NON-LINEAR MMSE ESTIMATION AND SBS-MAP RECEIVERS

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***Abstract* – For the first time it is here shown that Symbol-by-Symbol Maximum A Posteriori (SbS-MAP) receivers are able to generate Non-Linear Minimum Mean Square Error (NL-MMSE) estimates of the transmitted symbols.**

I. INTRODUCTION

SbS-MAP receivers have the appealing feature of being able of generating a kind of *soft information* that can be considered intermediate between hard-decisions and A Posteriori Probabilities (APPs): the NL-MMSE estimates of the transmitted symbols. This result is not a surprise since MMSE estimation is defined through an "*a posteriori*" expectation functional. Nevertheless, the fact that one can generate NL-MMSE estimates through an SbS-MAP receiver has never been clearly pointed out in the current literature.

The availability of NL-MMSE estimates of the transmitted symbols is very useful in many applications, especially in those applications where it is necessary to mitigate the effects of wrong hard decisions. This has been recently pointed out in [1], although no method for computing the NL-MMSE was given.

II. A GENERAL MODEL OF THE OBSERVATIONS

The general model of a signal transmitted over a noisy and dispersive time-invariant channel is here considered. The random data sequence $\{s(k)\}$, constituted by M-ary generally complex i.i.d. equiprobable symbols, is transmitted over a linear channel whose time-invariant equivalent L-long discrete-time impulse response is denoted by $\{g(k)\}$. Thus, the ISI-impaired noisy sequence observed at the output of a baud-rate sampled whitened matched receiving filter can be modeled by the usual relationship:

$$y(i) = \sum_{k=0}^{L-1} g(k)s(i-k) + v(i) \equiv G^T x(i) + v(i)\,, \qquad (1)$$

where G is the L-long impulse response vector of the ISI channel, $x(i) \equiv [s(i) \ldots s(i-L+1)]^T$ is the corresponding channel-state vector and $\{v(i)\}$ is a complex zero mean Gaussian noise sequence. The L-variate random sequence $\{x(i)\}$ is a first-order Markov chain known as "state sequence" of the ISI channel and may assume $N \equiv M^L$ distinct values $\{\xi_i\}$.

III. NL-MMSE ESTIMATION AND APPS

The MMSE estimate of the symbol $s(i)$ on the basis of the observations from step 1 to step i is given by the following relationship:

$$\hat{s}_{MMSE}(i) \equiv E\left\{s(i) \mid y_1^i\right\} \equiv \sum_{k=1}^{M} s_k \Pr\left(s(i) = s_k \mid y_1^i\right), \qquad (2)$$

It is possible to prove that (2) can be re-written in the following form:

$$\underline{\hat{s}}_{NL-MMSE}(i;L) = \Xi \pi(i/i) \qquad (3)$$

where $\underline{\hat{s}}_{NL-MMSE}(i;L)$ is the vector containing the NL-MMSE estimates of the last L transmitted symbols, $\pi(i|i)$ is the vector of the APPs of the state sequence of the ISI channel and Ξ is a LxN matrix whose columns are constituted by the vectors $\{\xi_i\}$ of (2). The relationship in (3) shows that the NL-MMSE estimates of the last L transmitted symbols can be expressed as a function of the APPs of the state of the ISI channel.

IV. CONCLUSIONS

In the present contribution, we presented a new method for generating NL-MMSE estimation with an SbS-MAP receiver. This method makes the use of SbS-MAP receivers very appealing because they can generate three kinds of information: a hard-statistics based information (the hard-decisions), a soft-statistics based information (the APPs) and an intermediate case represented by the NL-MMSE estimates of the transmitted symbols.

In general, the use of "estimates" in place of "decisions" is useful whenever the reliability of the hard-decisions is low. In fact, a wrong hard-decision is certainly more harmful (to channel estimation and tracking or to systems with feedback) than an imperfect estimate on the transmitted symbol [2, 3]. Other useful applications that may be foreseen for the proposed technique are in the field of multi-user detection.

REFERENCES

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