A New Method of Summing Crosstalk from Mixed Sources: the Generalized FSAN Method

Stefano Galli, Kenneth J. Kerpez

Telcordia Technologies Inc., 445 South Street, Morristown, NJ 07960, USA

***Abstract* – Digital subscriber lines (DSL) are fundamentally limited by crosstalk. The case where all crosstalk is from the same type of DSL has been studied over the years and accurate models have been standardized. However, crosstalk from multiple different types of DSLs is a relatively new area of study and models of summing mixed crosstalk have only recently been postulated. In this contribution, a new method of summing crosstalk due to mixed sources is proposed. This new method consists of deriving lower bounds to the worst-case method, which is intrinsically too pessimistic. It has been ascertained that the exploitation of the Minkowski inequality yields a new model that exhibits appealing properties. Moreover, it is shown here for the first time that the FSAN method is a particular case of the Minkowski-bound method and not, as previously believed, a method with no "physical justification or mathematical soundness." Finally, simulation results that confirm the effectiveness of this new method are presented.**

I. INTRODUCTION

Telephone subscriber loops are organized in binder groups of 10, 25 or 50 pairs. The electrical signals in a wire pair generates a small electromagnetic field which surrounds the wire pair and induces an electrical signal into nearby wire pairs. This inductive and capacitive coupling (known as *crosstalk*) can be the largest noise impairment in a twisted pair and can substantially reduce DSL performance. For this reason, there have been many efforts towards modeling this phenomenon. There are well-know accurate models for the case of a single type of crosstalk, where all crosstalkers have the same power spectral density (PSD). The models usually describe the 1% worst-case crosstalk power-sum, which is such that no more than 1% of all pairs in all cables will receive more crosstalk than the model.

A non-trivial problem arises when modeling complex access network scenarios where there may be different types of interferers. As Zimmerman pointed out in [1], it would be fundamentally incorrect to assume that the powers of different types of crosstalkers are simultaneously added using the 1% worst-case crosstalk coupling model. This would implicitly assume that each of a multitude of different services is simultaneously using the worst pairs in a binder, which is physically impossible. This problem of determining a good model for the summation of mixed interferers is becoming more important now than ever since DSL services are now being deployed in the field.

Although several summation methods have been proposed, at the moment there is no methodology or sound mathematical approach to the definition of a model. The basic starting point is the 1% worst-case model for the case of one type of disturber, which is then manipulated in some arbitrary fashion to find a model that is less pessimistic than the method that simply adds up the worst case powers. The lack of a sound mathematical approach should be considered as a major problem and further research on this topic is certainly needed.

However, the solution of this problem is not a simple goal to achieve. This is due to several difficulties that naturally arise when dealing with mixed sources of crosstalk. First of all, when there are several different kinds of disturbers they actually have a joint probability distribution. Under the assumption of independent interferers, this joint probability can be factored into the product of the marginal probability density functions, but the determination of these marginal probability densities is itself a non-trivial problem. Although it has been found that a log-normal distribution can well model the distribution of pair-to-pair couplings, the distribution of the sum of log-normal random variables is only approximately log-normal and the determination of the exact distribution of the sum of log-normal variates is still an open mathematical problem. Secondly, even solving the problem of determining the exact probability density function of the joint probability distribution of power sums would still not be enough since another big problem that arises is how to consider a 1% worst case of a joint probability.

Due to these analytical difficulties, researchers have devoted their efforts to the determination of simple models based on the 1% worst-case model used for the case of one type of crosstalker. Fundamentally, there are two well-known methods for modeling the crosstalk due to mixed sources: the mean power spectral density (PSD) method, originally proposed by Zimmerman [1], and the FSAN method developed by the Full Service Access Network (FSAN) initiative xDSL working group [3]. Preliminary work has shown that the FSAN method is superior to the mean PSD method, and so the FSAN method has recently been adopted by ANSI accredited committee T1E1.4 for use in spectrum management calculations [2]. From a merely theoretical point of view, it is difficult to state which method is better. Although the common goal achieved by these two methods is to be less pessimistic than the method that simply adds up the worst cases, there is no mathematical rule that gives any guideline on how much "less pessimistic" the model should be. For this reason, an answer to the problem of deciding which model is better can be given only on the basis of computer simulations.

In the present paper, a new approach is proposed for the modeling of crosstalk due to mixed interferes. This new approach consists of deriving a model by applying bounding techniques to the worst-case model, which is intrinsically too pessimistic. The use of mathematical bounds is a feasible and sound approach to the problem and allows us to retain the 1% worst-case models as a starting point and, moreover, to bypass the analytical difficulties mentioned earlier. In particular, the new summing method proposed here is based on the Minkowski inequality. The appealing feature of this new model is that it depends on a parameter that can be chosen accordingly to the desire of conveying more or less pessimism into the model. Another important result is that it is proven here that the FSAN method is a particular case of the method derived using the Minkowski inequality. This is an important result because it gives a mathematical validation of the FSAN method, which is commonly believed to be a model with no physical justification or mathematical soundness [3]. Moreover, a computer simulation analysis was also performed in order to assess the accuracy of the crosstalk summation methods and to compare them with the proposed one.

II. CROSSTALK LOSS MODELS

Crosstalk can be the largest noise impairment in a twisted pair and often substantially reduces DSL performance. Theoretical as well as experimental studies have ascertained that good models for the Power Spectral Densities (PSDs) of the NEXT and FEXT due to only one disturber are given by the following expressions:

$$Next(f) = S(f) X_N f^{1.5} \tag{1}$$

$$Next(f) = S(f) X_F f^{2} l \left|H(f)\right|^{2} \tag{2}$$

where f is the frequency in Hz, $S(f)$ is the PSD of the disturbing signal, l is the loop length in feet, $H(f)$ is the transfer function of the loop and X_N and X_F are constants determined by measurements. The increase of NEXT as $f^{1.5}$ with frequency may be found on the basis of theoretical considerations when some assumptions such as perfect line termination and uniform line characteristics are taken into account. Empirically derived crosstalk models fit a power of f in the range of 1.3 to 1.7.

As far as the computation of the constants X_N and X_F is concerned, the commonly accepted method is to consider the 1% worst case. This is done by considering the maximum value of the overall crosstalk power with a confidence of 99% or, equivalently, choosing an interference power that is likely to be exceeded in 1% or less of cases. In this case, the PSD models of NEXT and FEXT for n interfering signals of the same kind become:

$$Next[f,n] = S(f)\ X_N\ f^{1.5}\ n^{0.6} \tag{3}$$

$$Fext[f,n,l] = S(f) X_F\ f^{2}\ l \left|H^{2}(f)\right|\ n^{0.6} \tag{4}$$

where $X_N = 8.5 \cdot 10^{-15}$ and $X_F = 7.8 \cdot 10^{-21}$.

As mentioned in the Introduction, with several different kinds of disturbers there is actually a joint probability distribution and there is no unique way of defining a 1% worst case of a joint probability.

In this case a metric should be defined and, in the authors' opinion, a reasonable choice for the metric would be to use a metric that summarizes the effects of all the interferers on the actual system performance. A logical choice would be to consider the Signal-to-Noise Ratio (SNR), which represents the natural index of the impact of the overall crosstalk power on the system performance. In this way, the search for the 1% worst case is done on the set of SNRs and not on the crosstalk power sums. This method has been adopted for evaluating the "Monte Carlo NEXT summation method" (see Section IV) and, although time-consuming, should be considered as the most accurate way of determining the actual performance of a system.

A straightforward way to extend the model in (3), (4) is to simply sum the 1% worst-case crosstalk power contributions of each kind of disturber. For example, if we consider the case of NEXT interference given by K different systems we would obtain the following PSD for the interference:

$$Next_{NM}[f, n_1, \ldots, n_K] = \left[\sum_{i=1}^{K} S_i(f)\ n_i^{0.6} \right] X_N\ f^{1.5} \tag{5}$$

where n_i is the number of systems with PSD $S_i(f)$ (i=1, …, K). Each term is a 1% worst case, so this "naive" method would predict a distinctly pessimistic level of interference because it assumes that all the interfering systems simultaneously use the worst disturbing pairs.

III. A NEW CROSSTALK SUMMATION METHOD: THE GENERALIZED FSAN OR MINKOWSKI-BOUND METHOD

The exploitation of the Minkowski inequality allows us to derive a model for the summation of mixed sources that is always less pessimistic than the naive model in (5) and moreover, is dependent on a parameter. By changing this parameter, the tightness of the bound can be varied so that the optimism or the pessimism of the model can be chosen a priori.

Let $0<\lambda<1$ and a_{jk} be a set of nonnegative numbers for $1 \le j \le J$ and $1 \le k \le K$. The Minkowski inequality is expressed as follows:

$$\left[\sum_{j=1}^{J} \left(\sum_{l=1}^{L} a_{jl} \right)^{1/\lambda} \right]^{\lambda} \le \sum_{l=1}^{L} \left(\sum_{j=1}^{J} a_{jl}^{1/\lambda} \right)^{\lambda} \tag{6}$$

For the sake of simplicity, let us consider the special case of J=L=2 and a_{ij}=0 ($\forall i \neq j$). After some simple algebra eq.(6) boils down to the following inequality:

$$a_{11} + a_{22} \ge \left(a_{11}^{1/\lambda} + a_{22}^{1/\lambda} \right)^{\lambda} \tag{7}$$

Now, posing $a_{11} = S_1(f)n_1^{0.6}$ and $a_{22} = S_2(f)n_2^{0.6}$, we can exploit the inequality in (7) to compute a lower bound to the pessimistic model in (5):

$$\begin{aligned} &Next_{NM}[f, n_1, n_2] = \\ &= S_1(f)\, X_N\, f^{1.5}\, n_1^{0.6} + S_2(f)\, X_N\, f^{1.5}\, n_2^{0.6} \\ &\geq \left[\left(S_1(f)\, n_1^{0.6} \right)^{1/\lambda} + \left(S_2(f)\, n_2^{0.6} \right)^{1/\lambda} \right]^{\lambda} X_N\, f^{1.5} \end{aligned} \tag{8}$$

In this way, by exploiting the Minkowski inequality, we can derive the following model for *K* different sources:

$$\begin{aligned} &Next_{Minkowski}[f, \lambda, n_1, ..., n_K] = \\ &= \left[\sum_{i=1}^{K} \left[S_i(f)\, n_i^{0.6} \right]^{1/\lambda} \right]^{\lambda} X_N\, f^{1.5} \end{aligned} \tag{9}$$

The “amount” of pessimism that can be conveyed in the model depends on the value of the parameter λ, which can be chosen accordingly to the desire of a more or less conservative model. As λ tends to 1, the model in (9) tends to be more pessimistic (it asymptotically tends to the naive model in (5)) while, as λ tends to 0, the model tends to be more optimistic.

By choosing the particular value λ=0.6, we obtain:

$$\begin{aligned} &Next_{Minkowski}[f, \lambda = 0.6, n_1, ..., n_K] = \\ &= \left[\sum_{i=1}^{K} \left[S_i(f)\, n_i^{0.6} \right]^{1/0.6} \right]^{0.6} X_N\, f^{1.5} \end{aligned} \tag{10}$$

This is exactly the FSAN method. This method was first proposed by members of CSELT at the FSAN meeting in Bern in February 1998. After a little refinement, the idea was presented to ANSI T1E1.4 in June 1998 at Huntsville. It was stated in [3] that there is “no simple physical justification for this model.” However, this is not correct since the FSAN method is a particular case of the Minkowski-bound method, which has been derived on the basis of sound mathematical properties.

The problem of finding an optimal value for λ in (9) is a non-trivial problem since there is no cost function with respect to which one can pursue optimization. For this reason, it may be best to choose the optimal value for the parameter λ on the basis of Monte Carlo simulations. As a rule of thumb, if λ>0.6 the Minkowski-bound method will be more pessimistic than the FSAN method, whereas for λ<0.6 it will be more optimistic.

The effectiveness of the Minkowski-bound method depends on the value of λ but also on the number of different crosstalk sources *K*. A simple way of understanding the dependence of the amount of crosstalk predicted by the model as a function of *K* is to consider the function

$$\varphi_{Minkowski}[\lambda, n_1, ..., n_K] = \left[\sum_{i=1}^{K} \left(n_i^{0.6} \right)^{1/\lambda} \right]^{\lambda} \tag{11}$$

For a given value of λ and *K*, this function returns the amount of crosstalk (normalized to $X_N \cdot f^{1.5}$) predicted by the model in (9) when all the disturbing sources exhibit a flat spectrum of unitary value in the frequency band of interest. More simply, it can be considered as the deterministic weight to the crosstalk prediction given by eq.(9), whereas the contribution due to the different PSDs and to the stochastic part is neglected. Function (11) has been averaged over all the possible combinations of disturbers n_i (i=1, …, *K*; $0<n_i<25$, $\forall i$) and is plotted in Fig. 1, having considered the FSAN curve as the reference curve at 0 dB. The plot in Fig.1 shows that for small *K* all the methods exhibit a similar deterministic weight for crosstalk prediction but, as *K* increases, two important facts can be noticed. First, the naive method becomes very pessimistic as *K* increases so that it is confirmed that alternative summation methods are really necessary. Second, the flexibility given by the proposed bound in the choice of λ can be effectively used to limit the amount of predicted crosstalk and this flexibility becomes more important as *K* increases.

IV. SIMULATION RESULTS AND CONCLUSIONS

Computer simulations were run to ascertain the accuracy of the Minkowski crosstalk combination method with different values of λ. Each simulation generates many samples of same-binder pair-to-pair NEXT couplings, which in dB are generated by a log-normal model:

$$\begin{aligned} &\Pr(\text{pair} - \text{to} - \text{pair NEXT coupling} \leq x \text{ dB}) = \\ &= \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2 / 2\sigma^2}\, dx \end{aligned} \tag{12}$$

with σ = 9.0, $\mu = 164.0 - 15\log_{10}(f)$, and *f* is the frequency in Hz. Two methods were run: the “Minkowski” NEXT summation method for several values of λ and the “Monte-Carlo” NEXT summation method. The NEXT PSD is calculated according to the simulated summation method and, then, the SNR at the decision point is calculated. The SNRs of HDSL, and HDSL2 are calculated with ideal DFE equations with standard PSD shapes [2].

The “Minkowski” NEXT summation method.

For each disturber type with number of disturbers n_i, and n_i > 0, n_i pair-to-pair NEXT loss values are randomly and independently generated and power-summed. This is done repeatedly until there are 2000 samples of power-sums of each disturber type. From each set of 2000 power-sums, the 1% worst-case power-sum is calculated and, then, multiplied by the PSD of the NEXT of that disturber type, and so for disturber number *i*, this product equals $S_i(f)\, n_i^{0.6}$. These are summed by the Minkowski-bound method and normalized as $x_N \, f^{3/2}\left(\Sigma\left(S_i(f)\, n_i^{0.6}\right)^{1/\lambda}\right)^{\lambda}$ which is the crosstalk sum used to compute the NEXT PSD. After having computed the NEXT PSD, the SNR at the decision point is calculated as previously mentioned.

The "Monte-Carlo" NEXT summation method.

For each disturber type with number of disturbers n_i, and $n_i > 0$, n_i pair-to-pair NEXT loss values are randomly and independently generated and power-summed. These are each multiplied by a normalized crosstalk transfer function and the disturber's PSD, and are then power-summed to equal the crosstalk noise used to compute a single SNR value. This is done repeatedly until there are 2000 SNR values. The set of SNRs are ordered into ascending order, and the 1% worst-case SNR (point number 20) is then equal to the Monte-Carlo SNR, which is close to the actual 1% worst SNR.

Preliminary simulations found that the FSAN method is generally too pessimistic. So, the Minkowski-bound method with two values of $\lambda < 0.6$ was simulated, and the results are in Tables I and II for HDSL and HDSL2. In these Tables the difference between the SNR computed with the Minkowski NEXT summation method and the SNR computed by the Monte-Carlo method is shown in dB. Negative numbers in the tables indicate that the NEXT summation method is pessimistic, and positive numbers indicate that the NEXT summation method is optimistic.

For the HDSL simulations in Table I, the Minkowski method improved the average accuracy by about 0.4 dB, and for the HDSL2 simulations in Table II, the Minkowski method with $\lambda = 0.1$ improved the average accuracy by about 0.2 dB; compared to the standard $\lambda = 0.6$ FSAN method.

The simulation results show that only a moderate improvement in the accuracy is obtainable passing from λ=0.6 to λ=0.1. However, the improvement might be much higher if the number of disturbing services K is increased. In fact, as Fig. 1 shows, the gap between the FSAN method and the Minkowski-bound method for λ<0.6 becomes bigger when K increases. These results tend to indicate that the FSAN method might be too pessimistic when *many* different disturbing sources are considered and that the proposed method could be a good candidate for the summation of crosstalk due to mixed sources. In particular, the effectiveness of the proposed method is higher for the case of *many* different sources that is certainly the most likely scenario that will be found in the future.

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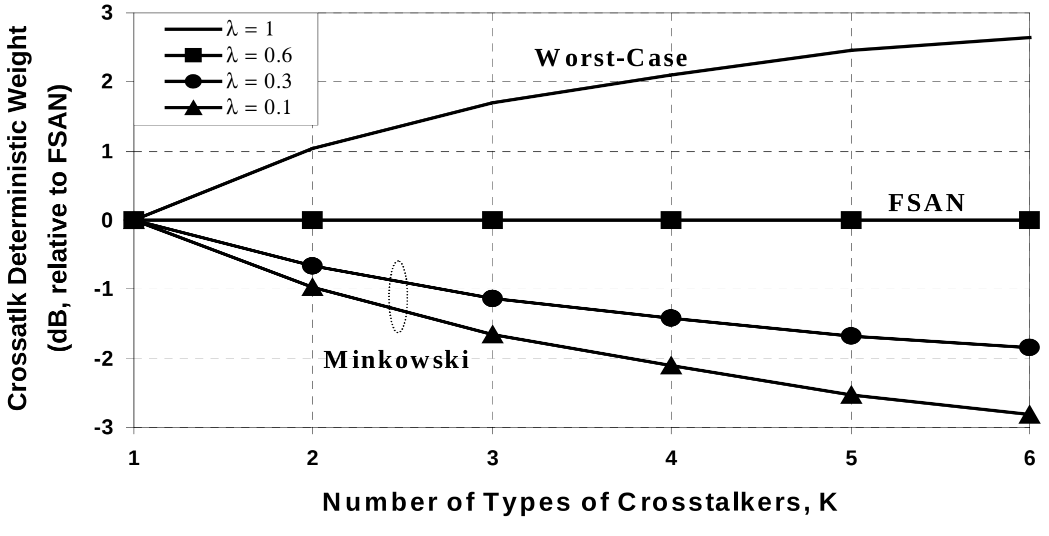

Figure 1 - Plot of function (11) averaged over all the possible combinations of disturbers for K ranging from 1 to 6 and different values of λ. In particular, the values λ=0.6 and λ=1.0 correspond to the FSAN and worst-case summation methods, respectively.

SNR difference (dB)	**λ = 1.0 (naive sum)**	**λ = 0.6 (FSAN)**	**λ = 0.3**	**λ = 0.1**
Average difference (dB)	-1.66	-1.12	-0.84	-0.74
Avg. difference in absolute (dB)	1.70	1.16	0.89	0.81
Maximum difference (dB)	0.29	0.29	0.29	0.29
Minimum difference (dB)	-3.36	-2.68	-2.61	-2.59

Table I - Statistics of the difference (in dB) between the SNR computed with the Minkowski NEXT summation method and the SNR computed by the Monte-Carlo method, for different values of λ. Upstream HDSL with mixed ISDN, HDSL, and downstream ADSL NEXT.

SNR difference (dB)	**λ = 1 (naive sum)**	**λ = 0.6 (FSAN)**	**λ = 0.3**	**λ = 0.1**
Average difference (dB)	-3.00	-2.67	-2.54	-2.50
Avg. difference in absolute (dB)	3.01	2.69	2.56	2.52
Maximum difference (dB)	0.26	0.29	0.56	0.66
Minimum difference (dB)	-5.81	-5.47	-5.33	-5.30

Table II - Statistics of the difference (in dB) between the SNR computed with the Minkowski NEXT summation method and the SNR computed by the Monte-Carlo method, for different values of λ. Downstream HDSL2 with mixed ISDN, T1, and upstream HDSL2 NEXT.