AN APPROACH TO BLIND DECONVOLUTION BASED ON SECOND-ORDER "SOFT" STATISTICS

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***Abstract* - In this paper we present a new blind equalizer that achieves identification of a channel by exploiting only second-order statistics of the observations. The novelty of the proposed approach is that the receiver accomplishes channel identification by using soft-statistics; roughly speaking, it consists of an Abend-Fritchman type [11] Maximum A Posteriori (MAP) equalizer that feeds a nonlinear Kalman-like channel-estimator with the *soft-statistics* constituted by the A Posteriori Probabilities (APPs) of the channel-state sequence. So, since the receiver employs second-order statistics only, it achieves channel identification with fewer symbols than most techniques based on higher-order statistics.**

I. INTRODUCTION

Digital communication is often impaired by Intersymbol Interference (ISI) due to a frequency selective behavior of the channel which imposes limitations on the data transmission rate. An attractive approach to channel identification and equalization, referred to as blind equalization or blind deconvolution, is to perform channel identification without resorting to known training-data.

A pioneering self-recovering adaptive equalizer was proposed by Sato [1] more than twenty years ago. Sato's approach consisted in the definition of new nonconvex cost functions different from the traditional quadratic Mean Square Error (MSE) functionals usually employed for trained equalizers. The work by Sato has been further developed among others by Godard [2], Benveniste and Goursat [3], Picchi and Prati [4] and Shalvi and Weinstein [5]. The approaches followed in the cited papers are similar and are all based on the computation of suitably defined higher-order statistics of the observations. The major problem inherent to these approaches is a *slow convergence* and a *high residual Mean Square Error* (MSE).

For a long time, the identification of nonminimum-phase channels has been accomplished using higher-order statistics. Recently [6], a new blind channel equalizer based on second-order statistics that exploits the cyclostationarity of the oversampled communication signal has been proposed. The contribution in [6] has represented a new approach to the problem of blind deconvolution and stimulated further research [7], [8].

In this paper we present a blind equalizer following a new approach based on a suitably developed channel estimator fed by an SbS-MAP detector. More in detail, the proposed receiver exploits the second-order moments only but *does not require* any oversampling of the observation process; moreover, the SbS-MAP equalizer feeds a Kalman-like non linear estimator with the *more informative soft-statistics* constituted by the APPs of the channel state instead of the *less informative hard-decided* data so as to obtain fast convergence to the unknown channel-impulse response. Simulations have shown the feasibility of identification of both nonminimum phase channels and channels exhibiting deep spectral nulls. Moreover, an additional feature of the channel-estimator embedded in the proposed blind equalizer is that it generates an indicator of the reliability of the achieved channel estimates so that it is able to decide automatically to switch to a decision-driven operating mode to further speed up and refine the estimation process.

II. THE MODEL OF THE COMMUNICATION SYSTEM

The considered discrete-time complex baseband equivalent data transmission system is reported in Fig.1, where the discrete-time ISI-channel accounts for the combined effects of transmitting filter, noisy time-dispersive analog waveform channel, whitened-matched receiving filter and baud-rate sampler [9,Sect6.3]. The random data sequence $\{s(k) \in A \equiv \{s_1,...,s_M\}\}$, constituted by M-ary generally complex i.i.d. equiprobable symbols, is transmitted over a linear channel whose unknown time-invariant equivalent L-long discrete-time impulse response is denoted by $\{g(k), 0 \le k \le L\text{-}1\}$. Thus, the ISI corrupted noisy random sequence observed at the input of the blind equalizer can be modeled as

$$y(i) = \sum_{k=0}^{L-1} g(k)s(i-k) + v(i) \equiv \equiv G^T x(\mathbf{i}) + v(i) \equiv z(i) + v(i),\ i \ge 1 \quad (1)$$

where $G \equiv [g(0)...g(L-1)]^T$ is the unknown L-long impulse response vector of the ISI channel, $x(\mathbf{i}) \equiv [s(i)\ ...\ s(i-L+1)]^T$ is the corresponding channel-state vector and $\{v(i)\}$ is a complex zero mean Gaussian noise sequence whose uncorrelated components exhibit a common variance equal to $(N_o/2)$ (see [9,Sect. 6.3]). Since the observation model in (1) exhibits a bilinear form in the unknown channel impulse-response and the unknown transmitted symbols, unambiguous deconvolution generally cannot be achieved; for this reason, we resort to using differential encoding and decoding to resolve the phase ambiguity (however, we point out that this could be avoided when the transmitted stream $\{s(k)\}$ is not zero-mean).

The L-variate random sequence $x(\mathbf{i}) \in A^L \equiv \{\xi_1, ..., \xi_N\}$ is a first-order Markov chain known as "state sequence" of the ISI channel [1]; it may assume $N \equiv S^L$ distinct values $\{\xi_i, 1 \le i \le N\}$, corresponding to the L-long ordered sub-sequences of constellation symbols, that is:

$$\xi_j \equiv \left[s_1^{(j)}\ ...\ s_L^{(j)}\right]^T,\ 1 \le j \le N, \quad (2)$$

where $s_i^{(j)} \in A$ is the i-th component of the j-th determination ξ_j of the channel-state.

From a statistical point of view, the homogeneous Markov chain $\{x(i)\}$ is described by the corresponding NxN transition probability matrix Φ (with elements defined as: $\phi_{rj} \equiv P(x(i+1)=\xi_r / x(i)=\xi_j)$) and by the vector: $\pi(1) \equiv N^{-1} (\underline{1}_N)$ of the probabilities of the initial state ($\underline{1}_N$ denotes the column vector constituted by N unit elements).

Now, following [10], without loss of generality it can be assumed that the N determinations of $\{x(i)\}$ are constituted by the N unit vectors $\{u_i \in \mathbb{R}^N\} \equiv U$ of the Euclidean space $\mathbb{R}^N$. So, by adopting the basis U, we can equivalently rewrite (1) as

$$y(i) = G^T M x(i) + v(i),\ x(i) \in U\ , \qquad (3)$$

where M is the LxN mapping matrix with columns constituted by the vectors $\{\xi_i\}$ of (2), i.e. $M \equiv [\xi_1, ..., \xi_N]$. Furthermore, the chain $\{x(i) \in U\}$ admits the following representation [10]:

$$x(i+1) = \Phi x(i) + w(i+1),\ x(i) \in U\ , \qquad (4)$$

where the N-variate real sequence $\{w(i) \in \mathbb{R}^N\}$ constitutes a "Martingale Difference Sequence" (in the sense of [10]).

The computation of the (L-1)-delayed MAP-detected sequence $\{\hat{s}_{MAP}(k-L+1/k)\}$ requires the computation of the APPs $\left\{P\left(s(i-L+1)=\alpha_r | y_1^i\right),\ 1 \le r \le M\right\}$ of the transmitted stream; an application of the "Total probability theorem" shows that the latter is directly related to the APPs of the Markov chain $\{x(i)\}$, (that is, to $\left\{\pi(i/i) \equiv \left[P(x(i)=u_1 | y_1^i) ... P(x(i)=u_N | y_1^i)\right]^T\right\}$), as in the following [1]:

$$P\left(s(i-L+1)=\alpha_r | y_1^i\right) = \sum_{j \in U(r)} P\left(x(i)=u_j | y_1^i\right), \qquad (5)$$

where $U(r)$ indicates the subset of U defined by the following relationships ($1 \le r \le M$):

$$U(r) \equiv \left\{u_j \in U:\ \xi_j \in A(r)\right\} \text{ and } A(r) \equiv \left\{\xi_j \in A^L:\ \alpha_{L-1}^{(j)} = \alpha_r\right\}. \qquad (6)$$

III. The Proposed Blind Channel Estimator

An application of the so-called "MD representation theorem" [10] allows us to obtain from (3), (4) the following recursive expression for the nonlinear MMSE filtered estimates of the unknown channel impulse-response vector G:

$$\begin{aligned} \hat{G}(i) &\equiv E\left\{G / y_1^i\right\} = \\ &= \hat{G}(i-1) + K(i)\left[y(i) - \hat{G}^T(i-1)\, M\pi(i/i-1)\right], \end{aligned} \qquad (7)$$

where the following relationships also hold:

$$\begin{aligned} \pi(i/i-1) &\equiv \left[P(x(i)=u_1 | y_1^{i-1}) ... P(x(i)=u_N | y_1^{i-1})\right]^T = \\ &= \Phi \pi(i-1/i-1) \end{aligned}, \qquad (8)$$

$$S_X(i) \equiv M\left[diag(\pi(i/i-1)) - \pi(i/i-1)\, \pi^T(i/i-1)\right] M^H, \qquad (9)$$

$$\begin{aligned} \sigma^2(i) &\equiv N_O + Trace\left\{S_X^*(i) S_g(i-1)\right\} + \\ &+ \pi^T(i/i-1) M^H S_g^*(i-1) M \pi(i/i-1) + \\ &+ \hat{G}^T(i-1) S_X(i) \hat{G}^*(i-1) \end{aligned} \qquad (10)$$

$$K(i) \equiv \left[S_g(i-1)\, M^* \pi(i/i-1)\right] / \sigma^2(i)\ , \qquad (11)$$

$$D(i) \equiv diag\left\{exp\left\{-\left\|y(i) - \hat{G}^T(i) M u_j\right\|^2 / N_O\right\}\right\} \qquad (12)$$

$$1 \le j \le N$$

$$\pi(i/i) \equiv [D(i)\pi(i/i-1)] / \left[(\underline{1}_N)^T D(i) \pi(i/i-1)\right], \qquad (13)$$

$$\begin{aligned} S_g(i) &\equiv E\left\{\left[G - \hat{G}(i)\right]\left[G - \hat{G}(i)\right]^H / y_1^i\right\} = \\ &\cong \left[I_{LxL} - K(i) \pi^T(i/i) M^T\right] S_g(i-1) \end{aligned} \qquad (14)$$

From eq.(7) it can be seen that the channel estimator utilizes the *soft-statistics* constituted by the APPs sequence $\{\pi(i/i)\}$ in place of the conventional *hard-statistics* constituted by the MAP-detected data sequence and this represents the main novelty of the proposed blind equalizer. Furthermore, the presented estimator collapses into a standard decision-driven Kalman-filter just by posing $S_X(i)$=0 and $M\pi(i/i-1) = [\hat{s}(i)\ ...\ \hat{s}(i-L+1)]^T$ in eq.(10). This allows the receiver to switch easily to a decision-driven Kalman operating mode to speed up convergence and refine the estimation once a good reliability in the channel estimation has been achieved during the APP-driven operating mode. A good indicator of the achieved reliability of the channel estimate turns out to be the trace of matrix $S_g(i)$ of eq.(14), that is proportional to the average MSE of the channel estimation.

In summary, from the foregoing we can summarize the proposed algorithm for blind deconvolution as below reported:

1. Initialization:, $\pi(1/0) = \frac{1}{N}[1..1]^T$, $S_g(0/0) = \frac{1}{L} I_{LxL}$, $\hat{G}(0/0) = [10...0]^T$, ($I_{LxL}$ is the unitary square matrix of dimension LxL).
2. Soft operating mode:

- Starting from $\pi(i-1/i-1)$, $\hat{G}(i-1)$, $S_g(i-1)$ computed at the previous step, the predicted APPs $\pi(i/i-1)$ of the channel state $x(i)$ are computed as in eq.(8);
- $S_X(i)$ is computed as in eq.(9);
- $\sigma^2(i)$ is computed as in eq.(10);
- The observation-dependent Kalman gain $K(i)$ is computed as in eq.(11);
- The nonlinear MMSE channel estimate $\hat{G}(i)$ is computed as in eq.(7);
- The APPs $\pi(i/i)$ are computed as in eq.(13);
- The error covariance matrix $S_g(i)$ for the channel estimate is computed as in eq.(14);
- The trace of $S_g(i)$ is computed and used to decide whether to switch eventually to the hard operating mode.

3. Hard operating mode:

- From the previous step, $\hat{G}(i-1)$ and $S_g(i-1)$ are available; moreover, on the basis of (5), hard-decision on the transmitted symbols are directly obtained from $\pi(i-1/i-1)$ according to the usual MAP rule;
- The same steps that appear in 2) are followed provided that $S_X(i)$ is set to zero in eq.(9) and the product $M\pi(i/i-1)$ is replaced by the vector containing the last L hard decisions.

4. Data Recovery mode:

- At this point the receiver can start to recover the data. Since the channel estimator uses the APPs, an Abend-Fritchman like [11] MAP detector can be directly implemented according to (5); however, once the channel has been identified any equalization strategy could also be used.

On the basis of the value assumed by the trace of $S_g(i)$ the receiver can decide to skip the Hard Operating Mode and directly start to recover the data; in fact, when the SNR is too low the decisions may not be reliable and it might happen that the estimate of the channel achieved in the Soft Operation Mode degrades.

IV. Simulation Results

The channels we considered in our simulations are listed in Tab.I; channel A has a deep spectral null and channel B is a nonminimum-phase channel. The trajectory of the estimated first channel tap $g(0)$ of channel A during the Soft Operating Mode is shown in Fig.2 as a function of the number of observed samples for SNRs of 10 dB and 20 dB. It can be seen that 200 samples are generally enough to achieve a good identification of the channel.

In the previous Sections we pointed out that the trace of the filtered error covariance matrix $S_g(i)$ defined in (14) is an effective indicator of the reliability of the achieved channel-estimates and can be used to switch from the Soft Operating Mode to the Hard Operating Mode. In Fig. 3 this trace is plotted versus the number of observed samples for different values of the SNR. In our simulations we experienced that a trace value lower than 0.01 corresponds to a good "status" of channel-identification; in Fig.3 it can be seen that higher SNRs require shorter time-intervals to achieve good channel estimates.

The effects of using a Hard Operating Mode after a preliminary Soft Operating Mode are shown in Fig.4. The Bit Error Rate of the system is plotted as a function of the number of observed samples used during the Soft Operating Mode for different values of the SNR; the performance improvement obtained by adding 200 additional symbols in the Hard Operating Mode is also shown. The simulations have been carried out on channel A for differentially-encoded BPSK-modulated data with MAP detection with decoding delay $D=L-1=2$; the results are averaged over 1000 independent trials. In the figure it can be verified that the Hard Operating Mode enhances the performances and this effect is more evident at high SNRs.

Finally, in Figs.5-6 the performance of the proposed blind MAP equalizer is shown and compared to the ideal case of known channel for the links A and B, respectively. Simulations refer to a BPSK modulated system with differential encoding and are averaged over 1000 independent trials; for SNRs values ranging from 6 to 8, dB the receiver used 500 symbols in the Soft Operating Mode, while for SNR values ranging from 9 to 16, the receiver used 300 symbols in the Soft Operating Mode and 200 in the Hard Operating Mode.

V. Conclusions

A novel blind equalizer has been presented. It exploits second-order statistics only for channel-identification purposes and does not require oversampling of the received signal. The proposed channel-estimator present in the proposed blind-equalizer exhibits a Kalman-like structure and is fed by the soft statistics constituted by APPs of the channel-state sequence in place of the usual hard-decided data. It allows quick identification of nonminimum-phase channels and channels exhibiting deep spectral nulls within a few hundred symbols. An interesting feature of the proposed blind equalizer is that it can work easily in a dual mode, that is, a *Soft Operating Mode* using the APPs and a *Hard Operating Mode* using the hard decided data. Furthermore, the receiver can also decide automatically whether it is convenient to switch from the first mode to the second one by testing the trace of the error covariance matrix of the channel estimate. Simulations have shown that the proposed blind channel equalizer can achieve performances similar to the ones obtained for the ideal case of known channel.

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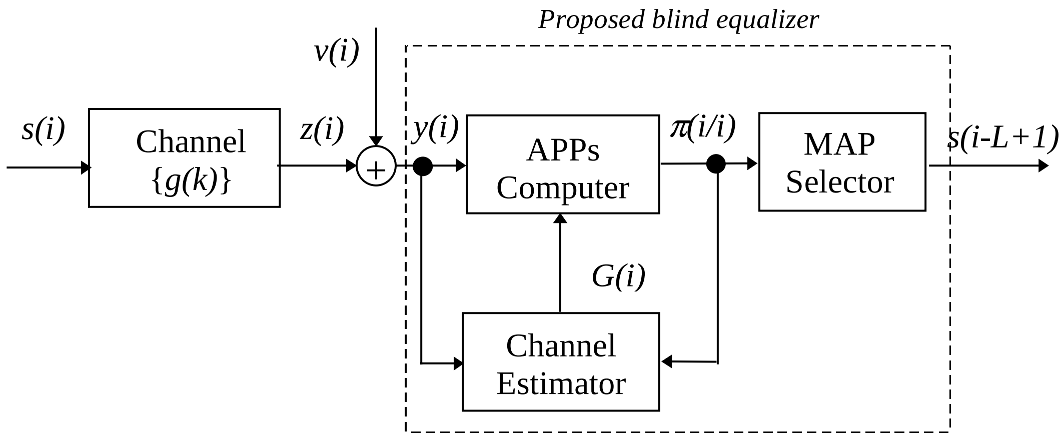

Fig. 1 - Discrete-time low-pass version of the considered ISI-impaired communication system.

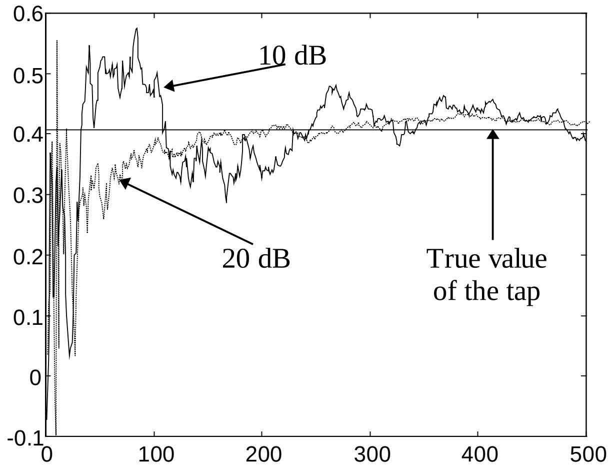

Fig. 2 - Trajectory of the estimate of the tap $g(0)$ of Channel A as a function of the number of the observed samples for different values of the SNR (dB); BPSK modulation with differential encoding has been adopted in the simulations.

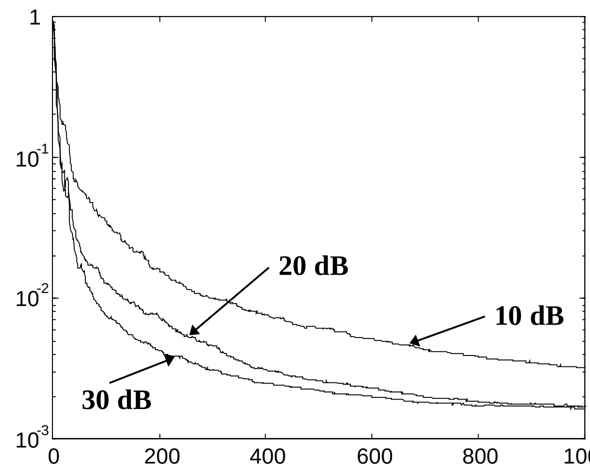

Fig. 3 - Averaged trace of $S_g(i)$ of eq.(14) as a function of the number of the observed samples for different values of SNR (dB). The channel used for the simulation is the Channel A and the results are averaged over 1000 independent trials. BPSK modulation with differential encoding has been used in the simulation.

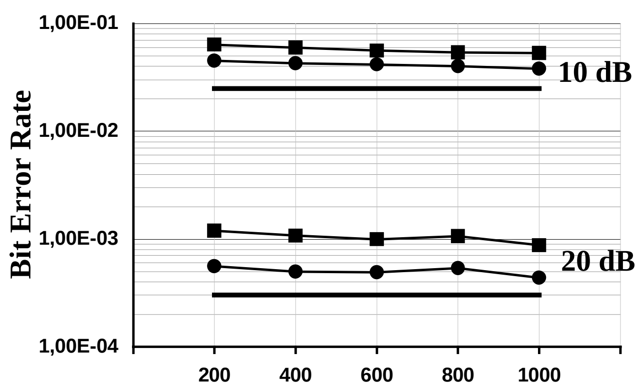

Fig. 4 - Steady-state values of the Bit Error Rate of the system of Fig.1 for the Channel A as a function of the number of the observed samples used during the Soft Operating Mode for different values of the SNR (—■—); for comparison purposes, the

performance improvement obtained by adding 200 symbols in the Hard Operation Mode is shown (—●—). As a bench mark, the bit error rate obtained for the ideal case of MAP equalization of known channel is also shown (——).

	g(0)	g(1)	g(2)	g(3)
Ch. A	0.408	0.816	0.408	0
Ch. B	0.67	0.47	0.26	-0.52

Tab.I - List of the impulse responses of the tested ISI channels.

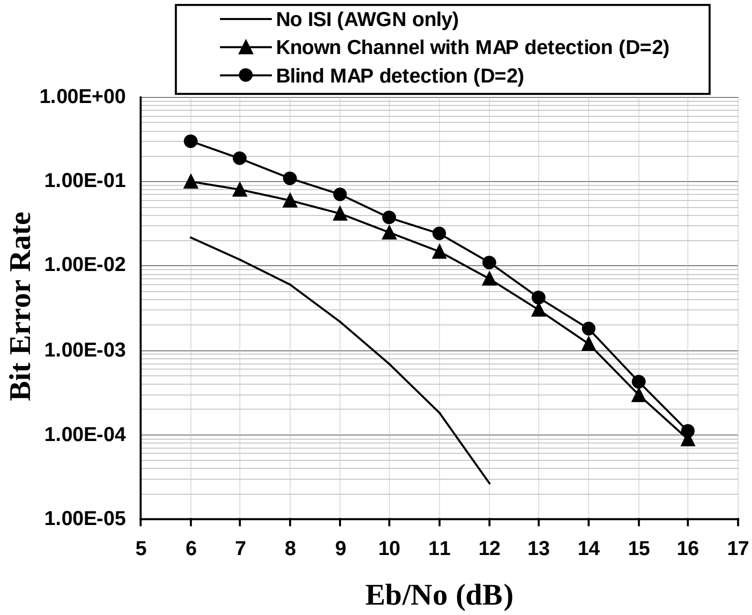

Fig. 5 - Steady-state values of the Bit Error Rate for Channel A with BPSK signaling and MAP equalization with decoding delay *D*=2.

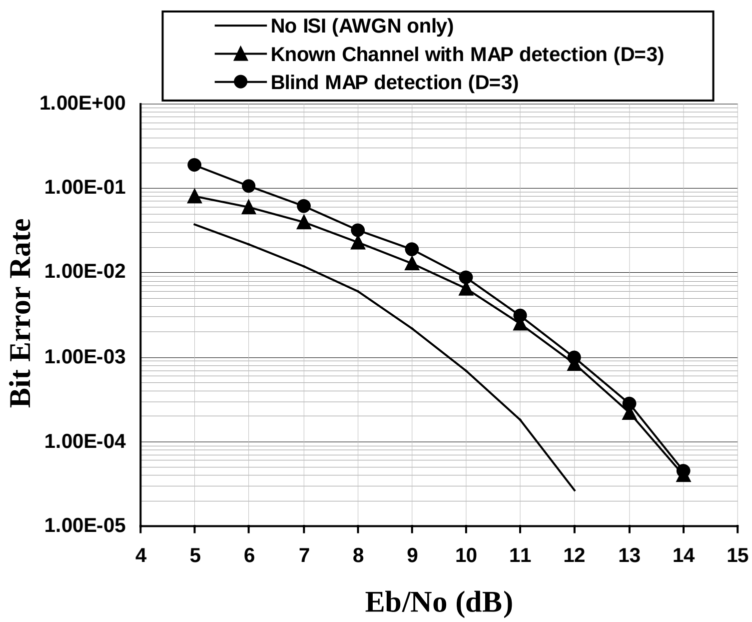

Fig. 6 - Steady-state values of the Bit Error Rate for Channel B with BPSK signaling and MAP equalization with decoding delay *D*=3.