The Problem of Summing Crosstalk from Mixed Sources

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***Abstract*—Digital subscriber lines (DSL) are fundamentally limited by crosstalk. The case where all crosstalk is from the same type of DSL has been studied over the years and accurate models have been standardized. However, crosstalk from multiple different types of DSLs is a relatively new area of study and models of summing mixed crosstalk have only recently been postulated. The goal of this paper is twofold: firstly, to point out the lack of this subject in the current literature; secondly, to propose a new method of summing crosstalk due to mixed sources. This new method consists of deriving a lower bound to the worst-case method, which is intrinsically too pessimistic. Moreover, it is shown here for the first time that the Full Service Access Network (FSAN) method is a particular case of the proposed method.**

***Index Terms*—Crosstalk, Digital Subscriber Lines.**

I. Introduction

CROSSTALK can be the largest noise impairment in a twisted pair and can substantially reduce DSL performance. For this reason, there have been many efforts toward modeling this phenomenon. There are well-known accurate models for the case of a single type of crosstalk, where all crosstalkers have the same power spectral density (PSD). The models usually describe the 1% worst-case crosstalk power-sum, which is such that no more than 1% of all pairs in all cables will receive more crosstalk than the model.

A nontrivial problem arises when modeling complex access network scenarios where there may be different types of interferers. As Zimmerman pointed out in [1], it would be fundamentally incorrect to assume that crosstalk from different sources is simultaneously added using the 1% worst-case crosstalk coupling model. This would implicitly assume that each of a multitude of different services is simultaneously using the worst pairs in a binder, which is physically impossible. This problem of determining a good model for the summation of mixed interferers is becoming important now that many different DSL services are being deployed in the field.

Fundamentally, there are two well-known methods for modeling the crosstalk due to mixed sources: the Mean Power Spectral Density (MeanPSD) method, originally proposed by Zimmerman [1], and the FSAN method developed by the Full Service Access Network (FSAN) initiative ×DSL working group [3]. Preliminary work has shown that the FSAN method is superior to the MeanPSD method, and so the FSAN method has recently been adopted by ANSI accredited committee T1E1.4 for use in spectrum management calculations [2].

Although some summation methods have been proposed, at the moment there is no methodology or sound mathematical approach to the definition of a model. The basic starting point is the 1% worst-case model for the case of one type of disturber, which is then manipulated in some arbitrary fashion to find a model that is less pessimistic than the method that simply adds up the worst case powers. For example, it has been stated in [3] that there is "no simple physical justification" for the FSAN method. The lack of a sound mathematical approach should be considered as a major problem and further research on this topic is certainly needed.

In the present paper, a new approach is proposed for the modeling of crosstalk due to mixed interferers. This new approach consists of deriving a model by applying the Minkowski inequality bound to the worst-case model, which is intrinsically too pessimistic. The appealing feature of this new model is that it depends on a parameter that controls the tightness of the bound and, therefore, can be chosen accordingly to the desire of conveying more or less pessimism into the model. Another important result is that it is proven here for the first time that the FSAN method is a particular case of the method derived using the Minkowski inequality. This is an important result because it gives a mathematical validation of the FSAN method, which is commonly believed to be a model with no physical justification or mathematical soundness [3]. Finally, a computer simulation analysis was also performed in order to assess the accuracy of existing crosstalk summation methods and to compare them with the new approach.

II. Crosstalk Loss Models

Theoretical as well as experimental studies have ascertained that good models for the Power Spectral Densities (PSDs) of the NEXT due to only one disturber are given by the following expression:[1]

$$\text{Next}[f] = S(f) X_N f^{1.5} \tag{1}$$

where f is the frequency in hertz, $S(f)$ is the PSD of the disturbing signal, and X_N is a constant determined by measurements. The increase of NEXT as $f^{1.5}$ with frequency may be found on the basis of theoretical considerations when some assumptions such as perfect line termination and uniform line characteristics are taken into account.

[1]Due to space limitations, only the problem of NEXT modeling is here addressed. However, the proposed model of NEXT can be straightforwardly applied to FEXT as well.

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The commonly accepted method for computing the constant X_N is to consider the 1% worst case. This is done by considering the maximum value of the overall crosstalk power with a confidence of 99% or, equivalently, choosing an interference power that is likely to be exceeded in 1% or less of cases. In this case, the PSD model of NEXT for n interfering signals of the same kind becomes

$$\text{Next}[f, n] = S(f) X_N f^{1.5} n^{0.6} \tag{2}$$

where $X_N = 8.5 \cdot 10^{-15}$.

When several different kinds of disturbers are considered, a joint probability distribution has to be taken into account and there is no unique way of defining a 1% worst case of a joint probability. In this case a metric should be defined and, in the authors' opinion, a reasonable choice for the metric would be to use a metric that summarizes the effects of all the interferers on the actual system performance. A logical choice is to consider the Signal-to-Noise Ratio (SNR), which represents the natural index of the impact of the overall crosstalk power on the system performance. In this way, the search for the 1% worst case is done on the set of SNRs and not on the crosstalk power sums. This metric has been adopted for evaluating the "Monte Carlo NEXT summation method" (see Section IV) which, although time-consuming, should be considered as the most accurate way of determining the actual performance of a system.

A straightforward way to extend the model in (2) is to simply sum the 1% worst-case crosstalk power contributions of each kind of disturber. For example, in the case of K different systems we would obtain

$$\text{Next}_{NM}[f, n_1, \cdots, n_K] = \left[\sum_{i=1}^{K} S_i(f) n_i^{0.6}\right] X_N f^{1.5} \tag{3}$$

where n_i is the number of systems with PSD $S_i(f)$ $(i = 1, \cdots, K)$. Each term is a 1% worst case, so this "naive" method would predict a distinctly pessimistic level of interference because it assumes that all the interfering systems simultaneously use the worst disturbing pairs.

III. New Crosstalk Summation Method: The Generalized FSAN or Minkowski-Bound Method

The exploitation of the Minkowski inequality allows us to derive a model for the summation of mixed sources that is always less pessimistic than the naive model in (3) and moreover, is dependent on a parameter. By changing this parameter, the tightness of the bound can be varied so that the optimism or the pessimism of the model can be chosen *a priori*.

Let $0 < \lambda < 1$ and a_{jk} be a set of nonnegative numbers for $1 \le j \le J$ and $1 \le k \le K$. The Minkowski inequality is expressed as follows:

$$\left[\sum_{j=1}^{J}\left(\sum_{l=1}^{L} a_{jl}\right)^{1/\lambda}\right]^{\lambda} \le \sum_{l=1}^{L}\left(\sum_{j=1}^{J} a_{jl}^{1/\lambda}\right)^{\lambda}. \tag{4}$$

For the sake of simplicity, consider the special case of $J = L = 2$ and $a_{ij} = 0$, $\forall i \ne j$. After some simple algebra eq. (4) boils down to the following inequality:

$$a_{11} + a_{22} \ge \left(a_{11}^{1/\lambda} + a_{22}^{1/\lambda}\right)^{\lambda} \tag{5}$$

Now, posing $a_{11} = S_1(f) n_1^{0.6}$ and $a_{22} = S_2(f) n_2^{0.6}$, we can exploit the inequality in (5) to compute a lower bound to the pessimistic model in (3):

$$\begin{aligned}\text{Next}_{NM}&[f, n_1, n_2] \\ &= S_1(f) X_N f^{1.5} n_1^{0.6} + S_2(f) X_N f^{1.5} n_2^{0.6} \\ &= \left(S_1(f) n_1^{0.6} + S_2(f) n_2^{0.6}\right) X_N f^{1.5} \\ &\ge \left[\left(S_1(f) n_1^{0.6}\right)^{1/\lambda} + \left(S_2(f) n_2^{0.6}\right)^{1/\lambda}\right]^{\lambda} X_N f^{1.5}.\end{aligned} \tag{6}$$

In this way, by exploiting the Minkowski inequality, we can derive the following model for K different sources:

$$\begin{aligned}\text{Next}_{\text{Minkowski}}&[f, \lambda, n_1, \cdots, n_K] \\ &= \left[\sum_{i=1}^{K}\left[S_i(f) n_i^{0.6}\right]^{1/\lambda}\right]^{\lambda} X_N f^{1.5}, \quad \text{for any } 0 < \lambda < 1\end{aligned} \tag{7}$$

The "amount" of pessimism that can be conveyed in the model depends on the value of the parameter λ, which can be chosen accordingly to the desire of a more or less conservative model. As λ tends to 1, the model in (7) tends to be more pessimistic [for $\lambda = 1$ the model in (7) exactly coincides with the naive model in (3)] while, as λ tends to 0, the model tends to be more optimistic. Now, by choosing the particular value $\lambda = 0.6$, we obtain:

$$\begin{aligned}\text{Next}_{\text{Minkowski}}&[f, \lambda = 0.6, n_1, \cdots, n_K] \\ &= \left[\sum_{i=1}^{K}\left[S_i(f) n_i^{0.6}\right]^{1/0.6}\right]^{0.6} X_N f^{1.5}\end{aligned} \tag{8}$$

This is exactly the FSAN method [3], so that the FSAN can be considered as a particular case of the Minkowski-bound method. It is worth pointing out that $\lambda = 0.6$ is the only value of λ that gives the right asymptotical value when there are several identical disturbers, which is a very desirable property. However, lower values of λ have the advantage of producing less pessimistic results and, moreover, lower values of λ also ensure a better average accuracy despite of the above mentioned problem (see next section).

IV. Simulation Results

Computer simulations were run to ascertain the accuracy of the Minkowski crosstalk combination method with different values of λ. Each simulation generates many samples of same-binder pair-to-pair NEXT couplings, which in dB are generated by a log-normal model:[2]

$$\Pr(\text{pair-to-pair NEXT coupling in dB} \le x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}\, dx. \quad (9)$$

with $\sigma = 9.0$, $\mu = 164.0 - 15\log_{10}(f)$, and f is the frequency in Hz. Two methods were run: the "Minkowski" NEXT summation method for four values of λ and the "Monte-Carlo" NEXT summation method. The NEXT PSD is calculated according to the simulated summation method and, then, the SNR at the decision point is calculated. The SNRs of HDSL and HDSL2 are calculated with ideal DFE equations with standard PSD shapes [2].

The "Minkowski" NEXT Summation Method: For each disturber type with number of disturbers n_i, and $n_i > 0$, n_i pair-to-pair NEXT loss values are randomly and independently generated and power-summed. This is done repeatedly until there are 2000 samples of power-sums of each disturber type. From each set of 2000 power-sums, the 1% worst-case power-sum is calculated and, then, multiplied by the PSD of the NEXT of that disturber type, and so for disturber number i, this product equals $S_i(f)n_i^{0.6}$. These are summed by the Minkowski-bound method and normalized as $X_N f^{3/2} (\sum(S_i(f)n_i^{0.6})^{1/\lambda})^{\lambda}$ which is the crosstalk sum used to compute the NEXT PSD. After having computed the NEXT PSD, the SNR at the decision point is calculated as previously mentioned.

The "Monte Carlo" NEXT Summation Method: For each disturber type with number of disturbers n_i, and $n_i > 0$, n_i pair-to-pair NEXT loss values are randomly and independently generated and power-summed. These are each multiplied by a normalized crosstalk transfer function and the disturber's PSD, and are then power-summed to equal the crosstalk noise used to compute a single SNR value. This is done repeatedly until there are 2000 SNR values. The set of SNRs are ordered into ascending order, and the 1% worst-case SNR (point number 20) is then equal to the Monte-Carlo SNR, which is close to the actual 1% worst SNR.

The Minkowski-bound method was simulated for the case of three mixed sources ($K = 3$) and for several values of λ, and the results are shown in Tables I and II. Negative numbers in the tables indicate that the NEXT summation method is pessimistic, and positive numbers indicate that the NEXT summation method is optimistic. For the HDSL simulations in Table I, the Minkowski method improved the average accuracy over the FSAN one by about 0.28 and 0.38 dB for $\lambda = 0.3$ and $\lambda = 0.1$, respectively. For the HDSL2 simulations in Table II, the Minkowski method with $\lambda = 0.3$ and $\lambda = 0.1$ improved the average accuracy by 0.13 and 0.17 dB compared to the standard $\lambda = 0.6$ FSAN method.

[2]The power-sum of log-normal components is not exactly log- normally distributed, although it is close. A number of intermediate simulations were run to numerically find good values of the parameters μ and σ. The chosen values of μ and σ gave the best overall compromise fit to both the mean NEXT loss and the 1% worst-case NEXT loss.

TABLE I
STATISTICS OF THE DIFFERENCE BETWEEN THE SNR COMPUTED WITH THE MINKOWSKI NEXT SUMMATION METHOD AND THE SNR COMPUTED BY THE MONTE-CARLO METHOD, dB. UPSTREAM HDSL WITH MIXED ISDN, HDSL, AND DOWNSTREAM ADSL NEXT ($K = 3$)

SNR difference (dB)	λ = 1.0 (naive sum)	λ = 0.6 (FSAN)	λ = 0.3	λ = 0.1
Average difference (dB)	-1.66	-1.12	-0.84	-0.74
Avg. difference in absolute (dB)	1.70	1.16	0.89	0.81
Maximum difference (dB)	0.29	0.29	0.29	0.29
Minimum difference (dB)	-3.36	-2.68	-2.61	-2.59

TABLE II
STATISTICS OF THE DIFFERENCE BETWEEN THE SNR COMPUTED WITH THE MINKOWSKI NEXT SUMMATION METHOD AND THE SNR COMPUTED BY THE MONTE-CARLO METHOD, dB. DOWNSTREAM HDSL2 WITH MIXED ISDN, T1, AND UPSTREAM HDSL2 NEXT ($K = 3$)

SNR difference (dB)	λ = 1 (naive sum)	λ = 0.6 (FSAN)	λ = 0.3	λ = 0.1
Average difference (dB)	-3.00	-2.67	-2.54	-2.50
Avg. difference in absolute (dB)	3.01	2.69	2.56	2.52
Maximum difference (dB)	0.26	0.29	0.56	0.66
Minimum difference (dB)	-5.81	-5.47	-5.33	-5.30

V. CONCLUSIONS

In this short paper a new method for summing crosstalk has been proposed. The proposed method is based on the derivation of a lower bound to the worst-case method that is intrinsically too pessimistic and, moreover, is dependent on a parameter λ that determines the tightness of the bound. In this way, the optimism or the pessimism of the model can be chosen a priori by changing this parameter. The problem of finding an optimal value for λ is currently investigated by the authors. However, this is indeed a nontrivial problem since it appears that there is no cost function with respect to which one can pursue optimization. It was also shown for the first time that the FSAN method is a particular case of the proposed method and not, as commonly believed, a method with no physical justification. Finally, simulation results have shown that a moderate improvement in average accuracy is obtainable passing from $\lambda = 0.6$ to lower values of λ. Other simulation results [4] tend to indicate that more sensible improvements are obtained for the case of $K > 3$, which is a very likely scenario in the near future.

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