Model Parameter Estimation for 2D Noncausal Gauss-Markov Random Fiel

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An original procedure for estimating the model of a noncausal Gauss-Markov Random Field (GMRF) bservations is proposed. Starting from a suitable 'local' on of the field and taking into account the symmetry the so-called 'potential fields' [3] describing the GMRF, uation system relating the model parameters to the on-stationary) 2D autocorrelation function (acf) of the ld is derived. Its solution for a known (or estimated) acf s the parameter estimates of the GMRF. The unkown the eventually present observation noise can be also intly with the model parameters.

SUMMARY

e-index 2D zero-mean Gaussian random process{x($\underline{s}$) R^1, d over a rectangular lattice I and constituting a noncausal ogeneous GMRF with respect to (wrt) an assigned 'support eighbourhood system' [3,4]) η(d) admits the 'innovations n' [1,2]

$$\sum_{\eta(d)} \phi(\underline{r})\, x(\underline{s} + \underline{r}) + u(\underline{s}), \quad \underline{s} \quad I\_(\eta(d)). \qquad (1)$$

t η(d) is assumed symmetric and constituted by an even ints 2L(d) [4]; I_(η(d)) is the set of 'internal points' of I wrt ^{1}, $\underline{r}$ η(d)} are the so-called 'field potentials', related as [2] to the acf {$R_u(\underline{r})$} of the 2D stationary zero-mean ovations process' {u($\underline{s}$) R^1, $\underline{s}$ I_(η(d))}, with variance

obvious symmetry property $R_u(\underline{r}) = R_u(-\underline{r})$ we have: $\phi(\underline{r})$ d). This allows to partition of the support region η(d) in the), $\eta_-(d) \subset \eta(d)$, each constituted by L(d) sites and such that en -$\underline{r}$ $\eta_-(d)$ for every $\underline{r}$ η(d). In this way (1) can be

$$\sum_{\_(d)} \phi(\underline{r})\, [\, x(\underline{s} + \underline{r}) + x(\underline{s} - \underline{r})\,] + u(\underline{s}), \quad \underline{s} \quad I\_(\eta(d)). \qquad (2)$$

esentation of the GMRF in (2) is then completed by e associated boundary conditions (b.c.), i.e. the statistics of vector constituted by the r.v.s extracted from the random t the boundary points of the lattice I.

assumed that the GMRF is corrupted by a 2D stationary

Walker equations for the parameter identification of 1D processes. From the field potentials, writing (3) for $\underline{s}$ I_(any $\underline{m}$ $\eta_-(d)$ such that $\phi(\underline{m}) \neq 0$, the noise variance calculated; finally, the parameter K_u is computed from (3) = $\underline{0}$.

The illustrated parameter estimation procedure is *full* fact it is valid for GMRFs defined on both *finite or infinit* for *any kind* of assumed boundary conditions, periodic or their influence being embedded in the acf of the field itself can be easily particularized to the case when the noise va known, or when the observation noise is absent.

Comparing the proposed solution to alternative method the literature, some improvements can be outlined. M having exploited the symmetry $\phi(\underline{r}) = \phi(-\underline{r})$ gives an alg with *half size* with respect to the system in [1]. On the o procedure in [4] is based on an iterative search algorithm, computational complexity proportional to the size of the fi proposed solution is based on a 'local' description of the GM does *not* involve time-consuming iterative search algor complexity is *independent* from the the size of the field. proposed approach the variance of the observation noise *together* with the model parameters while the procedures requires that it is known (or separately estimated). The re computer simulations of the above procedure are reported in [7].

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