

# New Insight Into The Performance Of TCM Schemes Over Fast Time-Variant Multipath-Degraded Faded Environments

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**Abstract** - The performance Symbol-by-Symbol (SbS) Abend-Fritchman like combined equalizers/decoders [5] performing diversity reception and coherent detection of Trellis-Coded-Modulated (TCM) is analyzed for the case of fast time-variant, ISI-corrupted, Rayleigh-faded data-channels. New upper-bounds for the resulting decoding error probability are derived and a related novel criterion for the design of TCM schemes optimally matched to the statistical properties of the transmission channel is also presented.

## I. GOALS OF THIS RESEARCH

To date, most analytical results about TCM schemes for faded environments have concerned the case of flat (i.e., ISI-free) slow and fast-faded channels [1,2]. Minor attention has been devoted to the case of slow-faded ISI-channels [3] and very little is known about TCM performances on fast-faded ISI-channels; neither does any good criterion seem available for designing TCM encoders matched to such a kind of multipath-degraded environment.

Moreover, the results reported in literature assume the presence of Maximum Likelihood Sequence Estimators (MLSEs) at the receiver side. Simulation results [4] recently have demonstrated that SbS detectors can offer better performance on ISI-degraded faded environments and today several low complexity algorithms make the implementation of such equalizers/decoders very attractive [4,5]. Nevertheless, analytical performance bounds for SbS detectors are yet not available and only simulation results have been reported about SbS detectors' performance [4,5].

The purpose of this contribution is twofold. First of all, an analytical upper-bound for the performance of the above adaptive SbS equalizers/decoders on fast-faded ISI channels is presented. Then, a related criterion for the effective design of actual good TCM schemes is formulated.

## II. THE PROPOSED PERFORMANCE-BOUND AND THE RELATED CRITERION FOR THE DESIGN OF TCM SCHEMES

Let us assume that the (complex) encoded sequence  $\{s(i) \in \mathcal{C}^L\}$  generated by a TCM encoder is transmitted over a Div-diversity link, where the Div branches are affected by multipath-phenomena introducing ISI of length  $(L_C - 1)$ . The Div multipath-phenomena are modeled as Wide-Sense-Stationary, mutually independent, fast time-varying, Rayleigh-distributed processes sharing a common autocorrelation function (ACF)  $\{\underline{R}_G(t)\}$  whose lags are  $L_C \times L_C$  matrices. Thus, the  $L_C$ -variate baud-rate sampled (complex) observed sequence  $\{\underline{y}(i)\}$  at the input of the adaptive SbS combined equalizer/decoder can be modeled as:  $\underline{y}(i) = \mathbf{G}(i)\underline{s}(i) + \underline{v}(i)$ , where  $\mathbf{G}(i)$  is the random vector of the Div channel impulse responses at the  $i$ -th epoch,  $\{\underline{s}(i) \in \mathcal{C}^{L_C}\}$  is the  $L_C$ -variate sequence collecting step-by-step the last  $L_C$  transmitted TCM-samples and  $\{\underline{v}(i)\}$  is a Div-variate (complex) white Gaussian noise sequence representing the thermal-noises.

Now, moving from the union-Bhattacharyya bound and jointly exploiting the ACF of the fading processes and the statistical properties of the combined SbS equalizer/decoder, it can be proved

that for high levels of the average received signal-to-noise ratio  $\bar{\gamma}$  the steady-state value of the decoding error probability  $\bar{P}_e$  can be bounded as in the following:

$$\bar{P}_e \leq \left[ (1/\bar{\gamma})^R \right]^{Div} \cdot B(\{\underline{s}(i)\}; \{\underline{R}_G(t)\}), \quad (\bar{\gamma} \rightarrow \infty) \quad (1)$$

where  $R$  is the rank of the covariance block-matrix built-up by the matrix-lags of the ACF of the channel fading processes and  $B(\cdot; \cdot)$  is a suitably-defined coefficient jointly depending on the set of TCM-codewords  $\{\underline{s}(i)\}$  and on the ACF  $\{\underline{R}_G(t)\}$  of the fading processes. Concerning the bound in eq.(1), the following remarks are in order.

1) The bound holds even when the interleaving is imperfect or absent, and when the employed TCM scheme is non-uniform.

2) The rank  $R$  of eq.(1) is the *unique key-parameter* dominating the asymptotical performance of any employed TCM scheme. This parameter depends on the ACF of the channel fading processes only and *fully summarizes*, for coding purposes, the "diversity" exhibited by the transmission link.

3) Roughly speaking, the term  $B(\cdot; \cdot)$  in eq.(1) is a suitable combination of average pairwise Euclidean-distances between TCM codewords, with weight-coefficients *depending* on the ACF of the channel fading processes.

4) From the foregoing, it follows that an *efficient criterion* for the optimal design of TCM schemes consists in selecting the TCM codewords so as to minimize the coefficient  $B(\cdot; \cdot)$  present in the bound of eq.(1).

Illustrative examples have been developed which confirm the effectiveness of the above criterion for the design of good TCM encoders for some actual ISI-corrupted fast time-varying digital links.

## REFERENCES

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