# A NOVEL APP-BASED SOFT-OUTPUT ADAPTIVE EQUALIZER WITH ENHANCED CHANNEL TRACKING CAPABILITY

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## Abstract

*A novel adaptive equalizer that exploits only the soft-information given by the A Posteriori Probabilities (APPs) of the states of the ISI channel for channel estimation, tracking and equalization is presented. Moreover, the channel tracking capability of the receiver is enhanced by exploiting the knowledge of the power-delay profile of the link and, therefore, by taking into account an unequal distribution of the powers of the channel taps. This is done by modifying the channel estimator algorithm and by conveying into the covariance matrix of the channel estimation error the available information on the powers of the taps of the low pass equivalent sampled input-delay spread function.*

## 1. Introduction

It is well known that the performance of an adaptive receiver can be greatly enhanced if the channel tracking capability of the channel estimator can be improved. In particular, in [2] a new channel estimator that is fed by the A Posteriori Probabilities (APPs) of the states of the ISI channel (instead of the less informative and less reliable hard-decisions) and that is able of generating *zero-delayed channel estimates* was proposed. Several simulation results have shown that the adaptive receiver presented in [2] (based on this new *soft channel estimator* and on a Symbol-by-Symbol Maximum A Posteriori (SbS-MAP) equalizer for data detection) outperformed other conventional adaptive receivers.

A further improvement of the adaptive receiver presented in [2] was proposed in [9] where a Maximum Likelihood Sequence Equalizer (MLSE) is used in place of the SbS-MAP one. So doing, the receiver proposed in [9] yielded better performances than those obtained in [2] because the MLSE is able to operate at the largest decision delay (i.e., the TDMA-slot length), something an SbS-MAP receiver cannot do at a reasonable computational complexity. The basic idea behind the receiver proposed in [9] was to exploit the advantages of both MLS and MAP receivers in a combined form.

In the present contribution, the receiver in [9] has been modified in two ways. First, the tasks of channel estimation, tracking and equalization are now accomplished by exploiting *only* the APPs. Secondly, the channel estimator has been modified in order to take into account an unequal distribution of the powers of channel taps.

The fact that the APPs of the states of ISI channel can be efficiently used for several purposes has been recently remarked in [12], where it was also shown that Non-Linear Minimum Mean Square Error estimates of the transmitted symbols can be generated on the basis of the APPs.

Moreover, besides designing optimum strategies of channel estimation and equalization, the channel tracking capability of a receiver can also be enhanced if some a priori knowledge on the power delay profile of the link were known. Usually, the channel estimators embedded in adaptive receivers do not take into account a non-constant power-delay profile of the radio link. However, if an information on the power delay profile is available to the receiver, the channel tracking algorithm can take into account this information. In practice, a feedback channel is always present in a cellular environment, for example for power control purposes [7]. Moreover, in almost every system (such as, UMTS or IS-95) there is an initial "acquisition" phase where the Mobile Unit (MU) monitors the link and, for every sensed Base Station (BS), performs a rough estimate of the delay profile for the initialization of the fingers of the rake receiver [7,8].

Finally, the proposed method has a general validity and can be applied to all the algorithms that rely on the Recursive Least Squares (RLS) principle.

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## 2. The considered adaptive receiver

Referring to a TDMA-based digital link impaired by time-variant multipath phenomena and AWGN, the baud-rate sampled complex sequence $\{r(i)\}$ received at the output of the equivalent low-pass randomly time-variant ISI channel can be modeled as [1]:

$$r(i) = \sum_{m=0}^{L-1} g(i;m)a(i-m) + w(i) = \\ = G^T(i)\sigma(i) + w(i), \qquad (1)$$

where the transmitted sequence $\{a(i)\in B\}$ is constituted by $M$-ary complex independent identically distributed symbols taken from an assigned modulation constellation $B$ and $\{g(i;m),\ 0\leq m\leq L-1,\ i\geq 1\}$ is the $T_S$-sampled time-variant impulse-response of the overall link (including the transmitting filter, the multipath-faded radio channel and the receiving filter). An application of the so-called *Martingale Difference Representation Theorem* [3] allows us to derive the following nonlinear Kalman-like channel estimator (for more details on the analytical derivation of (2) see [2]):

$$\hat{G}(i/i) \equiv E\left\{G(i)\,|\,r_1^i\right\} = \\ = \hat{G}(i/i-1) + C_G(i)\left[r(i) - \hat{r}(i/i-1)\right]. \qquad (2)$$

The filtering gain $C_G(i)$ and the one-step MMSE prediction $\hat{r}(i/i-1)$ of the observation $r(i)$ in eq.(2) depend on $\pi(i/i)$, the vector of the APPs of the states of the ISI channel.

For each received sample $r(i)$ the APP computer computes $\pi(i/i)$ and feeds it to the channel estimator for updating the channel estimate $\hat{G}(i/i)$ and to the MLS equalizer for the computation of the branch metrics $-\ln\left[\pi(i/i)\right]$. The channel estimator feeds the APPs computer with zero delayed filtered channel estimates. At the end of the TDMA-slot, the VA outputs the entire estimated sequence on a per-slot basis.

The receiver *does not utilize* hard decided data for channel estimation and tracking so that unreliable "tentative" decisions are not generated. Since channel estimation is performed independently from data-detection, the receiver can build the VA trellis with more reliable zero-delayed channel estimates (in parallel with channel tracking) and, therefore, output the entire decided sequence with a decision delay equal to the length of the TDMA-slot [9]. So doing, the proposed receiver yields better performances than those obtained in [2] because the MLS equalizer operates at the largest decision delay, i.e., the TDMA-slot length.

The receiver described in [9] can be further improved if the VA trellis is built using the APPs, i.e. the soft output of the SbS-MAP receiver. More in detail, these soft outputs are converted to $-\ln(\pi(i/i))$ for the computation of the branch metrics of the VA algorithm (see Fig.1). This last feature makes the adaptive receiver here proposed particularly suited for concatenated equalization and trellis coded systems. In fact, it is well known that the use of soft-outputs from the inner detector can make the outer decoder reach nearly optimum performance. Soft-output equalization techniques can yield to a noticeable performance improvement in adaptive receivers and may be considered as a good candidate for high-speed wireless data-transmission [5].

## 3. The exploitation of the knowledge of the power delay profile

The model usually adopted to describe the behavior of the radio channel refers to the so-called *input-delay spread function* $g_c(t;\tau)$ [10]. The function $g_c(t;\tau)$ is the response of the channel at the observation time $t$ when the excitation is a pulse at time $(t-\tau)$. It is useful to include the effects of the time-invariant transmitting and receiving filters in the channel impulse response, so that the input-delay spread function $g(t;\tau)$ generally considered describes the ordered cascade of the time-invariant shaping filter, the time-variant radio channel and the time-invariant receiver filter. It is reasonable to assume that the input delay spread function $g(t;\tau)$ be causal and limited in time. This time limitation depends on the delay spread ($T_m$) and on the signaling period $T_S$:

$$L = \left\lfloor T_m / T_S \right\rfloor + 1$$

Now, let us assume that $g(t;\tau)=0$ for $\tau<0$ and $\tau>LT_S$. The quantity $g(i;m)$ that appears in the observation sequence (1) is the so-called low-pass equivalent $T_S$-sampled input-delay spread function. The latter is obtained taking into account the interpolation effects of a raised-cosine-like receiving filter, i.e. convolving the continuous low-pass input-delay spread function of the channel with a waveform of the form $\sin x/x$ (if, for the sake of simplicity, we assume zero roll-off) and, then, sampling at the signaling period $T_S$. The power of the $j$-th tap of the power-delay profile of the low-pass equivalent $T_S$-sampled input-delay spread function can be calculated as follows:

$$\mathrm{P}_j = \sum_{i=1}^{Lc} P^{(i)}\left(Ca\left[\pi f_S\left(jT_S - \tau^{(i)}\right)\right]\right)^2 ,$$

where: $L$ is the length of the channel impulse response (in multiples of the signaling period $T_S$) assumed at the receiving side, $L_c$ is the number of discrete paths in the power delay-profile, $P^{(i)}$ and $\tau^{(i)}$ are the power and the delay of the $i$-th path of the power delay-profile, $T_s=1/f_s$ is the signaling time and $Ca[x]=\sin(x)/x$.

In adaptive equalization, it is the low-pass equivalent $T_S$-sampled input-delay spread function $g(i;m)$ that is estimated (and, eventually, tracked) in order to perform channel equalization. However, adaptive receivers do not usually exploit the fact that every link exhibits a particular power-delay profile that influences the power of the channel taps of the low-pass equivalent $T_S$-sampled input-delay spread function. In fact, channel estimators typically assume that the link is constituted of equal powered channel taps even though this is not true. This might be explained by the fact that it is in general difficult to have an a priori knowledge of the power-delay profile of a link but, in the case of cellular communications, this information can be obtained on the basis of the geographical localization of the base station. In fact, if a base station is situated in a city, the power delay profile of the link will be somewhat different from the one pertaining to a link where the base station is located on a hill. Once the mobile unit is tied to a particular base station, it can be informed of the most likely power-delay profile in that area. On the basis of this information, the receiver can tune its channel tracking algorithm and adapt it to that particular link. A way to take into account an unequal distribution of the power of the taps is to convey the information on the power of the taps of the low-pass equivalent $T_S$-sampled input-delay spread function into the covariance matrix of the channel estimation error.

For example, for the channel estimator embedded in the proposed adaptive receiver (as well as in all the channel estimators based on the RLS principle) the updating of this covariance matrix is done in the following way (see [6] for more details on the analytical derivation):

$$\begin{aligned} &S_G(i/i-1) \equiv \\ &\equiv E\left\{\left[G(i)-\hat{G}(i/i-1)\right]\left[G(i)-\hat{G}(i/i-1)\right]^H \mid y_1^{i-1}\right\}= \\ &= E\left\{G(i)G^H(i)-\hat{G}(i/i-1)\hat{G}^H(i/i-1) \mid y_1^{i-1}\right\}= \\ &= \Lambda S_G(i-1/i-1)\Lambda^T+2\left[R_G(0)-\Lambda R_G(0)\Lambda^T\right], \end{aligned} \quad (4)$$

$$\begin{aligned} &S_G(i/i) \equiv \\ &\equiv E\left\{\left[G(i)-\hat{G}(i/i)\right]\left[G(i)-\hat{G}(i/i)\right]^H \mid y_1^{i}\right\}= \\ &= \left[I_{LxL}-C_G(i)\pi(i/i)^T M^T\right]S_G(i/i-1), \end{aligned} \quad (5)$$

where $S_G(i-1/i-1)$ is the filtered covariance matrix of the estimation error at step ($i$-1). In the derivation of the above relationship we have assumed that:

- the process $\{G(i)\}$ is a first order Markovian process that obeys the following model:

  $G(i+1)=\Lambda G(i)+d(i+1)$,

  where $\Lambda$ is a $L$x$L$ real matrix and $\{d(i)\}$ is a stationary innovation sequence;
- the second order statistical description of the processes $\{G(i)\}$ and $\{d(i)\}$ is given by the following relationships:

$$\begin{aligned} &E\left\{G(i)G^H(i)\right\}\equiv 2E\left\{G_r(i)G_r^T(i)\right\}\equiv \\ &\equiv 2E\left\{G_i(i)G_i^T(i)\right\}=2R_G(0) \end{aligned};$$

$$E\left\{d(i)d^H(i)\right\}=2\left[R_G(0)-\Lambda R_G(0)\Lambda^T\right].$$

The above assumptions are typically employed in the derivation of Kalman-like estimators. Moreover, the model has a general validity since autoregressive models of higher orders can be treated similarly since we can represent higher order Markov processes as first order multivariate Markov processes.

If the fading process follows the Uncorrelated Scattering (US) model, the $L$ scalar processes that constitute the $L$-variate process $\{G(i)\}$ are mutually independent and the two real matrices $\Lambda$ and $R_G(0)$ are diagonal; moreover, if the channel taps are assumed equal powered the matrix $R_G(0)$ has the elements on its principal diagonal all equal.

Now, if we want to take into account the real power-delay profile of the link and the interpolation effects of the low-pass receiving filter, the $L$ processes are not mutually independent anymore and, moreover, they do not exhibit the same power. The information on the power of the channel taps is carried by the matrix $R_G(0)$. More in detail, its elements on the principal diagonal are now given by the following relationships (see eq.(3)):

$$R_G(j,j) = \frac{1}{2}\sum_{i=1}^{Lc} P^{(i)}\left(Ca\left[\pi f_S\left(jT_S-\tau^{(i)}\right)\right]\right)^2 .$$

In the above equation, the factor 1/2 arises because matrix $R_G(0)$ contains the power of either the real or the imaginary part of the normalized power of the complex tap.

As the simulations commented in the next Section will show, the knowledge of the distribution of the power of the channel taps of the low-pass equivalent $T_S$-sampled input-delay spread function yields to a moderate performance improvement.

## 4. Simulation results

The performance of the proposed adaptive receiver of Fig.1 has been tested via computer simulations and compared to those obtained by other conventional receivers. In all the carried out tests, the adopted modulation is BPSK, the preamble and slot lengths are $L_p$ and $L_s$ symbols and the throughput $\rho$ is set to 0.8 (as in the GSM standard). The case described in [11, Tabs.I-II], recommended by the GSM standard to test digital transmissions over Urban Typical (UT) and Hilly Terrain (HT) links. In all cases, the time correlation was modeled by the usual zero-order Bessel function $J_o(\cdot)$ [4,Sect.2.4.2].

On the basis of the given power-delay profiles, the performance of the receiver of Fig.1 with the modified channel tracking algorithm (labeled "Proposed (optimal Rg)") is shown in Figs. 2,3. In these typical cellular environments, the gap over the conventional hard-decided RLS-MLS receivers (see Fig.1 in [9]) is still noticeable. However, the performance gain is less impressive if the proposed receiver is compared to the one in [2]. Finally, we notice a moderate performance gain if the modified channel-tracking algorithm that exploits the information of the power-delay profile is used.

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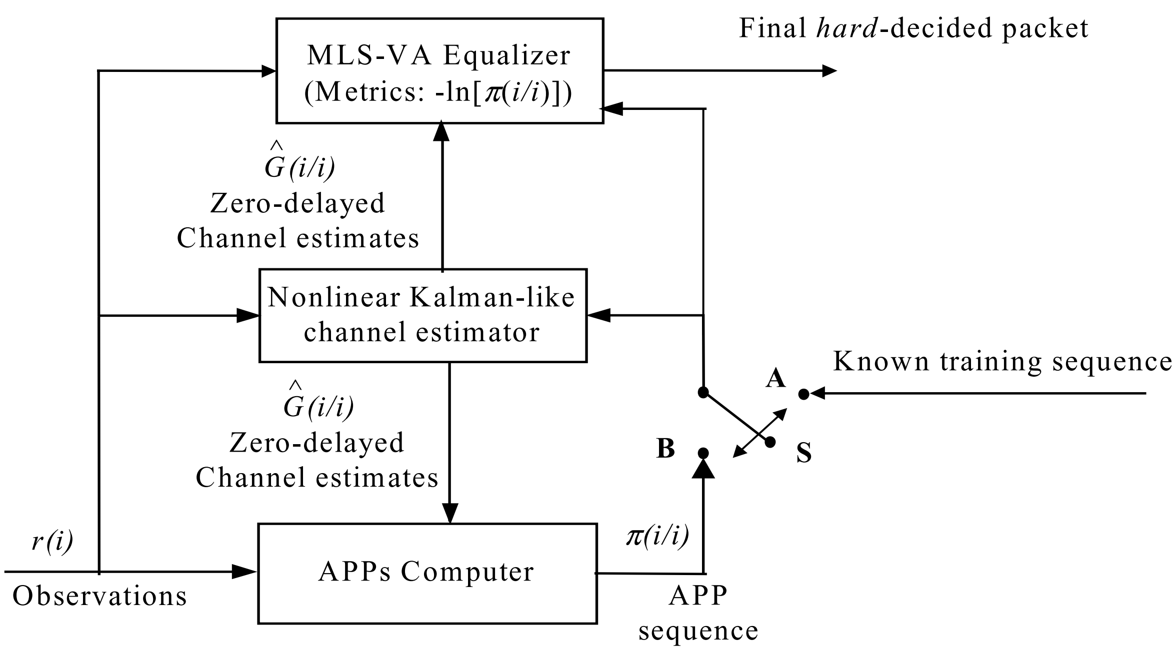


**Figure 1. Block diagram of the proposed adaptive receiver. The switch S is in position "A" during the training-intervals and in position "B" otherwise.**

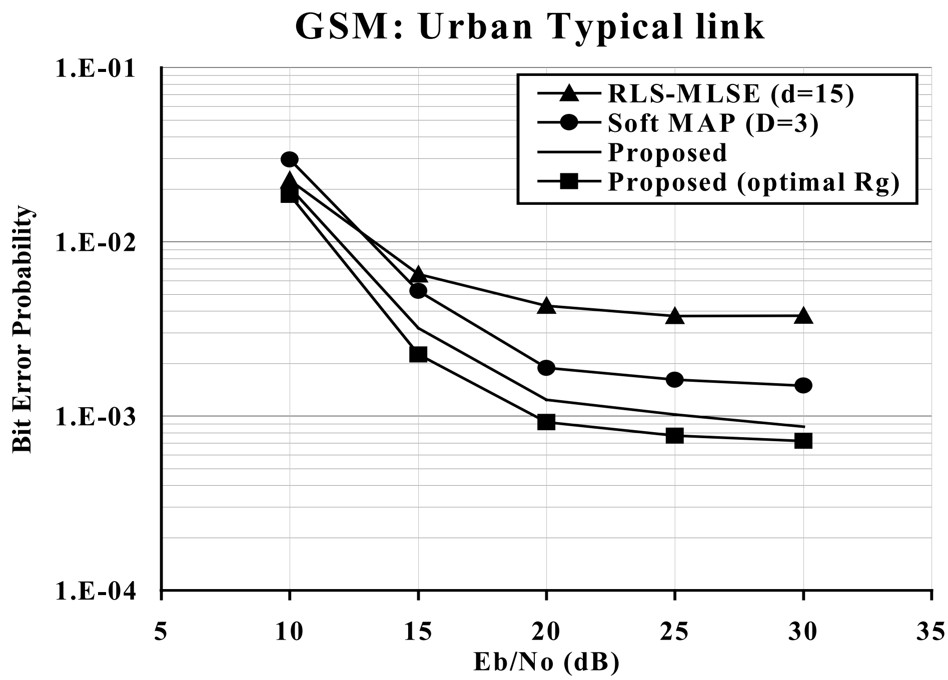


**Figure 2. Performance comparison on the GSM Urban Typical. The receivers considered are the conventional hard-decision based RLS-MLS (see Fig.1 in [9], the receiver in [2] that employs the *soft channel estimator* followed by an SbS-MAP equalizer (curve labeled "Soft-MAP"), the proposed receiver of Fig.1 (curve labeled "Proposed") and the proposed receiver with the algorithm that exploits the information of the power-delay profile (curve labeled "Proposed (optimal Rg)"). Parameters: $L_p$=18, $L_f$=90, $B_D T_S$ = $1.85 \cdot 10^{-4}$.**

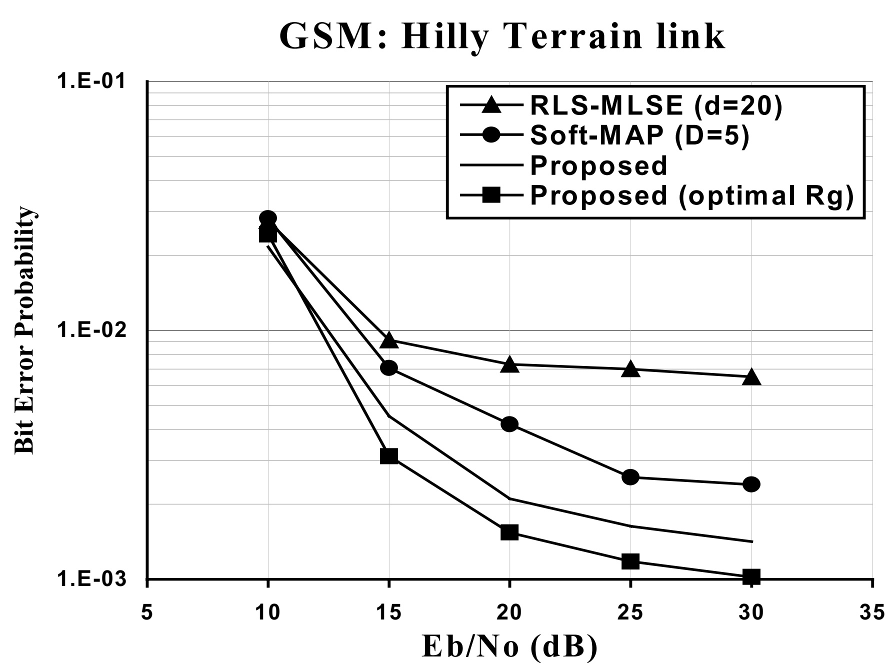


**Figure 3. Same as in Figure 2, with $L_p$=18, $L_f$=90, $B_D T_S$ = $3.695 \cdot 10^{-4}$.**