_{k,l}$, which is characterized by its autocovariance function

$$\text{Cov}[h_{k,l}, h_{m,n}] = 2\sigma_f^2 \delta[(k - m)T] \delta[(l - n)],$$

where $2\sigma_f^2$ is the average power in the diffuse component of the $l$th path. This piecewise-constant channel model approximates a special case of a Gaussian wide-sense-stationary uncorrelated-scattering (GWSSUS) channel, and it exhibits doubly selective Rayleigh fading. The time-correlation function $d$ is characterized by the half-power bandwidth, $B_d$, of its Fourier transform, and the normalized half-power bandwidth $D_T = B_dT$ is referred to as the Doppler spread of the channel. For a given channel-symbol rate, the Doppler spread is proportional to the system's carrier frequency and the relative velocity of the transmitter and the receiver.

### III. System Model

The DS packet transmission format includes an acquisition preamble and a data payload. If the system is designed to support adaptive rake reception, then the data payload includes $n_u$ update intervals, each consisting of a retraining sequence of $N_r$ chips followed by $n_P$ channel symbols of $N_c = T/T_c$ chips each. The transmitted signal for the data payload is given by

$$s(t) = \sqrt{\frac{\mathcal{E}_b}{T}} \sum_{k=0}^{n_u-1} [u(t)p_{N_r, T_c}(t - k(N_rT_c + n_P T))]$$

$$+ \sum_{t=0}^{s-1} (-1)^{k+t} u(t)$$

$$\times p_T(t - k(N_rT_c + n_P T) - N_{r} T_c - iT),$$

where $u$ is the DS spreading waveform with

$$\int_{-T}^{T} |u(t)|^2 dt = T,$$

and $p_T$ is the unit height rectangular pulse of duration $T$. The polarity of the $i$th data pulse is given by $b_i$ and $\mathcal{E}_b$ represents the energy per bit.

The signal is transmitted over a fading, multipath additive-white-Gaussian-noise (AWGN) channel to produce the received waveform

$$r(t) = \sum_{l=0}^{L-1} \sum_{k=-\infty}^{+\infty} h_{k, l} s(t - lt) p_T(t - kT - lt) + n(t),$$

where $(h_{k,l})$ is defined in Section II and $n(t)$ is the complex-valued AWGN process with power spectral density $N_0$. The signal-to-noise ratio (SNR) of the received signal is given by

$$\text{SNR} = \sum_{k=1}^{M} 2\sigma_k^2 \frac{\mathcal{E}_b}{N_0} \left( \frac{n_P N_c + N_r}{(n_P - 1)N_c} \right).$$

The righthand portion of the expression, in parentheses, accounts for the penalty incurred due to the length of the retraining sequence and the use of $n_p$ channel symbols to differentially encode $n_p - 1$ bits per update interval.

The received signal is passed through the correlators of the rake receiver with each tap synchronized to a distinct delay determined by the channel response at the time the delays are set. If fixed tap delays are considered, it is assumed that the $L$ taps are synchronized to the $L$ path delays with the greatest average signal power. If adaptive tap delays are considered, it is assumed that the receiver initially employs the $L$ paths that have the strongest signals at the start of the data payload. The tap delays are adapted after every elapsed update interval. At the start of the data portion of each update interval, the taps are synchronized to the $L$ paths that have the strongest signals at that instant.

The statistic generated at the output of tap $k$ in channel-symbol interval $i$ is given by

$$U_i^{(k)} = \int_{iT + t_i^{(k)}}^{(i+1)T + t_i^{(k)}} r(t) u^*(t - t_i^{(k)} - iT) dt,$$

where $t_i^{(k)}$ is the delay setting for tap $k$ during channel-symbol interval $i$. For DPSK modulation with noncoherent rake reception, tap outputs from two consecutive channel symbols are required to make a single bit decision. The post-detection, equal-gain square-law-combining rake receiver makes a binary decision based on the sign of the real-valued decision statistic

$$Z_i = \sum_{k=1}^{L} (U_i^{(k)} U_{i-1}^{(k)}) + U_i^{(k)} U_{i-1}^{(k)}.$$ (1)

### IV. Error Probabilities for the Rake Receiver

Analytical expressions are presented here for the average probability of bit error for the fixed-tap rake receiver operating over doubly selective Rayleigh-fading channels. The analysis is for paths with equal average power, but it can be generalized to consider unequal average power [8]. The average probability of error for the adaptive rake receiver is not amenable to expression in a simple form, and thus the performance results are obtained through simulation of the system. The analytical results for the fixed-tap rake receiver are used to test the correctness of the simulation.

Consider first the rake receiver with noncoherent combining and $L$ taps. It is assumed that the aperiodic autocorrelation function of the spreading sequence is ideal [9] and that any two tap delays differ by at least $T_c$. The probability of error for binary DPSK modulation and noncoherent rake reception with equal-gain square-law diversity combining is given by

$$P_b = 1 - \frac{1}{(4\sigma^2\sigma_B^2) \frac{L-1}{L-1}! \sum_{i=0}^{L-1} \frac{(2L - 2 - i)!}{(2\sigma_A^2) i!} \left( \frac{\sigma_B^2}{2\sigma_A^2} \right)^{2L-1-i}},$$

(2)
fixed-tap receivers with two and three taps.
that 2σ2 is the power in each of the L paths to which a
tap is synchronized. Equation (2) is used in the section on
numerical results for fixed-tap rake receivers with one, two,
and three taps, respectively. For example, for one tap and
two taps, respectively, Equation (2) reduces to

\[
\begin{align*}
\sigma_A^2 &= 2\sigma^2E_0T \left(1 + \sqrt{e^{-2\pi D_T}}\right) + N_0T, \\
\sigma_B^2 &= 2\sigma^2E_0T \left(1 - \sqrt{e^{-2\pi D_T}}\right) + N_0T, \\
\end{align*}
\]

and the duration of each retraining sequence. We consider
the performance of an adaptive rake receiver with two taps is compared with the performance of fixed-tap receivers with one, two,
and three taps. The second channel consists of three equal-strength, Rayleigh-fading paths, and
Doppler spreads of 0.001 and 0.01 are considered. For the
three-path channel, the performance of an adaptive rake re-
ceiver with two taps is compared with the performance of fixed-tap receivers with two and three taps.

The retraining sequences that are placed periodically
within the data payload of the packet are likely to be much
longer in duration than a channel symbol in order to ensure an
accurate estimate of the channel response. Thus, the fraction
of the total transmitted energy that is devoted to tap
retraining should be accounted for in comparing different sys-
tem designs, and that fraction depends on the update interval and the duration of each retraining sequence. We consider
two systems: an idealized system using a zero-length retrain-
ing sequence and a system in which a significant fraction of
the total transmitted energy is devoted to the retraining se-
quencies.

Consider first an idealized system in which a negligible
fraction of the energy in the packet is devoted to the retrain-
ing sequences, but each update interval begins with an accu-
rate estimate of the channel response. If the Doppler spread
of the channel is small and the tap-delay update interval is
not too large, fewer taps are required to achieve a given level of performance with the adaptive rake receiver than with the fixed-tap rake receiver. Correspondingly, the adap-
tive receiver provides much better performance than does the
fixed-tap receiver for a given number of taps. For example, for
the two-path channel with D_T = 0.001, updates of a single
tap as infrequent as one per 50 bits yields performance

that is 6 dB better than that obtained with one fixed tap and
is within 3 dB of that given by two fixed taps at P_b = 10^{-2}.
If two adaptive taps are employed, the receiver benefits from
both the tap-delay adjustments and the diversity protection
between tap updates that is provided by the use of multi-
ple taps. The performance of a system with two adaptive
taps is shown in Figure 1 for the three-path channel with
D_T = 0.001. If an update interval of 50 bits is used, the
two adaptive taps yield performance that is 2.5 dB better
than that provided by two fixed taps at P_b = 10^{-2}, and it
is within 0.8 dB of the performance provided by three fixed
taps.

At higher fade rates, in contrast, the performance of the
adaptive rake receiver degrades rapidly as the update interval is
increased. This is illustrated in Figure 2, for the three-
path channel and a Doppler spread of 0.01. Tap updates
that occur at the unrealistically high rate of one per two bits
allow the two-tap adaptive rake receiver to achieve nearly
the performance of the receiver with three fixed taps, but
if the update rate is increased to 25 bits, the performance
degraded by approximately 2 dB at P_b = 10^{-2}.

The sensitivity of adaptive rake reception to the fade rate
of the channel is illustrated in Figure 3, where the perfor-
mance of a receiver with one adaptive tap is shown as a
function of the update interval for two-path channels with
respective Doppler spreads of 0.01 and 0.001. The SNR is 12
dB. For the more slowly fading channel, most of the benefit
of an adaptive tap is obtained with a fairly large update in-
terval. In contrast, the update interval must be very small
in order to make effective use of an adaptive tap in the more
rapidly fading channel. Since a smaller update interval re-
results in a greater penalty in transmitted energy per channel
symbol, as discussed below, a given system can exploit adapt-
ive rake reception only if the fading rate is sufficiently low.

In any implementation of a system using an adaptive rake
receiver, the insertion of a retraining sequence of nonzero du-
ration into the packet is required for each update interval in
the data payload. Thus the fraction of the total transmitted
signal energy that is used for each update interval in the
data payload. The system using an adaptive rake receiver therefore suffers
a penalty in SNR relative to a system using a fixed-tap rake
receiver, due to the difference in the corresponding packet
formats. The use of an adaptive rake receiver is justified
only if the performance improvement that is achieved justi-
ifies the penalty in SNR.

For a given chip rate and a retraining sequence of specified
duration, a change in the tap-delay update interval requires
a change in either the data rate or the number of chips per
channel symbol, or both. If the number of chips per channel
symbol is varied to maintain a constant data rate as the up-
date interval is changed, then a fair comparison can be made
between systems using retraining sequences of the same du-
ration but with different update intervals. Consider a system
that uses a retraining sequence of 200 chips. Suppose that
the data rate is fixed so that one channel symbol is trans-
mitted per 20 chips, on average. If the update interval is
25 bits, then that interval consists of a 200-chip retraining
sequence followed by 25 channel symbols with 12 chips per channel symbol. If instead the update interval is 50 bits, then an update interval consists of a 200-chip retraining sequence followed by 50 channel symbols with 16 chips per channel symbol. In either case, the average channel-symbol rate is one bit per 20 chips. The performance of the system is shown in Figure 4 for several update intervals with a 200-chip retraining sequence and an average channel-symbol rate of one bit per 20 chips. Thus, the training sequence has a duration that is ten times the "nominal" channel-symbol duration. The three-path channel has a Doppler spread of 0.001 and the adaptive rake receiver employs two taps.

It is seen from Figure 4 that if the SNR penalty for the retraining sequence is accounted for, the results are somewhat different from those shown in Figure 1 for the idealized, zero-length retraining sequence. In particular, approximately the same performance is obtained at $P_t = 10^{-2}$ with update intervals of 50 bits or 100 bits. In addition, two adaptive taps with an update interval of 50 bits yields performance that is 1.5 dB poorer than that obtained with three fixed taps, as compared with a difference of only 0.8 dB with an idealized zero-length retraining sequence. In addition, the two adaptive taps yield performance that is only 1.8 dB better than that obtained with two fixed taps, as compared with a difference of 2.5 dB with a zero-length training sequence. Thus the SNR penalty for the system with an update interval of 50 bits is 0.7 dB at $P_t = 10^{-2}$. Clearly the penalty in SNR for the retraining sequence reduces the benefit of the adaptive rake receiver significantly, and it changes the selection of the optimal update interval for a given application. The cost of the retraining-sequence overhead is even more apparent if the fade rate is high. Consider performance with the three-path channel and a Doppler spread of 0.01. If there is no overhead, two adaptive taps are somewhat better than two fixed taps at $P_t = 10^{-2}$ if the update interval is small, as shown in Figure 2. If the 200-chip retraining sequence from the example is used, however, adaptive rake reception is no better than fixed-tap rake reception, regardless of the choice of the update interval, as shown in Figure 5.

Consider the implications of these results for specific systems. The packet formats are those of Examples 4 and 5. A system with a data rate of 25 kbps and a carrier frequency of 200 MHz is considered first. For this system, a mobile velocity of 81 mph corresponds to a normalized Doppler spread of 0.001. A Doppler spread of 0.01, on the other hand, corresponds to the unrealistic velocity of 810 mph. Thus, by proper selection of the update interval, the system can benefit from an adaptive rake receiver. If, instead, the carrier frequency is 900 MHz, the two normalized Doppler frequencies correspond to velocities of 18 mph and 180 mph, respectively, so that adaptive taps are beneficial for the mobile velocities encountered in most circumstances. Consider next a system with a data rate of 1 Mbps. At carrier frequencies of both 200 MHz and 900 MHz, even extremely high, unrealistic mobile velocities result in Doppler spreads that are well below the limit for effective adaptive rake reception. Hence, use of an adaptive rake receiver in these systems will perform well with any realistic mobile velocities.

VI. Conclusions

We have examined adaptive rake reception in a DS mobile packet-data communication system. The results show that adaptive rake reception is beneficial for systems with a fade rate that is low relative to the data rate. If the fade rate is high relative to the data rate, the trade-offs between performance and overhead due to retraining sequences prove too costly for adaptive rake reception. However, for many applications, adaptive rake reception is beneficial for the mobile velocities that are typically encountered.

REFERENCES


![Fig. 1. Performance with three Rayleigh-fading paths, $DF = 0.001$, and zero-length retraining sequence.](image-url)
Fig. 2. Performance with three Rayleigh-fading paths, $D_T = 0.01$, and zero-length retraining sequence.

Fig. 4. Performance with three Rayleigh-fading paths, $D_T = 0.001$, and 200-chip retraining sequence.

Fig. 3. Sensitivity to update interval for two Rayleigh-fading paths and one adaptive tap.

Fig. 5. Performance with three Rayleigh-fading paths, $D_T = 0.01$, and 200-chip retraining sequence.