A NOVEL REDUCED-COMPLEXITY MAP EQUALIZER USING SOFT-STATISTICS FOR DECISION-FEEDBACK ISI CANCELLATION

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***Abstract* - A novel version of the reduced-state Bayesian Maximum A Posteriori Probability /Decision-Feedback (MAP/DF) equalizer for ISI channels with long impulse responses is presented. The main feature of the proposed equalizer is that the soft-statistics generated by the MAP receiver are employed to recursively compute a suitable index of the actual reliability of the (soft) decisions feeding the DF filter. Therefore, in the presented equalizer the usual (over-optimistic) assumption of error-free decisions at the input of the feedback filter is relaxed and numerical results supporting the actual effectiveness of the proposed MAP/DF equalizer are provided for the so-called High-Bit-Rate Digital Subscriber Line (HDSL) test-loop # 4.**

I. MOTIVATIONS OF THE WORK

High-bit-rate data transmissions over waveform channels with large delay-spreads as those typically encountered in HDSL environments [8,9] are subject to severe ISI impairments which, in principle, could be well compensated by resorting to receivers based on Maximum-Likelihood Sequence Estimators (MLSEs) or Bayesian MAP Symbol Detectors (MAPSDs). However, the computational complexity of these receivers grows (at least) exponentially with the channel memory length, so that considerable efforts have been undertaken to develop reduced-complexity versions of the MLSE and MAP equalizers suitable for the application on transmission channels with large delay-spreads [2,3,4,7]. Various versions of reduced-state MLSEs which employ decision-feedback to "short" the channel memory and then lower the resulting receiver complexity are described, for example, in [1,2,3] and references therein. More recently, in [4,7] the decision-feedback approach has been also applied to reduce the complexity of the standard MAPSDs [5 and references therein]; the resulting MAP/DF equalizer reveals a structure similar to the DFE-MLSE of [3], the main difference being that a single feedback-filter is present in the MAP/DF receiver of [4,7] whereas in the DFE-MLSE of [3] each state of the trellis requires a feedback-filter.

All the above mentioned reduced-state DF-based equalizers share the common feature to attempt to cancel the tails of the channel ISI via *hard* decisions and furthermore have been derived under the over-optimistic assumption of *error-free* decisions feeding the feedback filter; so, they can incur error-propagation phenomena, specially over channels with non-minimum-phase long impulse responses as those characterizing some typical HDSLs [9].

Starting from the system-modelling of Sect.II and then following a Bayesian approach, in Sect.III of this contribution we present a novel version of the above mentioned reduced-state MAP Hard-Decision-Feedback (MAP/HDF) equalizer of [4,7] where the soft-statistics delivered by the MAPSD are used to enhance the suppression of the ISI tails. More in detail, the distinguishing features of the proposed MAP/Soft-Decision-Feedback (MAP/SDF) equalizer are twofold. First of all, the A Posteriori Probabilities (APPs) of the reduced-state of the ISI channel generated by the MAP receiver are employed to deliver low-delayed "soft" decisions which are utilized in the feedback-branch of the proposed equalizer for a more reliable cancellation of the ISI tails. Secondly, a suitable observation-depending index of the *actual reliability* of the decisions feeding the feedback-filter is also computed and then used *for updating on a per-step basis* the branch-metrics of the MAP/SDF equalizer. Therefore, since the proposed equalizer operates *without relying* on the usual assumption of error-free decisions at the input of the DF filter, its performance tends to be *less impaired* by error propagation phenomena, specially in data applications at medium SNRs; the numerical results reported in the final Sect.IV for some HDSL-type applications support this conclusion.

II. THE SYSTEM MODEL

The ISI-impaired noisy sequence $\{r(n)\in \mathbf{R},\ n\geq 1\}$ observed at the output of a baud-rate-sampled whitened matched receiving filter can be modelled by the usual relationship

$$r(n)=\sum_{m=0}^{L-1}h(m)a(n-m)+w(n)\,,\quad n\geq 1\,, \tag{1}$$

where $\{a(n)\in \mathrm{A}\}$ is the transmitted PAM[1] sequence with independent identically distributed (i.i.d.) zero-mean unit-variance outcomes taking value on an assigned q-ary unidimensional constellation $\mathrm{A}\equiv\{\alpha_1,\ldots,\alpha_q\}$ and $\{w(n)\}$ is a real zero-mean white Gaussian noise with variance $\mathrm{N_o}/2$. The ISI introduced by the transmission link is described by the L-long sampled impulse-response $\{h(m)\in \mathbf{R},\ 0\leq m\leq L\text{-}1\}$ which accounts for the combined effects of the front-end transmitting and receiving filters and the waveform transmission-channel.

Now, as in [1,2,3,4,7] we assume that a constraint on the receiver complexity is imposed so that only the first L^* out of the total number L of impulse-response coefficients are to be directly handled by the MAP section of the receiver while the effects of the remaining $L_{df}\equiv(L\text{-}L^*)$ terms of the channel-impulse response are to be compensated via a (suitably designed) L_{df} -long feedback filter. Thus, by splitting the overall channel-impulse response according to this assumption, the relationship in (1) can be equivalently rewritten as

$$r(n)=\sum_{m=0}^{L^*-1}h(m)a(n-m)+\sum_{m=L^*}^{L-1}h(m)a(n-m)+w(n) \tag{2}$$

$$=\mathbf{h}_1^{\mathrm{T}}\boldsymbol{\sigma}_1(n)+\mathbf{h}_2^{\mathrm{T}}\boldsymbol{\sigma}_2(n)+w(n)$$

$$=\mathbf{h}_1^{\mathrm{T}}\boldsymbol{\sigma}_1(n)+T(n)+w(n)\,, \tag{3}$$

where $\mathbf{h}_1\equiv[h(0),\ldots,h(L^*\text{-}1)]^{\mathrm{T}}$ and $\mathbf{h}_2\equiv[h(L^*),\ldots,h(L\text{-}1)]^{\mathrm{T}}$ are the partial vector impulse-responses of the channel, while $\boldsymbol{\sigma}_1(n)\equiv[a(n),\ldots,a(n\text{-}L^*\text{-}1)]^{\mathrm{T}}\in \mathrm{A}^{L^*}$ and $\boldsymbol{\sigma}_2(n)\equiv[a(n\text{-}L^*),\ldots,a(n\text{-}L\text{+}1)]^{\mathrm{T}}\in \mathrm{A}^{L_{df}}$ represent the corresponding channel state-vectors; therefore, the resulting term $T(n)\equiv \boldsymbol{h}_2^{\mathrm{T}}\boldsymbol{\sigma}_2(n)$ in eq.(3) constitutes the part of the channel impulse response to be compensated via the feedback mechanism. Furthermore, from the outset it follows that the above defined L^*-variate sub-state sequence $\left\{\boldsymbol{\sigma}_1(n)\in \mathrm{A}^{L^*}\equiv\left\{\boldsymbol{\xi}_1^{(1)},\ldots,\boldsymbol{\xi}_{\mathrm{N}_1}^{(1)}\right\}\subset \mathbf{R}^{L^*}\right\}$ is a first-order stationary Markov chain which may assume the $\mathrm{N}_1\equiv q^{L^*}$ outcomes $\left\{\boldsymbol{\xi}_k^{(1)},\ 1\leq k\leq \mathrm{N}_1\right\}$ corresponding to the L^*-long ordered sub-sequences of data symbols, these last being defined as

$$\boldsymbol{\xi}_k^{(1)}\equiv\left[\alpha_1^{(k)}\cdots\alpha_{L^*}^{(k)}\right]^{\mathrm{T}},\quad 1\leq k\leq \mathrm{N}_1\,, \tag{4}$$

($\alpha_i^{(k)}\in \mathrm{A}$ in (4) indicates the i-th component of the k-th outcome $\boldsymbol{\xi}_k^{(1)}$ allowed to the channel-state $\boldsymbol{\sigma}_1$(n)). Therefore, from the definition it stands out that the Markov chain $\{\boldsymbol{\sigma}_1(n),\ n\geq 1\}$ evolves as the state of a usual L^*-long right-shift-register and then it results statistically described by the resulting $\mathrm{N}_1\times\mathrm{N}_1$ transition probability matrix $\boldsymbol{\Phi}^{(1)}$ [5,6].

Now, having denoted by $\mathbf{r}_1^n\equiv\{r(1),\ldots,r(n)\}$, $n\geq 1$, the realization of the received sequence $\{r(n)\}$ in (1) up to the n-th step, the reduced-state MAP equalizer operating with a decision-delay D set to the memory length (L^*-1) of the "truncated" channel-impulse response[2] delivers the hard-decided data stream $\{\hat{a}(n\text{-}L^*\text{+}1),\ n\geq 1\}$ according to the usual MAP decision rule [5,6]

$$\begin{aligned}\hat{a}(n-L^*+1)=\alpha_r\ \textit{iff}\ \Pr\left(a(n-L^*+1)=\alpha_r\,|\,\mathbf{r}_1^n\right)>\\ >\Pr\left(a(n-L^*+1)=\alpha_j\,|\,\mathbf{r}_1^n\right),\forall j\neq r\end{aligned} \tag{5}$$

where $\left\{\Pr\left(a(n-L^*+1)=\alpha_k\,|\,\mathbf{r}_1^n\right),1\leq k\leq q\right\}$ are the

[1] The generalization of the presented results to QAM pass-band systems can be found in [11], and, due to space limitations, is no explicitly reported here.

[2] Although, in principle, decision-delays D higher than L^*-1 could be considered, simulation results and analytical bounds [6,7] confirmed that delays of the order of the memory length of the "truncated" channel are generally sufficient to give rise to reliable performance. So, in this contribution we directly focus on the case D = L^*-1.

APPs of the transmitted symbol $a(n-L^*+1)$ conditioned on the received sequence $\mathbf{r}_1^n$ which can be lumped to form the corresponding q-variate APP vector defined as:

$$\mathbf{p}^{(a)}(n) \equiv \begin{bmatrix} \Pr\left(a(n-L^*+1)=\alpha_1 \mid \mathbf{r}_1^n\right) \\ \vdots \\ \Pr\left(a(n-L^*+1)=\alpha_q \mid \mathbf{r}_1^n\right) \end{bmatrix}, \quad n \ge 1 . \qquad (6)$$

Now, an application of the Total Probability Theorem allows us to calculate the APPs of eqs.(5),(6) from those pertaining to the channel-state Markov chain $\{\boldsymbol{\sigma}_1(n)\}$ of eq.(3) via the usual relationship [6]

$$\Pr\left(a(n-L^*+1)=\alpha_m \mid \mathbf{r}_1^n\right)= = \sum_{\boldsymbol{\xi}_j^{(1)}\in \mathrm{A}(m)} \Pr\left(\boldsymbol{\sigma}_1(n)=\boldsymbol{\xi}_j^{(1)} \mid \mathbf{r}_1^n\right), \quad 1\le m \le q , \qquad (7)$$

where $\mathrm{A}(m) \equiv \left\{\boldsymbol{\xi}_j^{(1)} \in \mathrm{A}^{L^*} : \alpha_{L^*-1}^{(j)} = \alpha_m \right\}, 1 \le m \le q$, is the subset of A^{L^*} composed by the outcomes $\left\{\boldsymbol{\xi}_j^{(1)}\right\}$ with the (L^*-1)-th element equal to the constellation symbol α_m. Therefore, the computation of N_1-variate vector

$$\boldsymbol{\Pi}^{(1)}(n \mid n) \equiv \left[\Pr\left(\boldsymbol{\sigma}_1(n)=\boldsymbol{\xi}_1^{(1)} \mid \mathbf{r}_1^n\right)\cdots\Pr\left(\boldsymbol{\sigma}_1(n)=\boldsymbol{\xi}_{\mathrm{N}_1}^{(1)} \mid \mathbf{r}_1^n\right)\right]^{\mathrm{T}}$$

collecting the APPs of the channel sub-state $\boldsymbol{\sigma}_1(n)$ is sufficient to generate the MAP-decision in (5); this computation can be accomplished via the recursive algorithm presented in the next Section.

III. THE PROPOSED REDUCED-STATE MAP/SDF EQUALIZER

The APPs in (6) can be effectively exploited for a "soft" cancellation of the residual ISI-term $T(n)$ present in the received sequence of eq.(3) via a feedback-type mechanism; more in detail, by resorting to a suitable generalization of the usual Bayesian approach [5 and references therein], it can be proved that this task can be accomplished via the recursive updating of the following set of relationships:

$$\boldsymbol{\Pi}^{(1)}(n \mid n-1) \equiv \begin{bmatrix} \Pr\left(\boldsymbol{\sigma}_1(n)=\boldsymbol{\xi}_1^{(1)} \mid \mathbf{r}_1^{n-1}\right) \\ \vdots \\ \Pr\left(\boldsymbol{\sigma}_1(n)=\boldsymbol{\xi}_{\mathrm{N}_1}^{(1)} \mid \mathbf{r}_1^{n-1}\right) \end{bmatrix} = \boldsymbol{\Phi}^{(1)}\boldsymbol{\Pi}^{(1)}(n-1 \mid n-1), \quad n \ge 2, \qquad (8)$$

with $\boldsymbol{\Pi}^{(1)}(1 \mid 0) \equiv \frac{1}{\mathrm{N}_1}\mathbf{1}_{\mathrm{N}_1}$; [3]

$$\tilde{T}(n) \equiv \mathrm{E}\left\{T(n) \mid \mathbf{r}_1^{n-1}\right\} = \mathbf{h}_2^{\mathrm{T}}\tilde{\boldsymbol{\sigma}}_2(n), \quad n \ge 1; \qquad (9)$$

$$\mathrm{cov}_T(n) \equiv \mathrm{E}\left\{\left(T(n)-\tilde{T}(n)\right)^2 \mid \mathbf{r}_1^{n-1}\right\} = \mathbf{h}_2^{\mathrm{T}}\mathbf{Cov}_{\boldsymbol{\sigma}}^{(2)}(n \mid n-1)\mathbf{h}_2 , \quad n \ge 1 ; \qquad (10)$$

$$\delta(t;n) \equiv \exp\left\{-\frac{\left(r(n)-\mathbf{h}_1^{\mathrm{T}}\boldsymbol{\xi}_t^{(1)}-\tilde{T}(n)\right)^2}{N_o + 2\,\mathrm{cov}_T(n)}\right\}, \qquad (11)$$

$1 \le t \le \mathrm{N}_1$, $n \ge 1$;

$$\boldsymbol{\Delta}(n) \equiv diag\left\{\delta(1;n),\ldots,\delta(\mathrm{N}_1;n)\right\}, \quad n \ge 1; \text{[4]} \qquad (12)$$

$$\boldsymbol{\Pi}^{(1)}(n \mid n) = \frac{\boldsymbol{\Delta}(n)\boldsymbol{\Pi}^{(1)}(n \mid n-1)}{\mathbf{1}_{\mathrm{N}_1}^{\mathrm{T}}\boldsymbol{\Delta}(n)\boldsymbol{\Pi}^{(1)}(n \mid n-1)}, \quad n \ge 1 ; \qquad (13)$$

$$\tilde{a}(n-L^*+1 \mid n) \equiv \mathrm{E}\left\{a(n-L^*+1) \mid \mathbf{r}_1^n\right\} = \boldsymbol{\alpha}^{\mathrm{T}}\mathbf{p}^{(a)}(n), \quad n \ge 1 ; \text{[5]} \qquad (14)$$

$$\tilde{\boldsymbol{\sigma}}_2(n+1) \equiv \begin{bmatrix} \tilde{a}(n-L^*+1 \mid n) \\ \vdots \\ \tilde{a}(n-L+2 \mid n-(L-L^*)+1) \end{bmatrix} \text{[6]} = \mathbf{S}\tilde{\boldsymbol{\sigma}}_2(n) + \mathbf{e}\,\tilde{a}(n-L^*+1 \mid n), \quad n \ge 1; \qquad (15)$$

$$\mathrm{cov}_{\tilde{a}}(n) \equiv \mathrm{E}\left\{\left(a(n-L^*+1)-\tilde{a}(n-L^*+1 \mid n)\right)^2 \mid \mathbf{r}_1^n\right\} = \mathbf{A}^{\mathrm{T}}\mathbf{p}^{(a)}(n) - \left(\tilde{a}(n-L^*+1 \mid n)\right)^2 , \quad n \ge 1; \text{[7]} \qquad (16)$$

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[3] $\mathbf{1}_m$ indicates the m-variate column vector with unit elements.

[4] diag $\{\beta_1,\ldots, \beta_n\}$ indicates the $n\times n$ diagonal matrix with the elements $\{\beta_1,\ldots, \beta_n\}$ ordinately disposed along its main diagonal.

[5] $\boldsymbol{\alpha}\equiv[\alpha_1,\ldots, \alpha_q]^{\mathrm{T}}$ is the q-variate (column) vector which collects the q-elements of the modulation constellation A.

[6] $\mathbf{S}$ is the $(L-L^*)\times(L-L^*)$ square shift-type matrix defined as:

$$\mathbf{S} \equiv \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \end{bmatrix};$$

furthermore, $\mathbf{e} \equiv \left[1 \,\vdots\, \mathbf{0}_{(L-L^*-1)\times 1}\right]^{\mathrm{T}}$ is the $(L-L^*)$-variate first unit vector of $\mathbf{R}^{L-L^*}$. Finally, the recursion in (15) must be initialized by: $\tilde{\boldsymbol{\sigma}}_2(1) \equiv \left(\frac{1}{q}\boldsymbol{\alpha}^{\mathrm{T}}\mathbf{1}_q\right)\mathbf{1}_{L-L^*}$.

$$\mathbf{Cov}_{\boldsymbol{\sigma}}^{(2)}(n+1\,|\,n) \equiv \mathrm{E}\{[\boldsymbol{\sigma}_2(n+1) - \tilde{\boldsymbol{\sigma}}_2(n+1)]\cdot \cdot [\boldsymbol{\sigma}_2(n+1) - \tilde{\boldsymbol{\sigma}}_2(n+1)]^{\mathrm{T}} \,|\, \mathbf{r}_1^n\} = \quad [8] \quad (17)$$

$$= \mathbf{S}\,\mathbf{Cov}_{\boldsymbol{\sigma}}^{(2)}(n\,|\,n-1)\mathbf{S}^{\mathrm{T}} + \mathbf{e}\mathbf{e}^{\mathrm{T}}\operatorname{cov}_{\tilde{a}}(n), \quad n \geq 1.$$

Remark 1 (Soft-Decision-based versus Hard-Decision-based ISI tails cancellation). The conventional reduced-state decision-feedback MLSE and MAP equalizers of [1,2,3,4,7] share the common feature to employ *hard* decisions for the cancellation of the ISI tails *and*, furthermore, they assume *error-free* decisions at the input of the feedback filters. By fact, this assumption implies that in the conventional MAP/HDF equalizer of [4,7] a *hard*-decision-based estimate $\hat{\boldsymbol{\sigma}}_2(n) \equiv [\hat{a}(n-L^*)\cdots\hat{a}(n-L+1)]^{\mathrm{T}}$ of the sub-state $\boldsymbol{\sigma}_2(n)$ is built-up and then updated directly on the basis of the current *hard* decision $\hat{a}(n-L^*+1)$ delivered by the MAP equalizer via the following formula:

$$\hat{\boldsymbol{\sigma}}_2(n+1) = \mathbf{S}\hat{\boldsymbol{\sigma}}_2(n) + \mathbf{e}\,\hat{a}(n-L^*+1)\,. \qquad (18)$$

Hence, in the MAP/HDF equalizer the resulting estimate $\hat{T}(n)$ of the ISI term $T(n)$ affecting the observation (3) is simply computed as

$$\hat{T}(n) = \mathbf{h}^{(2)\mathrm{T}}\hat{\boldsymbol{\sigma}}_2(n)\,; \qquad (19)$$

then, under the assumption of *error-free* feedback-decisions, this estimate is employed to calculate the terms of the diagonal matrix $\boldsymbol{\Delta}^{(1)}(n)$ in (12) via the following relationship:

$$\delta(t;n) \equiv \exp\left\{-\frac{\left(r(n) - \mathbf{h}_1^{\mathrm{T}}\boldsymbol{\xi}_t^{(1)} - \hat{T}(n)\right)^2}{N_o}\right\}, \qquad (20)$$

$1 \leq t \leq \mathrm{N}_1,\ n \geq 1.$

Now, in the proposed MAP/SDF equalizer *soft-type non-linear* MMSE symbol estimates are used for the cancellation of the ISI tails (see eqs.(9), (11), (14), (15)); furthermore, the *actual reliability* of the estimate $\tilde{T}(n)$ in (9) employed to suppress the residual ISI term $T(n)$ is *explicitly* taken into account via the corresponding conditional error covariance of eq.(10) in the recursive branch-metrics updating of the MAP/SDF equalizer (see eqs.(10), (11), (16), (17)). ◊

IV. Simulation results on HDSL environments and conclusions

The performance of the proposed MAP/SDF equalizer has been tested by simulating the transmission of BPSK modulated data over the so-called HDSL test-loop # 4 channel considered in [8,9]. The BERs obtained for the proposed MAP/SDF equalizer and the MAP/HDF one are reported in Figs.1,2[9] for values of L^* ranging from 7 to 4; as a benchmark, on the same figures also the performance of the conventional hard-decision-driven ideal ZF-DFE is drawn, this last being constituted by a feedback filter with taps (exactly) set to the impulse response coefficients of the considered channel.

An examination of the reported numerical results shows that specially for SNRs ranging from 11 dB to 17 dB the proposed MAP/SDF equalizer exhibits noticeable BER reductions over the MAP/HDF one; the resulting BER gaps approach indeed about *one order of magnitude* at SNRs around 15-16 dB and for values of L^* ranging from 2 to 5.

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[7] $\mathbf{A} \equiv [(\alpha_1)^2 \cdots (\alpha_q)^2]^{\mathrm{T}}$ is the q-variate column vector which gathers the squared values of the q elements of the constellation A.

[8] The recursion in (17) for the $(L-L^*)\times(L-L^*)$ dimensional covariance matrix must be initialized by $\mathbf{I}_{L-L^*} - \left(\frac{1}{q}\boldsymbol{\alpha}^{\mathrm{T}}\mathbf{1}_q\right)^2 \mathbf{1}_{L-L^*}\,\mathbf{1}_{L-L^*}^{\mathrm{T}}$.

[9] We adopt the notations MAP/SDF[L^*/L] and MAP/HDF[L^*/L] to refer to a channel with L-long impulse response with the MAP sections of the equalizers of Figs.1,2 which model only the first L^* taps (i.e., 2^{L^*} trellis-states) and the corresponding DF filters accounting for the last (L-L^*) taps of the channel impulse response.

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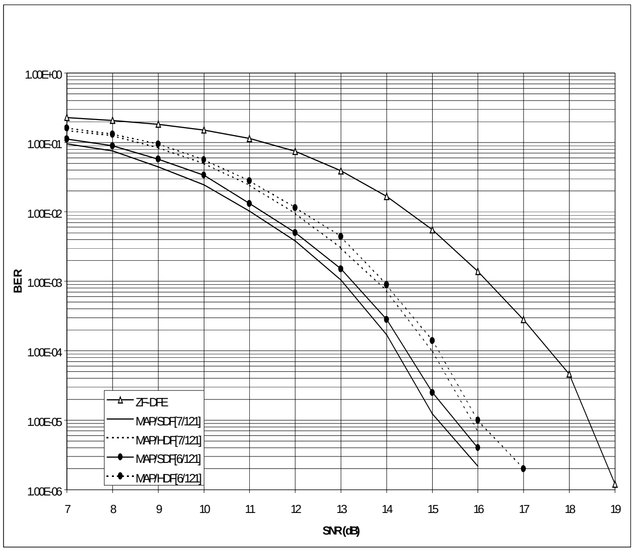

Fig. 1. Performance plots of the ZF-DFE, MAP/SDF[7/121], MAP/HDF[7/121], MAP/SDF[6/121] and MAP/HDF[6/121] for the considered HDSL test-loop # 4.

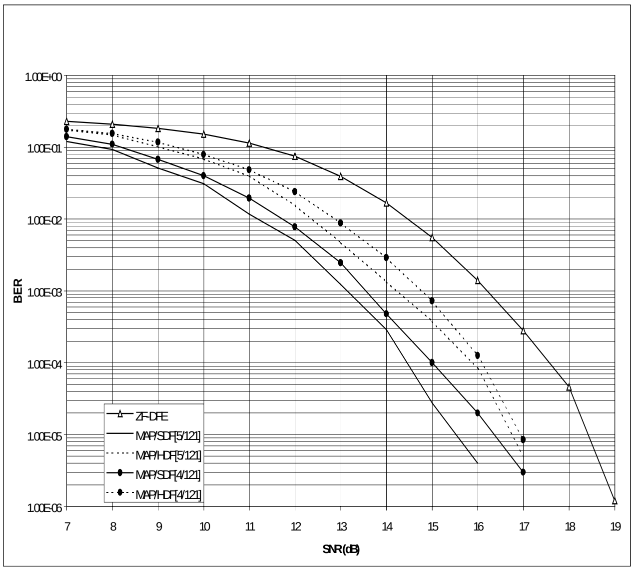

Fig. 2. Performance plots of the ZF-DFE, MAP/SDF[5/121], MAP/HDF[5/121], MAP/SDF[4/121] and MAP/HDF[4/121] for the considered HDSL test-loop # 4.