THE CUT-OFF RATE OF DIGITAL RADIO-LINKS IMPAIRED BY TIME-CORRELATED MULTIPATH PHENOMENA

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***Abstract* - The symmetric cut-off rate is derived for Quadrature-Amplitude-Modulated (QAM) block-coded data-transmissions over noisy links impaired by time-correlated multipath-phenomena which give rise to Rayleigh-distributed randomly time-variant Intersymbol Interference (ISI) effects. The presented formulas for the cut-off rate are new; they are derived from a random coding bound developed “ad hoc” for bounding the ensemble-averaged block-error probability of a Maximum Likelihood (ML) decoder supported by perfect Channel-State-Information (CSI). The assumption of time-correlated ISI phenomena introduces memory in the link and makes the underlying ISI-channel a continuous-state channel with memory.**

I. INTRODUCTION

Data-transmissions over microwave terrestrial channels are generally impaired by randomly time-variant multipath-phenomena which severely degrade the quality of the resulting links and the cellular land-mobile radio-channel as well as the ionospheric sky-wave channel are important examples of such kind of links. The degrading effects of these fading phenomena can be mitigated through various forms of channel-coding which constitute an effective means to introduce “time-diversity” in the transmitted data-streams.

As it is recognized today [2,5], [10, Chap.4], the potential benefits arising from coding-operations are well summarized by the cut-off rate of the underlying transmission-channel. In fact, this parameter dictates the limit to the practical maximum information-rate supported by the coding-channel and it is also representative of the performance (measured in terms of error-probability exponent) achievable through codes of assigned rates and block-lengths [2,4], [10, Sect.4.4]. Thus, the maximization of the cut-off rate is recognized today as an effective design-criterion for the optimization of key-parameters of the transmission system as, for example, code-rate, signaling period, modulation and demodulation formats [5], [8, Sect.5.2.9].

Due to its importance as index of the coding-capability of the overall transmission-system, a large body of published works has concerned the computation of the cut-off rate for several classes of practical channels. More in detail, comprehensive results for memoryless links are well known today and have been recently summarized in [10,Sect.4.5]. Furthermore, the important case of data-channels affected by time-invariant deterministic ISI has also been largely investigated; extensive results on this topic have been derived mainly following the general guidelines reported in [1, Sect.5.9] for coding of finite-state channels with memory and can be found, for example, in [2, 6, 7], [9, Sect.5.8].

Also the case of randomly faded data-links has been examined in a lot of works [10, Sect.4.6 and references therein]; however, this has been done for the most part by resorting to the *simplifying* assumption of *perfect interleaver* which gives rise to a *memoryless* coding-channel ([5], [10, Sect.4.6] and references therein). On the other hand, several communication systems present hard constraints on the allowed decoding-delay which make the assumption of perfect interleaving unrealistic. An important example of such systems is represented by the cellular mobile-radio link of [13], where the maximum decoding-delay is hardly limited when voice-communications are considered. In these cases, the time-correlation presented by the fading-phenomena introduces memory in the link which makes this last *continuous-state*; so, the above cited Gallager’s approach of [1, Sect.5.9] for the computation of the error-probability exponent must be generalized and the known results reported in [2,5,6,7], [9,Sect.5.8] are *no longer valid*.

An overview of the published works shows, indeed, that only fragmentary results have been reported to date about the coding-capability of data-links affected by time-correlated fading phenomena. More in particular, in [3,4] analytical expressions for the cut-off rate and the error-exponent have been developed for channels affected by time-correlated *flat-faded* (that is, ISI free) phenomena and some generalizations of these results for the case of systems with diversity-reception have been presented more recently in [11,12].

In the present paper, moving from an application of the union-Bhattacharyya bound followed by random-coding argumentations, we develop an ensemble-averaged performance bound for QAM block-coded data-transmissions over channels impaired by Rayleigh-distributed time-correlated time-variant ISI phenomena. Closed-form analytical formulas for the computation of this parameter are developed for ML decoding with perfect CSI by resorting to the assumption of independently-selected equi-distributed codeword-symbols; so, according to a current taxonomy [6,7], we qualify as “symmetric” the presented cut-off rate. However, due to the general assumptions made about the statistical behavior of the data-links here considered, the presented results *generalize* those previously published in [2,3] as well as those reported in [10, Sect.4.6] for systems with perfect interleaving.

II. THE SYSTEM-MODEL

An illustrative sketch of the considered communication system for the transmission of QAM block-coded data streams over noisy links impaired by randomly time-variant multipath phenomena is reported in Fig.1. The M-ary random source-message feeds a block-encoder of rate $R \equiv \log(M)/N$ whose output $\underline{x}_m(i) \equiv [x_m(1) \ldots x_m(N)]^T \in A^N$ is a codeword of length N constituted by QAM symbols from an assigned q-ary complex constellation $A \equiv \{\alpha_1, \ldots, \alpha_q\}$ (hereafter natural logarithms are used). So, the chip-period sampled ISIcorrupted noisy sequence $\{y(i)\in C^1, 1\le i\le N\}$ observed at the output of the transmission channel can be modeled as [8, Sect.6.3]

$$y(i)=\sum_{k=0}^{Lc-1} g(i;k)x_m(i-k)+n(i)\equiv \equiv z(i)+n(i),\ 1\le i\le N \tag{1}$$

where $\left\{g(i;k)\in C^1,\ 1\le i\le N,\ 0\le k\le L_C-1\right\}$ is the usual chip-period sampled "*input delay-spread function*" [8,Chap.7] of the equivalent low-pass complex system which takes into account the cascade of the transmission filter, multipath-impaired randomly time-variant waveform channel and receiving filter. Furthermore, $\left\{n(i)\equiv n_C(i)+jn_S(i)\right\}$ in (1) is a complex zero-mean white Gaussian noise sequence whose uncorrelated components share a common variance equal to $N_o/2$. Therefore, having indicated by ($1\le i\le N$ and $N_S\equiv q^{Lc}$):

$$\overline{s}(i)\equiv\left[x_m(i)\ \ldots\ x_m(i-L_C+1)\right]^T\in A^{L_c}\equiv \equiv A_S\equiv\left\{\overline{\sigma}_1,\ldots,\overline{\sigma}_{N_S}\right\}, \tag{2}$$

$$\overline{G}(i)\equiv\left[g(i;0)\ \ldots\ g(i;L_C-1)\right]^T\equiv \equiv\overline{G}_C(i)+j\overline{G}_S(i)\in C^{Lc}, \tag{3}$$

the channel-state and the channel-impulse response vectors at the i-th step respectively, eq.(1) can be equivalently rewritten in a more compact form as:

$$y(i)=\overline{G}^T(i)\overline{s}(i)+n(i)\equiv z(i)+n(i),\ 1\le i\le N\ . \tag{4}$$

Now, as pointed out in [1, Sect.4.6], due to the ISI effects the i-th channel-state value $\overline{s}(i)$ *jointly* depends on the transmitted codeword $\underline{x}_m$ and *also* on the starting-state $\overline{s}(0)$ so that we refer to it by adopting the more complete notation $\overline{s}(i)\equiv\overline{s}(i;\underline{x}_m,\overline{s}(0))$ which stresses this twofold dependence of the channel-state in (2) on $\underline{x}_m\in A^N$ and $\overline{s}(0)\in A_S$. Furthermore, as far as the statistical description of the multipath phenomena is concerned, we assume that the L_C-variate impulse-response sequence $\left\{\overline{G}(i)\in C^{Lc},\ 1\le i\le N\right\}$ in (3) constitutes a complex stationary zero-mean Gaussian random sequence with mutually uncorrelated components $\left\{\overline{G}_C(i)\in R^{Lc}\right\}$ and $\left\{\overline{G}_S(i)\in R^{Lc}\right\}$ which share a common autocorrelation sequence $\left\{\overline{R}_G(t)\equiv E\left\{\overline{G}_C(i)\overline{G}_C^T(i+t)\right\}\equiv E\left\{\overline{G}_S(i)\overline{G}_S^T(i+t)\right\}\right\}$ with real matrix lags of dimensions (L_CxL_C); as a consequence, the resulting multipath phenomena are Rayleigh-distributed and give rise to correlated ISI effects which span over (L_C-1) chip-periods (see eq.(1)).

Therefore, by indicating as $\underline{\Gamma}(N)\equiv\left[\overline{G}(1)^T\ldots\overline{G}^T(N)\right]^T\equiv\underline{\Gamma}_C(N)+j\underline{\Gamma}_S(N)$ the L_CN-variate complex vector which collects the channel impulse-responses until the N-th step, under the assumption of perfect CSI the ML detector of Fig.1 decides for the codeword which maximizes the channel-transition probability density $P(\underline{y}\,|\,\underline{x},\underline{\Gamma}(N))$, where $\underline{y}\equiv\left[y(1)\ldots y(N)\right]^T\in C^N$ is the N-variate vector which gathers the observations due to the transmitted codeword. Now, the link between the observed vector $\underline{y}$ and the transmitted codeword $\underline{x}_m$ can be obtained by ordering the N scalar relationships of eq.(1) in the matrix form below reported:

$$\underline{y}\equiv\Lambda(\underline{\Gamma}(N)\underline{s}(\underline{x}_m,\overline{s}(0))+\underline{n}\equiv\underline{z}(\underline{\Gamma}(N);\underline{x}_m,\overline{s}(0))+\underline{n}\ . \tag{5}$$

The N-variate vector $\underline{n}\equiv[n(1)\ \ldots\ n(N)]^T$ present in eq.(5) is constituted by the noise samples of eq.(1) and the NL_C-variate channel super-state: $\underline{s}(\underline{x}_m,\overline{s}(0))\equiv\left[\overline{s}(1;\underline{x}_m,\overline{s}(0))^T\ldots\overline{s}(N;\underline{x}_m,\overline{s}(0))^T\right]^T$ collects the N-long state sequence of eq.(2) obtained when the codeword $\underline{x}_m$ is transmitted starting from the initial-state $\overline{s}(0)$. Furthermore, $\Lambda(\underline{\Gamma}(N))\equiv diag\left\{\overline{G}^T(1),\ldots,\overline{G}^T(N)\right\}$ is an NxN diagonal block-matrix with the matrix elements along the main diagonal ordinately given by the above described channel impulse-response sequence $\left\{\overline{G}(i),\ 1\le i\le N\right\}$. Finally, the N-variate vector: $\underline{z}(\underline{\Gamma}(N);\underline{x}_m,\overline{s}(0))\equiv\Lambda(\underline{\Gamma}(N))\underline{s}(\underline{x}_m;\overline{s}(0))$ in (5) gathers the outputs of the noiseless part of the channel due to the transmission of $\underline{x}_m$ starting from $\overline{s}(0)$. So, as sketched in Fig.1, the relationship in (5) describes the transmission link on a *codeword-basis* and constitutes the basic system-model for the development of the performance bounds of the next Section.

III. The Ensemble-Averaged Error-Bound and the Symmetric Cut-Off Rate for QAM Block-Coded Data-Transmissions Impaired by Time-Correlated Multipath Phenomena

Similarly to [1, Sect.5.6] and [2, Sect.III], an application of the union-Bhattacharyya bound followed by random coding argumentations [1, Sect.5.5] allows us to bound the block-error probability $\overline{P}_E(\underline{\Gamma}(N))$ *averaged* over the ensemble of admissible codes and *conditional* on the fading realization $\underline{\Gamma}(N)$ as (the overbar in (6) indicates expectation over the ensemble of the randomly selected admissible codes)

$$\overline{P}_E(\underline{\Gamma}(N))\equiv\overline{P(\underline{x}_m\ne\underline{\breve{x}}|\underline{\Gamma}(N))}\le \le M\sum_{\underline{x}\in A^N}\sum_{\underline{x}'\in A^N}Q(\underline{x})Q(\underline{x}')\int_{\underline{y}\in C^N}\sqrt{P(\underline{y}|\underline{x},\underline{\Gamma}(N))P(\underline{y}|\underline{x}',\underline{\Gamma}(N))}\,d\underline{y} \tag{6}$$

where $Q(\underline{x})$, $\underline{x}\in A^N$, is an assigned probability measure on the overall set A^N of the N-long codewords. Now, as also pointed out in [1, p.99], for ISI channels the computation of the transition probabilities $P(\cdot|\cdot,\cdot)$ present in (6) requires that the starting-state $\overline{s}(0)$ of the channel be taken into account and this task is here accomplished through the following relationships:

$$\sqrt{P(\underline{y}|\underline{x},\underline{\Gamma}(N))P(\underline{y}|\underline{x}',\underline{\Gamma}(N))}= =\sqrt{\sum_{\overline{s}(0)\in A_S}\sum_{\overline{s}(0)'\in A_S}P(\underline{y},\overline{s}(0)|\underline{x},\underline{\Gamma}(N))P(\underline{y},\overline{s}(0)'|\underline{x}',\underline{\Gamma}(N))}\le \tag{7}$$

$$\le\sum_{\overline{s}(0)\in A_S}\sum_{\overline{s}(0)'\in A_S}\sqrt{P(\overline{s}(0))P(\overline{s}(0)')}\cdot \cdot\sqrt{P(\underline{y}|\underline{s}(\underline{x},\overline{s}(0)),\underline{\Gamma}(N))P(\underline{y}|\underline{s}(\underline{x}',\overline{s}(0)'),\underline{\Gamma}(N))} \tag{8}$$

where the bound (7) arises from the algebraic inequality $\sqrt{\sum_i a_i} \le \sum_i \sqrt{a_i}$ of [1, p.253]. So, after introducing the inequality (8) into the bound (6), due to the Gaussian distribution of the noise $\underline{n}$ in (5) we recognize that the integral present in (6) can be computed moving from known results about the integration of *N*-variate Gaussian distributions [8, Sect.4.3.2] and then we get

$$\overline{P}_E(\underline{\Gamma}(N)) \le M \sum_{\underline{x}\in A^N} \sum_{\overline{s}(0)\in A_S} \sum_{\underline{x}'\in A^N} \sum_{\overline{s}(0)'\in A_S} Q(\underline{x})Q(\underline{x}') \cdot \sqrt{P(\overline{s}(0))P(\overline{s}(0)')}\, B^{d_N^2(\underline{\Gamma}(N);\underline{x},\overline{s}(0);\underline{x}',\overline{s}(0)')} \qquad (9)$$

where $B \equiv exp\left(-1/4N_o\right)$ is the Bhattacharyya-parameter. As might be expected, eq.(9) looks like eq.(20) of [2] for deterministic ISI channels; indeed, similarly to eq.(21) of [2], the parameter

$$d_N^2(\underline{\Gamma}(N);\underline{x},\overline{s}(0);\underline{x}',\overline{s}(0)') \equiv$$
$$\equiv \left\| \underline{z}(\underline{\Gamma}(N);\underline{x},\overline{s}(0)) - \underline{z}(\underline{\Gamma}(N);\underline{x}',\overline{s}(0)') \right\|^2,$$

plays in eq.(9) the role of the squared Euclidean distance between the two *N*-long signal sequences: $\underline{z}(\underline{\Gamma}(N);\underline{x},\overline{s}(0))$ and $\underline{z}(\underline{\Gamma}(N);\underline{x}',\overline{s}(0)')$ received at the output of the noiseless part of the link of Fig.1 in correspondence with the pair of codewords/initial-states $(\underline{x},\overline{s}(0))$ and $(\underline{x}',\overline{s}(0)')$, when in both cases $\underline{\Gamma}(N)$ is the determination assumed by the *N*-long sequence of channel-impulse responses (see eq.(5)). The next task is to average the bound in (9) over the statistics of the fading process; this can be done by noting that $\underline{\Gamma}(N)$ in (9) is an $L_C N$-variate zero-mean complex Gaussian random variable whose mutually uncorrelated components $\underline{\Gamma}_C(N)$ and $\underline{\Gamma}_S(N)$ share the following (*NxN*) Toeplitz-type real covariance block-matrix:

$$0$$

where $\left\{\overline{R}_G(t),\, 0\le t\le N-1\right\}$ is the above introduced matrix autocorrelation sequence which describes the second-order properties of the multipath phenomena. So, the expectation of the bound in (9) with respect to the distribution of $\underline{\Gamma}(N)$ can be carried out again resorting to the above cited results about the integration of multivariate Gaussian distributions and the obtained expression takes on the following final form:

$$\overline{P}_E \equiv E_{\underline{\Gamma}(N)}\left\{\overline{P}_E(\underline{\Gamma}(N))\right\} \le$$
$$\le M \sum_{\underline{x}\in A^N} \sum_{\overline{s}(0)\in A_S} \sum_{\underline{x}'\in A^N} \sum_{\overline{s}(0)'\in A_S} Q(\underline{x})Q(\underline{x}')\sqrt{P(\overline{s}(0))P(\overline{s}(0)')} \cdot \psi(N_o,\underline{R}_G(N);\underline{x},\overline{s}(0);\underline{x}',\overline{s}(0)') \qquad (10)$$

with

$$\psi(N_o,\underline{R}_G(N);\underline{x},\overline{s}(0);\underline{x}',\overline{s}(0)') \equiv$$
$$\equiv \left\{ det\left[I + \frac{1}{2N_o}\underline{\Delta}^2(\underline{x},\overline{s}(0);\underline{x}',\overline{s}(0)')\underline{R}_G(N) \right] \right\}^{-1} \qquad (11)$$

Furthermore, in (11) the matrix

$$\underline{\Delta}^2(\underline{x},\overline{s}(0);\underline{x}',\overline{s}(0)') \equiv$$
$$\equiv diag\left\{ \left[\overline{s}(i;\underline{x},\overline{s}(0)) - \overline{s}(i;\underline{x}',\overline{s}(0)')\right]\left[\overline{s}(i;\underline{x},\overline{s}(0)) - \overline{s}(i;\underline{x}',\overline{s}(0)')\right]^H \right\} \qquad (12)$$

is an hermitian semidefinite-positive diagonal block-matrix whose *N* block-entries along the main diagonal constitute the (matrix) distance-spectra between the *N*-long channel-state sequences of eq.(2) obtained in correspondence of the pair codewords/starting-states $(\underline{x},\overline{s}(0))$ and $(\underline{x}',\overline{s}(0)')$.

Now, in principle, the bound (10) should be minimized with respect to the probability assignment $Q(\underline{x})$; however, even in the more simple case of deterministic ISI channels this task appears very hard to be accomplished [1, p.183], [2, p.355]. Therefore, similarly to [1, Sect.5.9], [2, Sect.III], we assume in the sequel that the *N* symbols of any codeword are *independently* selected with a time-invariant *uniform* probability so that: $Q(\underline{x}) \equiv q^{-N}$, for any $\underline{x}\in A^N$. Under this assumption, the bound (10) can be put in the usual exponential form

$$\overline{P}_E \le exp\left\{-N\left[R_o^*(N) - R\right]\right\},\ R \le R_o^*(N), \qquad (13)$$

where the symmetric cut-off rate for *N*-long codewords $R_o^*(N)$ is given by the following formula (in nats per channel-symbol):

$$R_o^*(N) = -\frac{1}{N} log\left\{ q^{-(2N+L_C)} \cdot \sum_{\underline{x}\in A^N} \sum_{\overline{s}(0)\in A_S} \sum_{\underline{x}'\in A^N} \sum_{\overline{s}(0)'\in A_S} \psi(N_o,\underline{R}_G(N);\underline{x},\overline{s}(0);\underline{x}',\overline{s}(0)') \right. \qquad (14)$$

The relationship (14) for the symmetric cut-off rate of block-coded modulated data-links impaired by time correlated Rayleigh-distributed multipath-phenomena represents the main result of the work; about it, some remarks are in order.

Remark 1. Due to the time-correlation exhibited by the fading phenomena, analogously to [3, eq.(11)] and [4, eq.(23)] the cut-off rate in (14) *depends* on the length *N* of the transmitted codewords; furthermore, the ISI phenomena require that the effects of the initial channel-state $\overline{s}(0)$ also be taken into account in (14). Finally, for increasing values of *N* the cut-off rate in (14) approaches zero and log*q* for $N_O \to \infty$ and $N_O \to 0$, respectively.

Remark 2 (The case of very fast time-varying fading channels with unbounded Doppler-spreads). When the Doppler-bandwidths approach infinity, the sequence $\{\overline{G}(i)\}$ of the channel impulse-responses in (4) is reduced to a white process and the above defined covariance matrix $\underline{R}_G(N)$ gives rise to a diagonal (block) matrix. So, in this case the time-correlation of the fading phenomena vanishes and the transmission link is reduced to a more conventional finite-state channel whose memory is induced by the ISI phenomena only. For such a kind of channels, the general approach developed in [1, pp.182-187] can be followed; so, a suitable application of Frobenius' theorem [1,p.184] gives rise in our case to the following performance bound:

$$\overline{P}_E \le \alpha\, exp\left\{-N\left[R_o^* - R\right]\right\},\ R \le R_o^*, \qquad (15)$$

with R_o^* independent of N and given by

$$R_o^* = -\log \lambda_{max}\,, \text{ with: } \alpha \equiv q^{Lc}\left(v_{max}^{(max)} / v_{max}^{(min)}\right) \tag{16}$$

Following the notation of [1, eq.(5.9.45)], [2, eq.(25)], [7, eq.(5)], in eq.(16) $v_{max}^{(max)}$ and $v_{max}^{(min)}$ are the maximum and minimum components of the right-eigenvector associated to λ_{max}; in turn, the latter is the maximum eigenvalue of a q^{2Lc}xq^{2Lc} nonnegative irreducible matrix A. The entries of this matrix are organized in the usual way described, for example, in [1, p.184], [2, p.355], [6, Fig.2]. Finally, it must be remarked that, as a consequence of the random feature of the fading phenomena here considered, the expressions in (17) for the elements of the matrix A *differ* from those reported in [2, eq.(24)], [6,eq.(7.a)], [7, eq.(8) and following text] for the case of *deterministic time-invariant* ISI channels.

Remark 3. (The case of static fading channels with zero Doppler-spreads). For fading channels with vanishing Doppler-bandwidths the random sequence $\{\overline{G}(i)\}$ in (4) becomes *i*-invariant so that the above defined covariance matrix $\underline{R}_G(N)$ approaches a singular (block) matrix. In this case it can be proved that the formula (14) still holds with the $\psi(\cdot,\cdot;\cdot,\cdot;\cdot,\cdot)$ functional replaced by $\left\{det\left[I+\frac{1}{2N_o}\underline{Tr}\left\{\underline{\Delta}^2(\underline{x},\overline{s}(0);\underline{x}',\overline{s}(0)')\right\}\overline{R}_G(0)\right]\right\}^{-1}$, where $\underline{Tr}\left\{\underline{\Delta}^2(\cdot)\right\}$ indicates the L_CxL_C matrix obtained by summing the N block-entries present along the main diagonal of the matrix $\underline{\Delta}^2(\cdot)$ in (12).

IV. SOME ILLUSTRATIVE EXAMPLES AND CONCLUSIVE REMARKS

The formulas for the symmetric cut-off rate developed in the previous Section have been evaluated for a typical land-mobile test-channel constituted by two equal-power mutually uncorerelated taps affected by multipath phenomena described by the following (matrix) autocorrelation sequence:

$$\overline{R}_G(t) = I_{2x2}\, J_o(2\pi B_D T_C\, t),\; 0 \le t \le N-1,$$

where $J_o(\cdot)$ is the usual zero-order Bessel function of the first kind and $B_D T_C$ indicates the Doppler-spread of the fading phenomena normalized by the chip-period T_C. BPSK, 4PSK and 8PSK modulated codewords of lengths N=32 and N=64 have been considered and the obtained cut-off rates (measured in bits per channel-symbol) are reported in Figs.2,3,4 for values of the product $B_D T_C$ equal to zero, $5\cdot 10^{-5}$ and $5\cdot 10^{-2}$, respectively. As Fig.2 shows, when the fading phenomena are "static" (that is, $B_D T_C$ vanishes) the cut-off rate performance is very bad and for Signal-to-Noise-Ratios (SNRs) $\overline{\gamma}_c$ below 30 dB no substantial improvements in the cut-off rate values are achieved by expanding the size q of the signal constellation. Moreover, according to the observation that channels affected by "static" fading phenomena exhibit vanishing Shannon capacity [4, Sect.II.B and references therein], the plots of Fig.2 show that the cut-off rate values decrease for increasing values of the block-length N.

Although a (small) improvement in the cut-off rate values can be noted, the same conclusions are to be drawn from the plots of Fig.3 which refer to the quasi-static multipath-faded channel in eq.(21) with $B_D T_C = 5\cdot 10^{-5}$.

However, an opposite trend is exhibited by the results reported in Fig.4, which depict the behavior of the fast-faded channel of eq.(21) with $B_D T_C = 5\cdot 10^{-2}$. In fact, in this case the gains in the SNR achievable by expanding the signal constellation are noticeable and increasing values of the cut-off rate are indeed obtained as the codeword-length N grows.

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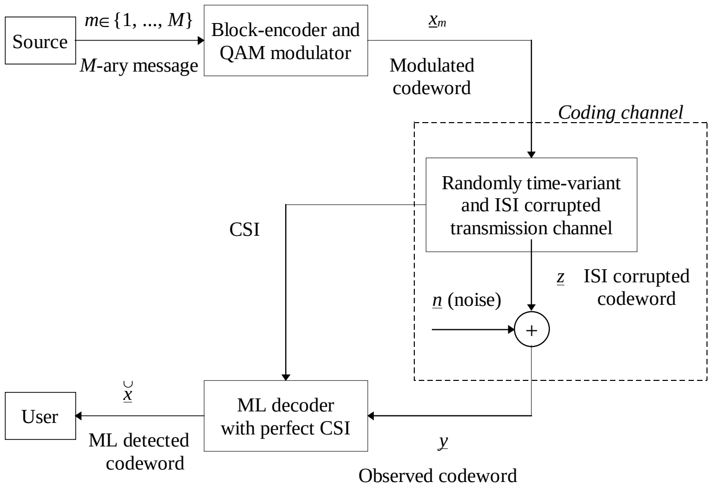

Fig. 1 An illustrative diagram of the coded-modulated system considered in the present work.

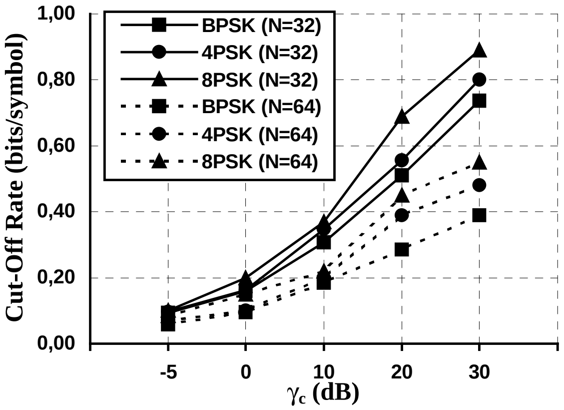

Fig. 2 Behavior of the cut-off rate (in bits per channel-symbol) versus the SNR $\bar{\gamma}_c$ for the two-path channel of eq.(21) with vanishing $B_D T_C$. $\bar{\gamma}_c$ is the average SNR per channel-symbol at the receiver site.

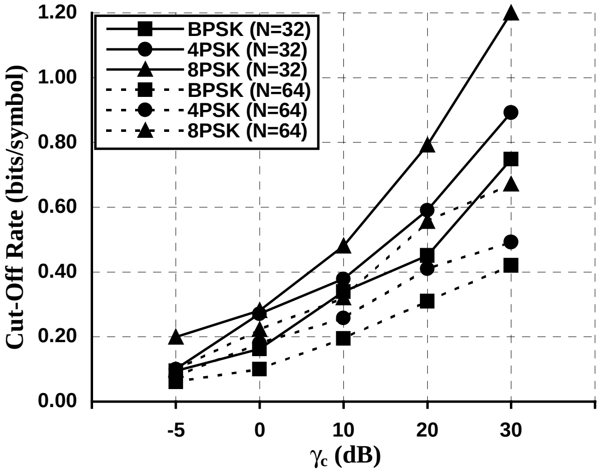

Fig. 3 The same as in Fig.2 for $B_D T_C$=5·10^{-5}.

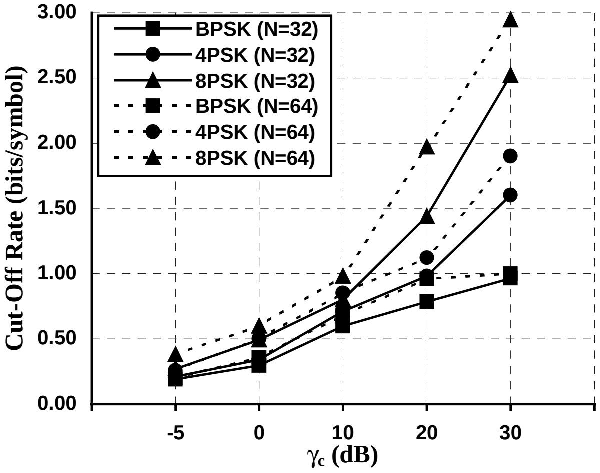

Fig. 4 The same as in Fig.3 for $B_D T_C$=5·10^{-2}.